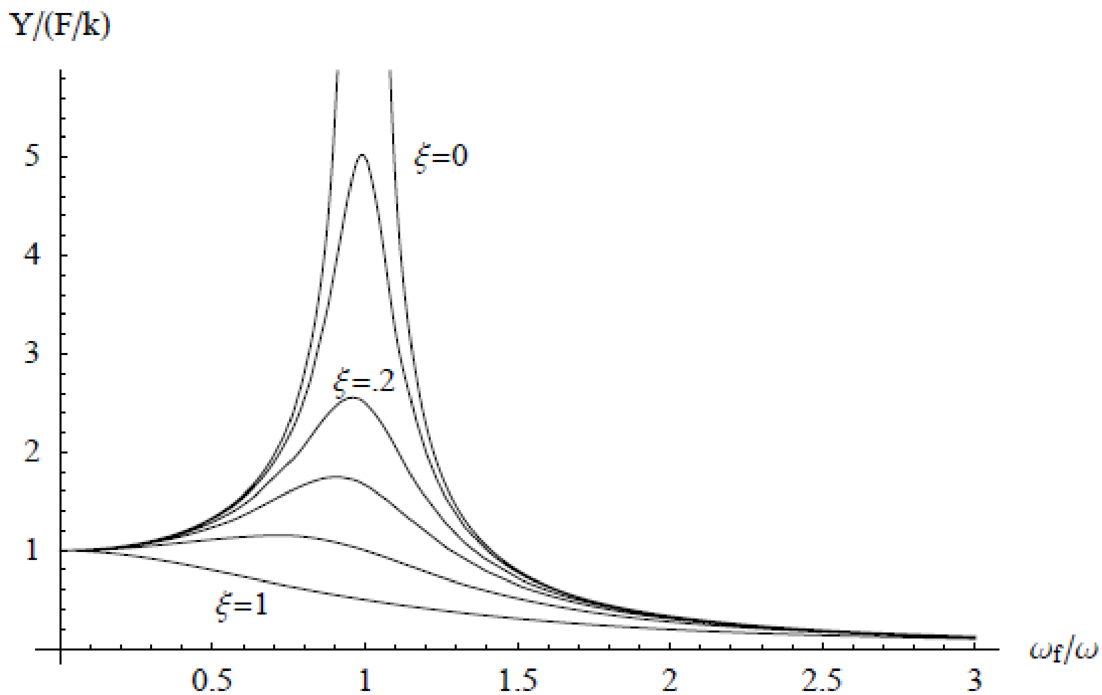
Lecture 9: Harmonic Loads (Con't)

Reading materials: Sections 3.4, 3.5, 3.6 and 3.7

1. Resonance

- The dynamic load magnification factor (DLF)

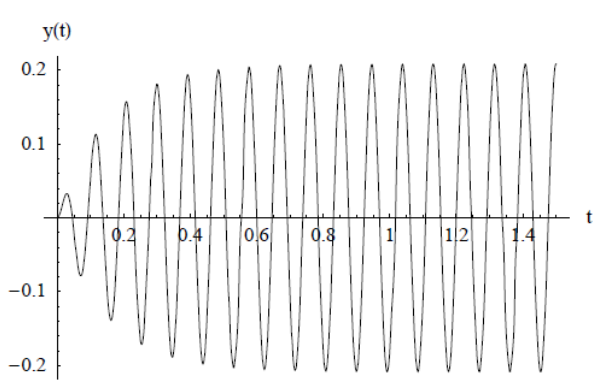
$$\text{DLF} = \frac{Y}{F/k} = \frac{1}{\gamma} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

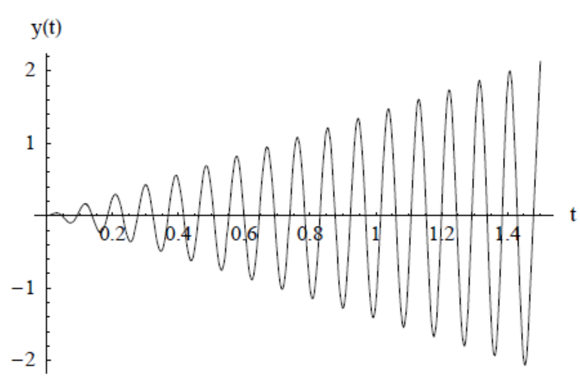


- The peak dynamic magnification occurs near r=1 for small damping ratios.

