Question #1: (33.33 points)

A soil deposit with a groundwater table located 2m beneath the ground surface is as shown in Fig. 1a. It is anticipated that due to excavations on an adjacent parcel of land the water-table will drop by 4m over a time span of one-month, and then remain at the lower level permanently. The owner of the parcel of land shown in Fig. 1a is concerned that the drop in water table could cause settlements on her land. As her consultant, using the information provided, calculate for her:

a. the ultimate settlements that might be expected due to the dropping of the water table;

b. how much settlement would be expected at:
   - t=1 year?
   - t=5 years?
   - t=20 years?

Note the non-dimensional time constants for varying degrees of average consolidation are given on page 4 of this exam.
Question #2: (33.33 points)

Figure 2 shows a homogeneous, sandy soil deposit with a horizontal ground surface. Before the strip load is applied, the stresses at point A are as follows: vertical stress $\sigma_v=90$ kPa; horizontal stress $\sigma_h=45$ kPa.

a. Compute the maximum shear stress at point A before the surface pressure is applied.

b. Using the information provided in Figure 1, compute the major and minor principal stresses at point A after the uniform strip load is applied.

c. What are the respective orientations of the principal planes at point A after the surface pressure is applied?

Stress increases at a general point P making angles $\alpha$ and $\beta$ with respect to the strip load are given by the following formulae:

\[
\Delta \sigma_{zz} = \frac{q}{\pi} \left[ \alpha + \sin(\alpha)\cos(\alpha + 2\beta) \right] \\
\Delta \sigma_{xx} = \frac{q}{\pi} \left[ \alpha - \sin(\alpha)\cos(\alpha + 2\beta) \right] \\
\Delta \tau_{xz} = \frac{q}{\pi} \left[ \sin(\alpha)\sin(\alpha + 2\beta) \right]
\]

Fig. 1.
**Question #3: (33.33 points)**

A sheetpile retaining wall is shown in Figure 3a, and the state of total stresses in the silty-sandy soil at point A are as shown. As part of a construction operation, a bracing force is pushing on the sheetpile wall as shown in Figure 3b, and this force leads to an increase in lateral stress in soil behind the retaining wall, while the vertical stress in the soil remains essentially constant.

a. For the conditions shown in Figure 3b, how large would the lateral stress need to become at point A to cause shear failure?

b. At shear failure at point A, what would be the orientation of the plane(s) on which shear failure occurs? (Use the pole method.)

c. What are the effective shear and normal stresses on the failure plane passing through point A?

---

Figure 3.
Bonus Question (10 points!!)

Explain in detail how one can go about estimating the permeability of a fine-grained soil from a 1-dimensional consolidation test.

Tabulated values of degree of consolidation U(%) versus non-dimensional time factor T_v in the one-dimensional consolidation model.

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Solution of Makeup 53:030 Soil Mechanics Midterm Exam #2, Fall Semester, 2005.

Question #1: (33.33 points)

a) 24 points

For the sand layer:

\[
\gamma_d = \frac{G_s \gamma_w}{1 + e} = \frac{2.65 \times 9.81 \text{kN} \cdot \text{m}^{-3}}{1 + 0.70} = 15.29 \text{kN} \cdot \text{m}^{-3}
\]

\[
\gamma_{sat} = \frac{\gamma_w (G_s + e)}{1 + e} = \frac{9.81 \text{kN} \cdot \text{m}^{-3} \times (2.65 + 0.70)}{1 + 0.70} = 19.33 \text{kN} \cdot \text{m}^{-3}
\]

For the clay layer:

\[
\gamma_{sat} = \frac{\gamma_w (G_s + e)}{1 + e} = \frac{9.81 \text{kN} \cdot \text{m}^{-3} \times (2.74 + 0.88)}{1 + 0.88} = 18.89 \text{kN} \cdot \text{m}^{-3}
\]

