

Lecture 3: Discrete Models

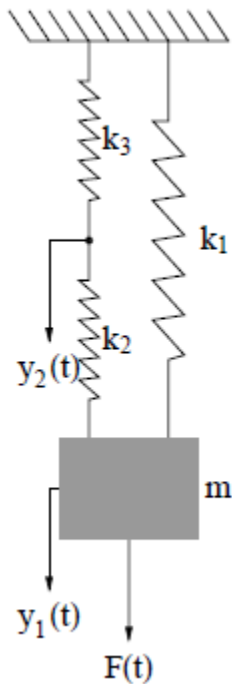
Reading materials: Sections 1.9, 1.10

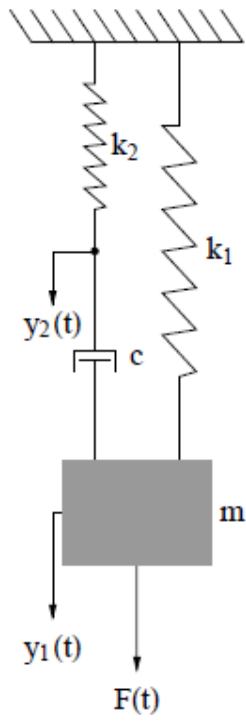
1. Introduction

Equations of motion for discrete spring-mass-dashpot systems can be written by considering equilibrium of forces acting on different masses in the system.

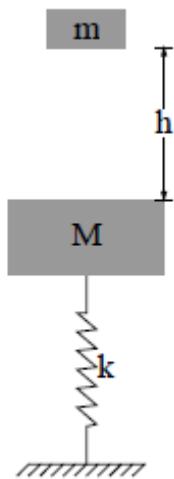
Equations of motion can also be derived via energy methods.

2. Spring in series and in parallel

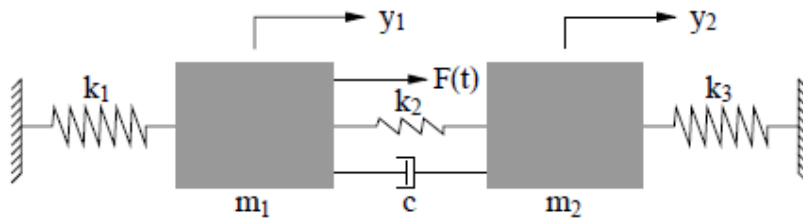




3. Mass dropping on a Spring-mass system



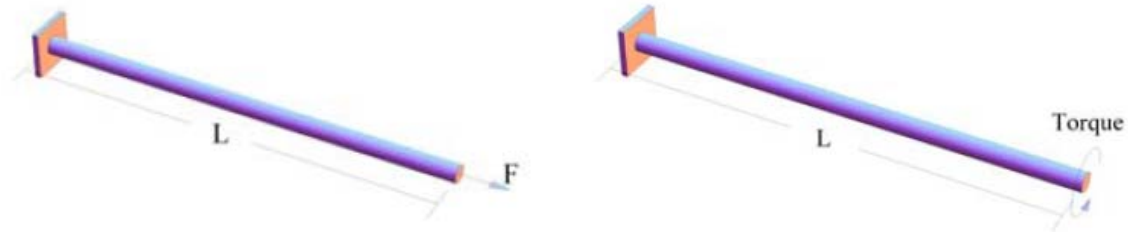
4. A two degree of freedom system



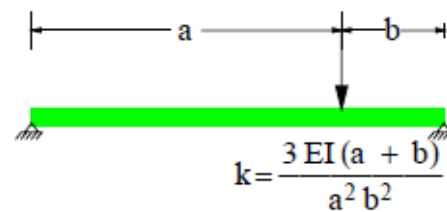
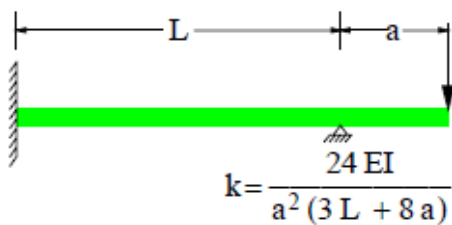
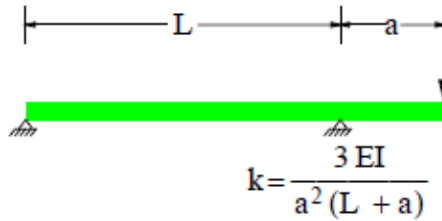
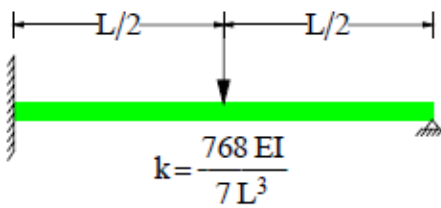
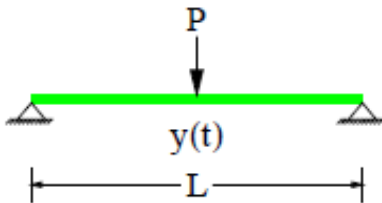
5. Equivalent Spring-Mass systems for framed structures

- Common framed structures are usually idealized as equivalent single or multi-degree of freedom spring-mass systems.
- The mass of the equivalent spring is equal to the total mass of the beam.
- The stiffness of the equivalent spring is determined from the static deflection produced by a concentrated load placed at the location where the mass is lumped.

6. Shafts subjected to axial forces and torques

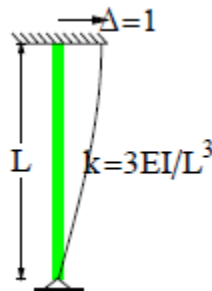
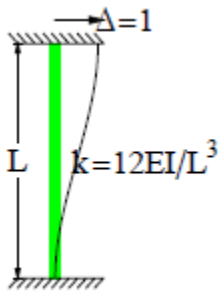


7. Beams



8. Frames (Optional)

- Simplified model of building frame considers lateral dynamic forces.
- The simplest lumped-mass model of a multi-story frame subjected to lateral dynamic loads is based on the assumption that the floor systems, and therefore girders (horizontal beams), are rigid. Thus the number of degrees of freedom for dynamic analysis is equal to the number of stories in the frame.
- The mass at each story level includes: dead loads based on the tributary area of the frames, weight of the girders, and half the weight of the columns and walls from the story above and below.
- The story stiffness is equal to the sum of stiffnesses of all columns in a given story.
- Rigid frame



Example 1

Define an equivalent two degrees of freedom spring-mass model for the two story frame. The dead load from the floors and the walls in lb/ft² (psf) is shown in the figure. The frame spacing is 20 ft and the columns have a moment of inertia = 133.2 in⁴ and $E = 30 \times 10^6$ lb/in²