- Damping is usually small in practical systems. Therefore, the resonance condition is usually defined as when the load frequency is same as the undamped natural frequency.

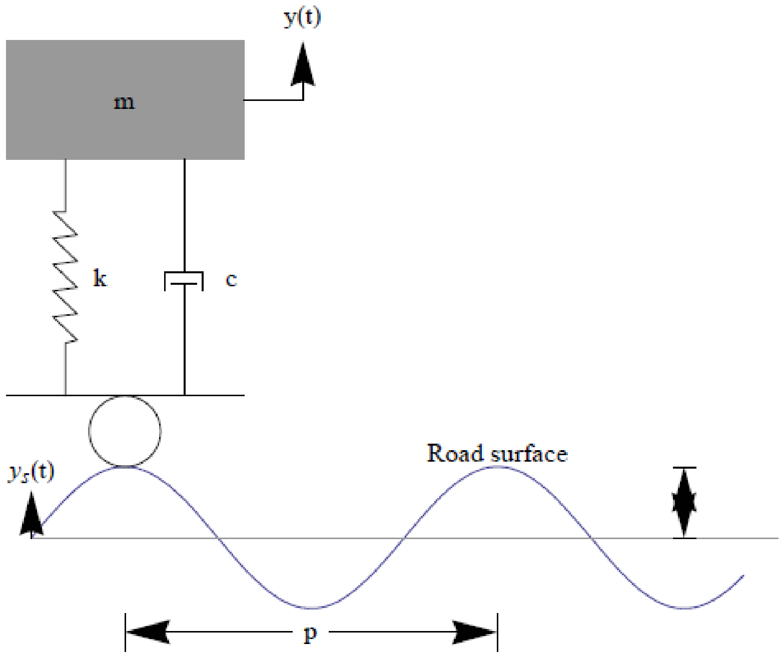
Example





2. Vehicle travelling over a rough terrain (Optional)

- The automobile is simply modeled as a single degree of freedom model.



- The road profile is idealized by a sine harmonic

$$y_s(t) = a \sin \omega_f t$$

$$\omega_f = V \frac{\text{km}}{\text{hour}} \times \frac{1}{p\,\text{km}} \times \frac{\text{hour}}{3600\,s} \times \frac{2\pi\,\text{rad}}{\text{cycle}}$$

- Equations of motion

$$m\ddot{y} + c(\dot{y} - \dot{y}_s) + k(y - y_s) = 0$$

$$m\ddot{y} + c\dot{y} + k y = c\dot{y}_s + k y_s$$

$$m\ddot{y} + c\dot{y} + k y = c\,\omega_f\, a \cos \omega_f t + k\, a \sin \omega_f t$$

- Solution is a superposition of sine and cosine harmonic loading

$$y_{\cos}(t) = \frac{2\,a\,r\,\xi\left(2\,r\,\xi\sin(r\,t\,\omega) - \left(r^2-1\right)\cos(r\,t\,\omega)\right)}{r^4 + \left(4\,\xi^2-2\right)r^2+1}$$

$$y_{\sin}(t) = -\frac{a\left(2\,r\,\xi\cos(r\,t\,\omega) + \left(r^2-1\right)\sin(r\,t\,\omega)\right)}{r^4 + \left(4\,\xi^2-2\right)r^2+1}$$

$$y(t) = -\frac{a\left(2\,\xi\cos(r\,t\,\omega)\,r^3 + \left(r^2\left(1-4\,\xi^2\right)-1\right)\sin(r\,t\,\omega)\right)}{r^4 + \left(4\,\xi^2-2\right)r^2+1}$$

- Maximum displacement

$$A = -\frac{2\,a\,r^3\,\xi}{r^4 + \left(4\,\xi^2-2\right)r^2+1}; \qquad B = -\frac{a\left(r^2\left(1-4\,\xi^2\right)-1\right)}{r^4 + \left(4\,\xi^2-2\right)r^2+1}$$