The current effective stress level at the center of the clay layer is:

\[
\sigma_v = 2m \times \gamma_d + 8m \times \gamma_{sat} + 5m \times \gamma_b
\]

\[
= 2m \times 15.29 \text{kN} \cdot \text{m}^{-3} + 8m \times (19.33 - 9.81) \text{kN} \cdot \text{m}^{-3} + 5m \times (18.89 - 9.81) \text{kN} \cdot \text{m}^{-3}
\]

\[
= 152.15 \text{kPa}
\]

The effective stress level at the center of the clay layer after the water level drops by 4m and the clay layer consolidates under the associated increase in stress will be:

\[
\sigma_v = 6m \times \gamma_d + 4m \times \gamma_{sat} + 5m \times \gamma_b
\]

\[
= 6m \times 15.29 \text{kN} \cdot \text{m}^{-3} + 4m \times (19.33 - 9.81) \text{kN} \cdot \text{m}^{-3} + 5m \times (18.89 - 9.81) \text{kN} \cdot \text{m}^{-3}
\]

\[
= 175.33 \text{kPa}
\]

The resulting consolidation settlements due to compression of the clay layer will be:

\[
S_c = \frac{H_o - \Delta e}{1 + e_o} = \frac{H_o - C_e \log \left( \frac{\sigma_v}{\sigma_o} \right)}{1 + e_o}
\]

\[
= \frac{10m}{1 + 0.88} \times 0.35 \times \log \left( \frac{175.33}{152.15} \right)
\]

\[
= 0.115 \text{m}
\]

b) 9 points

At t=1yr:

\[
T_y = \frac{t \times c_v}{(H_{dr})^2} = \frac{1 \text{yr} \times 2 \text{m}^2 \text{yr}^{-1}}{(5 \text{m})^2} = 0.08 \Rightarrow U = 0.32 \Rightarrow S_c = 0.32 \times 0.115 \text{m} = 0.037 \text{m}
\]
At $t=5$ yr:
\[
T_v = \frac{5 \text{yr} \times 2 \text{m}^2 \text{yr}^{-1}}{(5 \text{m})^2} = 0.40 \Rightarrow U = 0.70 \Rightarrow S_e = 0.70 \times 0.115m = .080m
\]

At $t=20$ yr:
\[
T_v = \frac{20 \text{yr} \times 2 \text{m}^2 \text{yr}^{-1}}{(5 \text{m})^2} = 1.60 \Rightarrow U = 0.985 \Rightarrow S_e = 0.985 \times 0.115m = .0112m
\]

Question #2: (33.33 points)

a) 11 points

\[
\sigma'_1 = \sigma'_v = 90kPa \\
\sigma'_3 = \sigma'_h = 45kPa
\]

The resulting effective stress Mohr’s Circle has a center at
\[
\sigma_e = \frac{1}{2}(90 + 45) = 67.5kPa, \text{ and a radius of } r = \frac{1}{2}(\sigma'_1 - \sigma'_3) = 22.5kPa
\]
\[
\tau_{max} = r = 22.5kPa.
\]

b) 11 points

At the point of interest A: \( \alpha = \tan^{-1}\left(\frac{10m}{5m}\right) = 1.107\text{rad}; \beta = \tan^{-1}\left(\frac{0m}{5m}\right) = 0\text{rad} \)

Thus:
\[
\Delta\sigma_{zz} = \Delta\sigma_v = \frac{100kPa}{\pi}\left[1.107 + \sin(1.107)\cos(1.107)\right] = 47.97kPa
\]
\[
\Delta\sigma_{xx} = \Delta\sigma_h = \frac{100kPa}{\pi}\left[1.107 - \sin(1.107)\cos(1.107)\right] = 22.5kPa
\]
\[
\Delta\tau_{xz} = \frac{100kPa}{\pi}\left[\sin(1.107)\cos(1.107)\right] = 25.46kPa
\]

