Viscoelastic Damping Characteristics of Indium–Tin/SiC Particulate Composites

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Overview

• Objectives: Materials that feature both high stiffness and high viscoelastic damping ($G \tan \delta$)

  What composite material structure can provide both properties?

2. Experimental Approach:
• Based on past experience, indium-tin has well-characterized stiffness/damping.
• Fabricate and test composites with “high” volume fractions of SiC particulate reinforcement.

• Modeling Approach:
• Unit cell analysis of particulate composites at high reinforcement volume fractions.
• Correspondence principle to predict effective stiffness and damping.
Essence of Unit-Cell Homogenization
(for heterogeneous, periodic media)

• On a given length scale at which the material is heterogeneous (micro scale), apply an average stress or average deformation to a detailed model (unit cell)

• For each loading, compute detailed, equilibrium microscale stress and deformations fields.

• Take the spatial average of the “microscale” stress and deformation fields, to get their “macroscopic” correspondent.

• Develop/calibrate a constitutive model that adequately relates the macroscale stresses and deformations.

• When performing analysis of the system on the “macroscale” use the “homogenized” constitutive model to represent the medium.
Micro-/Macro-scale Notation

- Periodic medium and unit cell

- Microscale stress/deformation

\[ \sigma(X) = \Sigma + \sigma^*(X); \]
\[ F(X) = \Phi + F^*(X); \]
\[ <\sigma^*(X)> = 0; \]
\[ <F^*(X)> = 0; \]
\[ F(X + n\lambda_i e_i) = F(X); \] periodicity of microscale deformation
\[ \sigma(X + n\lambda_i e_i) = \sigma(X); \] periodicity of microscale stress

- Averaging stress/deformation to find macroscale correspondents

\[ \Sigma = \langle \sigma \rangle = \frac{1}{V} \int_{\Omega_s} \sigma \, d\Omega_s; \]
\[ \Phi = \langle F \rangle = \frac{1}{V} \int_{\Omega_s} F \, d\Omega_s; \]
**PROCESS: Deformation-Controlled Loading of Unit-Cell**

- Specify an average state of deformation \( \Phi \) for the unit cell.
- Apply a consistent “homogeneous” displacement field \( u = \Phi \cdot X \) to unit cell.
- To achieve stress-field equilibrium on microscale, solve for the additive, periodic, heterogeneous displacement field \( u^*(X) \).
- Resulting equilibrium displacement field: \( u(X) = \Phi \cdot X + u^*(X) \)
- For each macroscopic state of deformation \( \Phi \), compute the corresponding macroscopic state of stress \( \Sigma \).
- Consider the \( \Sigma \) versus \( \Phi \) behavior of the unit cell model.
- Provide and calibrate a macro-scale constitutive model \( \Sigma = \Sigma(\Phi) \).
Symmetric, Conjugate, Macro Stress/Strain Measures

- Using conjugate macroscopic stress/strain measures ensures energy conservation between micro- and macro-scales.

- Nemat-Nassar (2000) demonstrated/used conjugacy between macroscale deformation gradient $\Phi$ and the macroscale nominal stress $\langle P \rangle$.

$$\langle P : \dot{F} \rangle = \langle P \rangle : \langle \dot{\Phi} \rangle$$

- It is preferred to develop constitutive models in terms of symmetric, macroscopic stress and deformation measures. Here, we use:

$$\hat{\Sigma} = \langle P \rangle \Phi^{-T};$$
$$\hat{E} = \frac{1}{2} [\Phi^T \Phi - I].$$

- These symmetric measures satisfy the following conjugacy relationship:

$$\hat{\Sigma} : \hat{E} = \langle P : \dot{F} \rangle = \langle S : \dot{E} \rangle$$
Elastic moduli of composite constituents

<table>
<thead>
<tr>
<th>Elastic Constants</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td>SiC</td>
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<tr>
<td>InSn</td>
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</tbody>
</table>

To realize high $G \tan \delta$, must achieve high volume fractions of particulates

- consider multiple sizes of spherical particles
- consider cubical particles
- past experience with Sn matrix shows that it does not “wet” SiC
(a) Exterior Faces

(b) Top Face & Dimensions

c) Shear Test: Proper Unit Cell Model and its Symmetric Model
Typical Unit–Cell Mesh for Particulate Composite

(a) Unit-cell

(b) Reinforcement only
Framework for discussing elastic anisotropy
Anisotropy of $E$ and $G$ for spherical reinforcement, 50% volume fraction.
Young's Modulus w.r.t normal direction, 50% One-Sized Particulate Composite
Min Shear Modulus w.r.t normal direction, 50% One-Sized Particulate Composite
Variation of homogenized elastic constants with orientation for different particulate-reinforced composites with 50% SiC particle volume fraction.

<table>
<thead>
<tr>
<th>Arrangement/Particles</th>
<th>Young’s modulus</th>
<th>Shear modulus</th>
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<tbody>
<tr>
<td></td>
<td>min</td>
<td>max</td>
</tr>
<tr>
<td>FCC/one-sized spherical particles</td>
<td>54.4</td>
<td>62.4</td>
</tr>
<tr>
<td>FCC/Two-sized particles</td>
<td>56.1</td>
<td>59.8</td>
</tr>
<tr>
<td>BCC/One-sized cubical particles</td>
<td>49.2</td>
<td>84.0</td>
</tr>
</tbody>
</table>
\[ G = G_0 (1 + A_1 (\phi - \phi^{A_2}) - \phi^{A_3}) + G_1 \phi^{A_4} \]

**Coefficients used to fit shear modulus versus volume fraction results.**

<table>
<thead>
<tr>
<th>Type of composite</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCC Two-sized spheres, Lower limit</td>
<td>2.522</td>
<td>5.223</td>
<td>5.223</td>
<td>5.223</td>
</tr>
<tr>
<td>BCC One-sized cubicles, Upper limit</td>
<td>7.172</td>
<td>4.584</td>
<td>0.364</td>
<td>4.584</td>
</tr>
<tr>
<td>BCC One-sized cubicles, Lower limit</td>
<td>2.7890</td>
<td>9.9849</td>
<td>9.9849</td>
<td>9.9849</td>
</tr>
</tbody>
</table>
Summary of Results

- With polymer matrix composites best performance is $G \tan \delta \sim 0.23$ GPa.

- With cubical SiC inclusions in InSn matrix, best $G \tan \delta \sim 2.7$ GPa

- With single-sized spherical SiC inclusions in InSn matrix, best $G \tan \delta \sim 1.6$ GPa

- With two-sized spherical SiC inclusions in InSn matrix, best $G \tan \delta \sim 1.7$ GPa