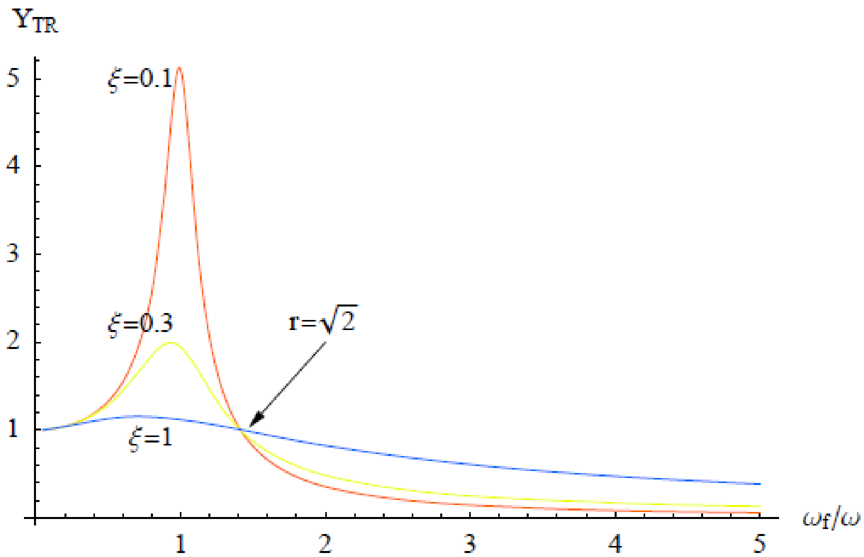
$$Y = \sqrt{A^2+B^2} = \sqrt{\frac{a^2\left(4\,r^2\,\xi^2+1\right)}{r^4 + \left(4\,\xi^2-2\right)r^2+1}}$$

- Displacement transmissibility: the ratio of maximum displacement with the input displacement a

$$\frac{Y}{a} = \sqrt{\frac{4\,r^2\,\xi^2+1}{r^4+\left(4\,\xi^2-2\right)r^2+1}}$$

- When the frequency ratio is greater than sqrt(2), the displacement transmitted is less than the static displacement. It means that the damping has a negative effect. In other words, lightly damped systems have lower displacement transmissibility than those with larger damping.

When the frequency ratio is less than sqrt(2), the displacement transmitted is larger than the static displacement. Larger damping values significantly reduce the displacement transmissibility.



Force transmitted to the automobile

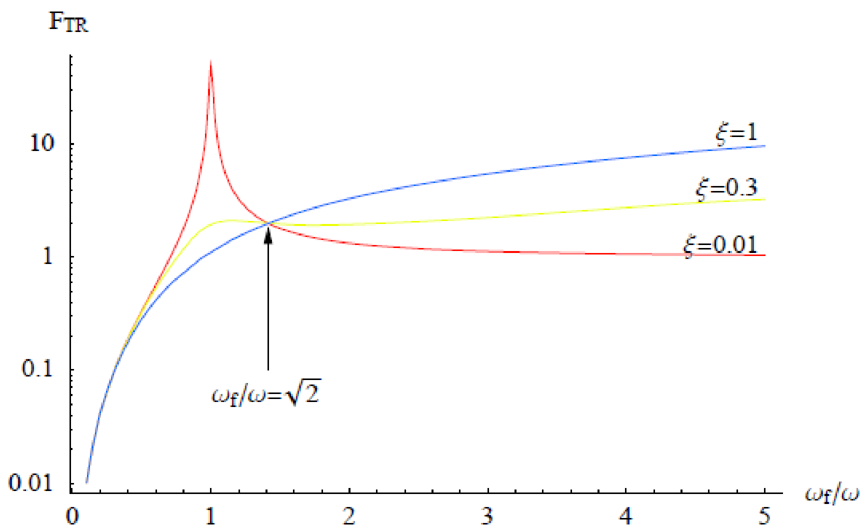
$$F(t) = m\,\ddot{y}(t) = \frac{a\,k\,r^2\left(2\,\xi\cos(r\,t\,\omega)\,r^3 + \left(r^2\left(1-4\,\xi^2\right)-1\right)\sin(r\,t\,\omega)\right)}{r^4+\left(4\,\xi^2-2\right)r^2+1}$$