Adding these stresses to the originals gives:
\[
\sigma_{zz} = \sigma_v + 47.97kPa = 137.97kPa \\
\sigma_{xx} = \sigma_h + 22.5kPa = 67.51kPa \\
\tau_{xz} = 25.46kPa \\
\sigma_e = \frac{1}{2}\left(\sigma_{xx} + \sigma_{zz}\right) = 102.74kPa \\
r = \left[\left(\frac{1}{2}\left(\sigma_{xx} - \sigma_{zz}\right)^2 + \tau_{xz}^2\right)\right]^{1/2} = 43.47kPa \\
\sigma_1 = \sigma_e + r = 146.21kPa \\
\sigma_3 = \sigma_e - r = 59.27kPa
c) 11 points

\[ \sigma_v = 137.97 \text{kPa} \]
\[ \tau = 25.5 \text{kPa} \]
\[ \sigma_h = 67.5 \text{kPa} \]

\[ 2\theta = \tan^{-1} \left( \frac{\sigma_v - \sigma_c}{\tau} \right) \]
\[ = \tan^{-1} \left( \frac{137.97 - 102.74}{25.46} \right) = 35.85^\circ \]
\[ \theta = 17.93^\circ \]

Accordingly, the major principal plane passing through point A makes an angle of 17.93 degrees counter-clockwise with respect to the horizontal.

The minor principal plane makes an angle of 17.93 degrees counter-clockwise with respect to the vertical.

Question #3: (33.33 points)

a) 11 points:

\[ \sigma_v = 85 \text{kPa}; \quad \sigma_h = 40 \text{kPa} \]

While the vertical stress remains constant, the horizontal stress increases until shear failure occurs. At failure, the horizontal stress will be the major principal stress, and the vertical stress will be the minor principal stress. Also at failure, the relation between the major and minor principal stresses is:

\[ \sigma_i = \sigma_s \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) + 2c \tan \left( 45^\circ + \frac{\phi}{2} \right) \]
\[ = 85 \text{kPa} \tan^2 \left( 55^\circ \right) + 2 \times 180 \text{kPa} \tan \left( 55^\circ \right) \]
\[ = 688 \text{kPa} \]

b) 11 points:
Using Mohr’s circle and the Pole Method, the orientation of the planes passing through the point of interest, and on which shear failure occurs can be computed as follows:

![Diagram showing Mohr's circle and Pole Method](image)

Orientation of the planes on which shear failure occurs at A:

\[ \theta = \pm \left( 45^\circ - \frac{\phi}{2} \right) = \pm 35^\circ \text{ wrt the horizontal} \]

(c) 11 points:

The normal and shear stresses acting on the failure planes can be computed as follows:

\[ \sigma^* = \sigma_c - r \cos(2\theta) = \frac{1}{2} (688 + 85) - \frac{1}{2} (688 - 85) \cos(70^\circ) \]

\[ = 283.2 \text{kPa} \]

\[ \tau^* = r \sin(2\theta) = \frac{1}{2} (688 - 85) \sin(70^\circ) \]

\[ = 283.1 \text{kPa} \]

Check: Is \( \tau^* = c + \sigma^* \tan(\phi) \)?

\[ 283.1 \text{kPa} = 180 \text{kPa} + 283.2 \text{kPa} \times \tan(20^\circ) \]

\[ = 180 \text{kPa} + 103.1 \text{kPa} \]

\[ = 283.1 \text{kPa} \]

Bonus Question (10 points)

How to measure the permeability from the 1-D consolidation test?

This can be done following the same procedure used in processing of the Lab 9 experimental data:
1) From the displacement versus time curve, note the time \( t_{50} \) required for 50% consolidation to occur.

2) Using the known drainage distance, estimate the coefficient of consolidation over the interval as:

\[
c_v = \frac{0.196(H_{dr})^2}{t_{50}}
\]

3) From consolidation theory, \( c_v = \frac{k(1+e_o)}{\gamma_v a_v} \), so re-arranging gives an expression for the soil permeability \( k \). The soil compressibility coefficient \( a_v \) over the interval needs to be estimated as \( a_v = \Delta e / \Delta \sigma_v \).