Maximum force and force transmissibility

$$A = \frac{2\,a\,k\,r^5\,\xi}{r^4+\left(4\,\xi^2-2\right)r^2+1}; \qquad B = \frac{a\,k\,r^2\left(r^2\left(1-4\,\xi^2\right)-1\right)}{r^4+\left(4\,\xi^2-2\right)r^2+1}$$

$$F_T = \sqrt{A^2+B^2} = \sqrt{\frac{a^2\,k^2\,r^4\left(4\,r^2\,\xi^2+1\right)}{r^4+\left(4\,\xi^2-2\right)r^2+1}}$$

$$\frac{F_T}{k\,a} = r^2\sqrt{\frac{4\,r^2\,\xi^2+1}{r^4+\left(4\,\xi^2-2\right)r^2+1}}$$



When the frequency ratio is greater than sqrt(2), the force transmitted is still larger than the static force. It is unlike the displacement transmissibility.

When the frequency ratio is less than sqrt(2), the increase in magnitude is not as severe as for the ratio less than sqrt(2). In this range, the damping has a negative effect. Lightly damped systems have lower force transmissibility than those with larger damping.

Example

3. Estimating damping from a harmonic load test (Optional)

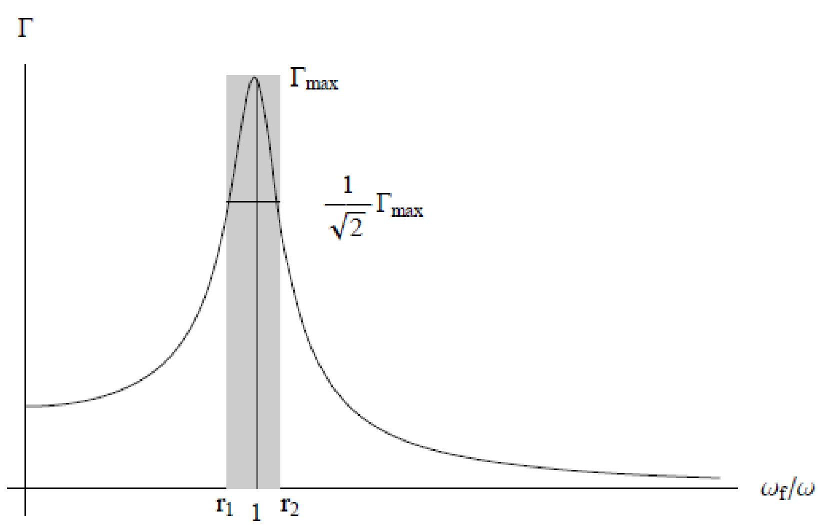
Dynamic load magnification factor (DLF)

$$\mathrm{DLF} \equiv \Gamma = \frac{Y}{F/k} = \frac{1}{\sqrt{(2\xi r)^2 + (1-r^2)^2}}$$

At resonance

$$\Gamma_r = \frac{1}{2\xi}$$

Half-power points



$$\frac{1}{\sqrt{2}}\Gamma_{max} = \frac{1}{\sqrt{2}}\frac{1}{2\xi} = \frac{1}{\sqrt{(2\xi r)^2 + (1-r^2)^2}}$$

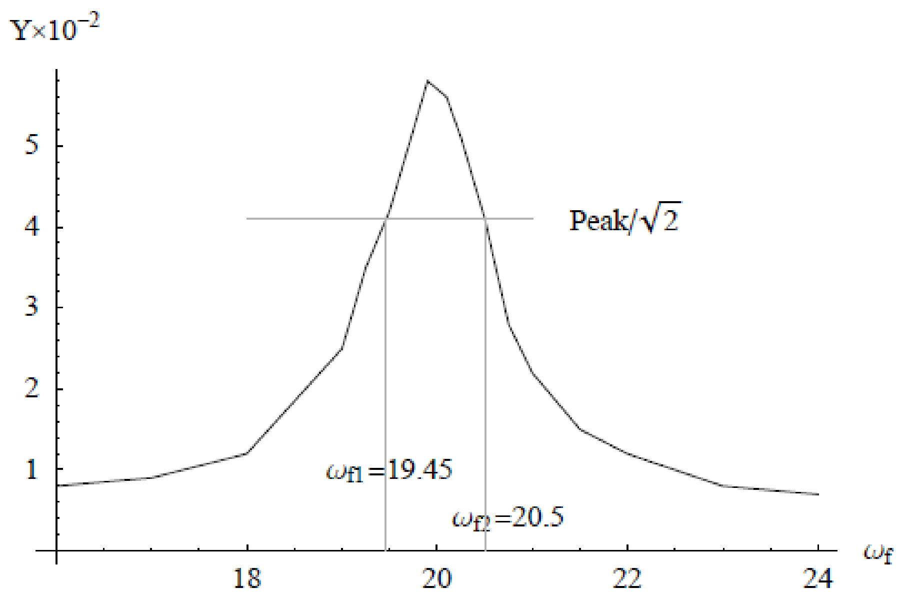
$$r_{1,2} = \sqrt{1 - 2\xi^2 \pm 2\xi\sqrt{1+\xi^2}}$$

$$r_2 + r_1 = \sqrt{1 - 2\xi^2 + 2\xi\sqrt{1+\xi^2}} + \sqrt{1 - 2\xi^2 - 2\xi\sqrt{1+\xi^2}} = 2 - 3\xi^2 + O[\xi]^3$$

$$r_2 - r_1 = \sqrt{1 - 2\xi^2 + 2\xi\sqrt{1+\xi^2}} - \sqrt{1 - 2\xi^2 - 2\xi\sqrt{1+\xi^2}} = 2\xi + O[\xi]^3$$

$$\frac{r_2 - r_1}{r_2 + r_1} = \xi \qquad\qquad \frac{\omega_{f2} - \omega_{f1}}{\omega_{f2} + \omega_{f1}} = \xi$$

Example

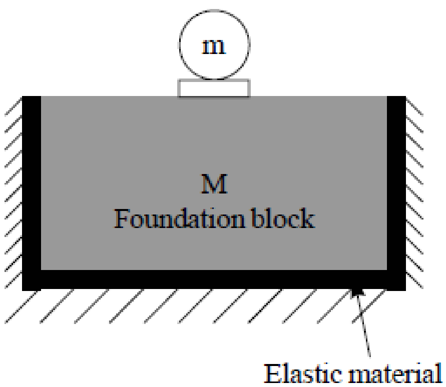


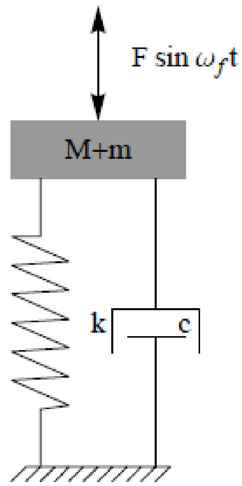
$$\xi = \frac{\omega_{f2} - \omega_{f1}}{\omega_{f2} + \omega_{f1}} = 0.0262829 = 2.63\%$$

4. Vibration isolation (Optional)

- Machines generating substantial dynamic forces are isolated by mounting them on massive blocks that rest on some other elastic materials, such as rubber, springs, cork, and soil.

- The system can be modeled as a single degree of freedom system.





- Spring constant

$$k = A\, k_s$$

- Equations of motion

$$(M+m)\,\ddot{y}(t) + c\,\dot{y}(t) + k\,y(t) = F \sin \omega_f\, t$$

- Dynamic force

$$f(t) = k\,y(t) + c\,\dot{y}(t) = m\,\omega^2\,y(t) + 2\,m\,\omega\,\xi\,\dot{y}(t)$$

$$\omega = \sqrt{\frac{k}{M+m}}$$

Steady state displacement

$$y_p(t) = \frac{F/k}{(2\xi\omega_f/\omega)^2+(1-\omega_f^2/\omega^2)^2}\left[\left(1-\omega_f^2/\omega^2\right)\sin\omega_f t-\left(2\xi\,\omega_f/\omega\right)\cos\omega_f t\right]$$

$$f(t) = \frac{F}{(2\xi\omega_f/\omega)^2+(1-\omega_f^2/\omega^2)^2}\left[\left(1-\frac{\omega_f^2}{\omega^2}+\frac{4\xi^2\omega_f^2}{\omega^2}\right)\sin\omega_f t-\frac{2\xi\,\omega_f^3}{\omega^3}\cos\omega_f t\right]$$

- maximum force

$$F_T = \frac{F}{(2\xi\omega_f/\omega)^2+(1-\omega_f^2/\omega^2)^2}\sqrt{\left(1-\frac{\omega_f^2}{\omega^2}+\frac{4\xi^2\omega_f^2}{\omega^2}\right)^2+\left(\frac{2\xi\,\omega_f^3}{\omega^3}\right)^2}$$

$$F_T = \frac{F\sqrt{1+(2\xi\omega_f/\omega)^2}}{\sqrt{(2\xi\omega_f/\omega)^2+(1-\omega_f^2/\omega^2)^2}}$$

- Force transmissibility

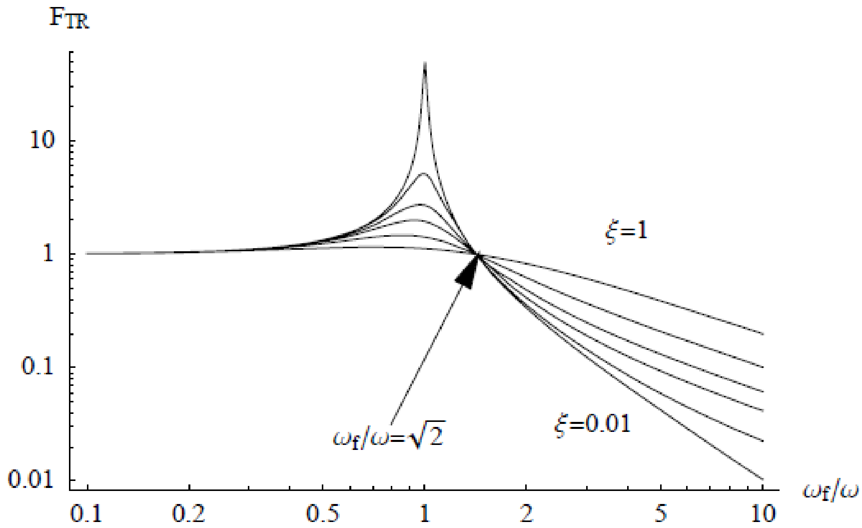
$$F_{\mathrm{TR}} = \frac{F_T}{F} = \sqrt{\frac{1+(2\xi\omega_f/\omega)^2}{(2\xi\omega_f/\omega)^2+(1-\omega_f^2/\omega^2)^2}}$$

- When the frequency ratio is greater than sqrt(2), the force transmitted is less than the static force. It is desirable to design foundation so that the frequency ratio at is beyond sqrt(2).

- Larger M is better since M increases ω decreases and the frequency ratio increases.

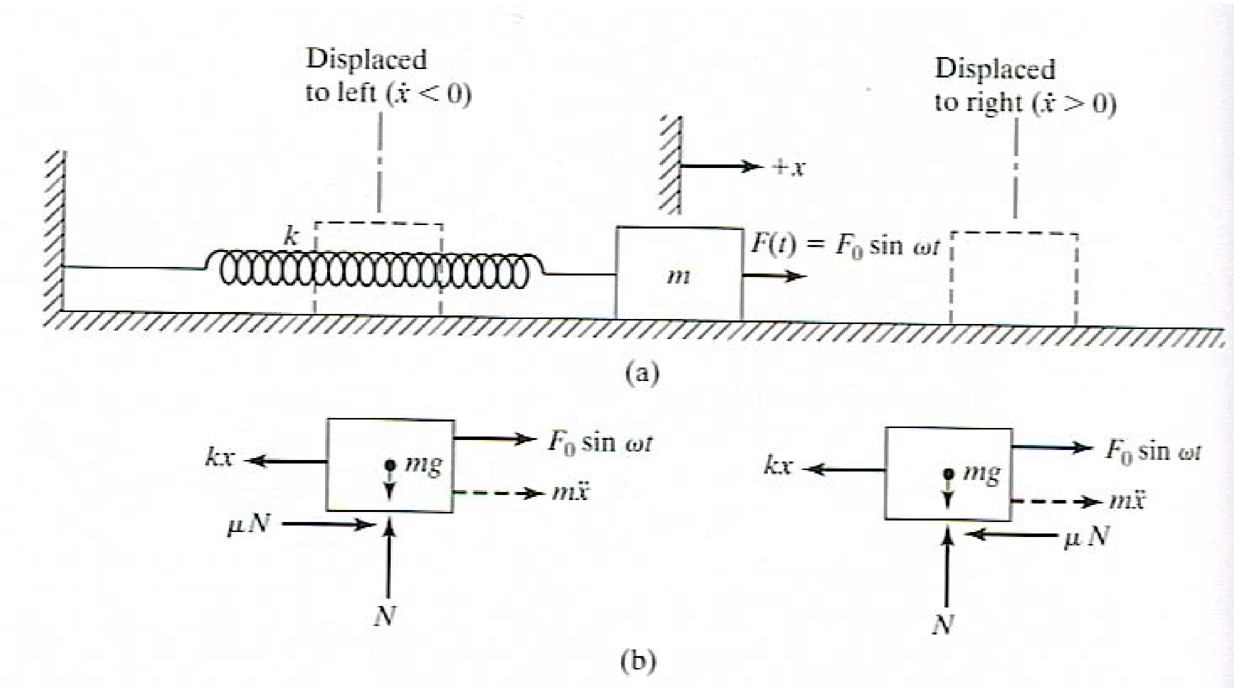
- The damping has a negative effect.

- Lightly damped systems have lower force transmissibility than those with larger damping.



Example: To design a foundation for a machine that weighs 300 kN. The concrete block supporting the machine is 2.5 x 12 m in plan. Determine its thickness so that only 30% of the static force is transmitted to the supporting structure of this machine. The density of concrete is 1300 kg/m^3. The block is placed on a bed of dry sand and gravel with a coefficient of compression of 0.3 MPa/m. The damping ratio is 3%. The machine operates at 240 rpm.

5. Forced vibration with Coulomb Damping (Optional)



6. Stability analysis (Optional)

A system is dynamically stable if the motion (or displacement) converges or remains steady with time.

If the amplitude of displacement increases continuously (diverges) with time, it is said to be dynamically unstable.

7. Self-excitation (Optional)

- The force acting on a vibrating system is usually external to the system and independent of the motion.

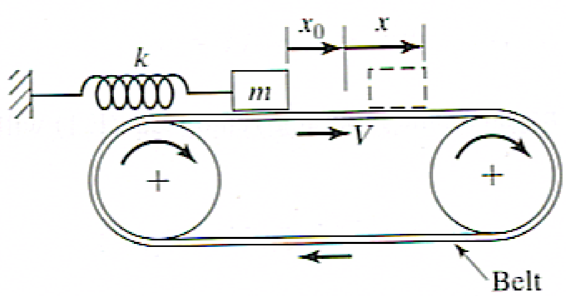
- There are systems for which the exciting force is a function of the motion parameters of the system, such as displacement, velocity, or acceleration.

- Such systems are called self-excited vibrating systems since the motion itself produces the exciting force.

- The motion diverges and the system becomes unstable if energy is fed into the system through self-excitation.

- The instability of rotating shafts, the flutter of turbine blades, the flow-induced vibration of pipes, and the automobile wheel shimmy and aerodynamically induced motion of bridges are typical examples of self-excited vibration.

- Example: instability of spring-supported mass on moving belt



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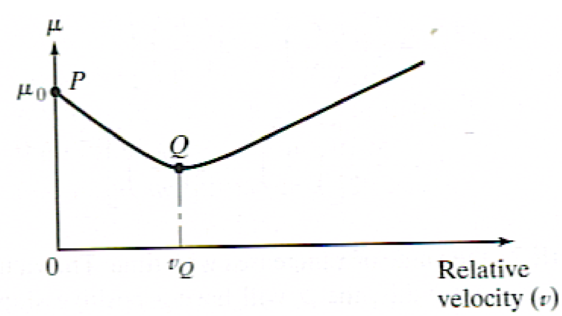

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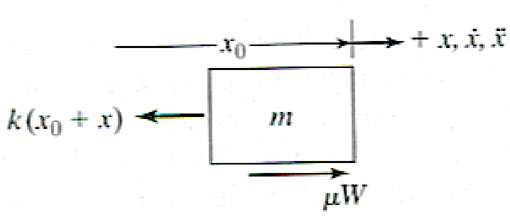

(c)