Uncertainties and CFD Code Validation

A new approach to computational fluid dynamics code validation is developed that gives proper consideration to experimental and simulation uncertainties. The comparison error is defined as the difference between the data and simulation values and represents the combination of all errors. The validation uncertainty is defined as the combination of the uncertainties in the experimental data and the portion of the uncertainties in the CFD prediction that can be estimated. This validation uncertainty sets the level at which validation can be achieved. The criterion for validation is that the magnitude of the comparison error must be less than the validation uncertainty. If validation is not accomplished, the magnitude and sign of the comparison error can be used to improve the mathematical modeling. Consideration is given to validation procedures for a single code, multiple codes and/or models, and predictions of trends. Example results of verification/validation are presented for a single computational fluid dynamics code and for a comparison of multiple turbulence models. The results demonstrate the usefulness of the proposed validation strategy. This new approach for validation should be useful in guiding future developments in computational fluid dynamics through validation studies and in the transition of computational fluid dynamics codes to design.

1 Introduction

Uncertainty considerations involved in using experimental data in validating the predictions of CFD codes (or, more generally, computer simulations) are discussed in this article. The word uncertainty is used in the following sense—the uncertainty associated with a measured quantity or a predicted quantity defines the ±U interval about that quantity within which we expect the true (but unknown) value of that quantity to lie 95 times out of 100. It is important to recognize that a validation is restricted to some range, typically the range of conditions of the data used in the (successful) validation effort. This is intended by the authors to be implicit in the discussions in this article.

The comparison process in attempting to validate predictions using experimental data is illustrated schematically in Fig. 1. The theoretical predictions of result r versus independent variable X from two models (or simulations) are shown along with experimental data points (Xr). In part (a) of the figure, no uncertainties are considered, and one might well be tempted to argue that Model 1 is superior to Model 2. The predictions from Model 1 seem to “capture the trend of the data” better than the predictions from the (simplistic, linearized, etc.) Model 2. If the uncertainties in the experimentally-determined values of the result r are considered (Fig. 1(b)), the perspective changes completely, and it is obvious that arguing for one method over another based on comparison with the experimental data is wasted effort since the predictions from both methods fall well within the data uncertainty.

Actually, Fig. 1(b) does not show the complete situation. For the data points, uncertainties in both the experimentally-determined r and the experimentally-determined value of the independent variable X should be considered, giving an uncertainty “box” around each experimental data point. Additionally, the prediction from a model should not be viewed as an infinitesimally thin r vs. X line, but rather as a “fuzzy band” that represents the prediction plus and minus the uncertainty that should be associated with the simulation/model/code. This is illustrated in Fig. 2, which also shows that, in general, uncertainties in both the data and the predictions can vary (sometimes dramatically) over the range of X. Figure 2 shows the variables and their uncertainties, but the comparison it shows is deceptive because it is two-dimensional. The independent variable X must be considered a vector (X) of n dimensions—fluid velocity as a function of position and time, V(x, y, z, t), for example—and the “box” around X will therefore be n-dimensional. The (total) uncertainty in r that should be used in a comparison should include the experimental uncertainty in r and the additional uncertainty in r arising from experimental uncertainties in the measurements of the n independent variables (this is developed in detail in Section 3).

Contributors to the prediction uncertainty U(r(X)) can be divided into two broad categories—numerical uncertainty and modeling uncertainty. In fact, ensuring that the modeling uncertainty is below some designated value is one purpose of CFD validation through comparisons with benchmark experimental data.

The validation strategy proposed in this article and discussed in detail in Section 3 views the situation from a new perspective, isolating the modeling uncertainty (which the authors do not know how to estimate) from the uncertainties that can be estimated (the data uncertainty and the non-modeling uncertainties in predictions). A direct calculation of the comparison error E (data minus prediction) is made and compared with a validation uncertainty Uv that is composed of the uncertainties in the experimental data and the portion of the uncertainties in the CFD prediction that can be estimated. This validation uncertainty Uv is the best resolution possible in the validation effort (i.e., it sets the “noise level” below which no discrimination is possible). If the absolute value of the calculated comparison error |E| is less than Uv, then validation is defined as being successful at the Uv level.

From the preceding discussion the authors believe it is evident that (1) the uncertainties in the data and in the predictions set the scale at which validation is possible, and (2) these uncertainties must be considered in determining if validation has been achieved. Obviously, these uncertainties should be considered in planning and implementing a computational/experimental research program for validating CFD codes, al-
though they typically have not been in the past. Figure 1(a) gives a qualitative view of the way most previous validation efforts have proceeded, with Fig. 1(b) being typical of the few approaches considering uncertainties at all.

This current work is part of a larger program (Rood, 1996) for developing and implementing a strategy for validation of Reynolds-Averaged Navier-Stokes (RANS) computational ship hydrodynamics codes. The program includes complementary computational ship hydrodynamics and towing tank investiga-

2 Uncertainties in Data and in Simulations

As stated earlier, the uncertainty $U$ associated with a measured quantity or a predicted quantity defines the $\pm U$ interval about that quantity within which we expect the true (but unknown) value of that quantity to lie 95 times out of 100. For detailed discussion of uncertainties associated with experimental data, the reader is referred to Coleman and Steele (1989, 1995). Although the uncertainty in an experimental data point originally comes from both bias (systematic) and precision (random) sources, the uncertainty is "fossilized" into a fixed quantity (a bias) once the value $(X, r)$ of the data point is recorded and reported. This is logical if one notes that the value of the data point is always the same amount different from the (unknown) true value each time the data point is used.

The uncertainties associated with the predictions of models, simulations, CFD codes, etc. and their role in the validation process have been receiving increasing attention in the last few years, and only a few representative references are cited here. Editorial policies have been set by professional journals (ASME, 1993; AIAA, 1994; Gresho and Taylor, 1994), and standards and guidelines (IAHR, 1994) and recommended practices (ITTC, 1996) have been specified by international organizations. The literature covers a broad range from governmental (Rood, 1996) and industrial (Melnik et al., 1994) perspectives to overall methodology (Coleman, 1996; Marvin, 1995; Mehta, 1996 (also AIAA, 1997); Oberkampf et al., 1995) and detailed application (Blottner, 1990; Rouche, 1997; Zang, 1992). The 1993 ASME Symposium on Quantification of Uncertainty in Computational Fluid Dynamics (Celik et al., 1993) provides a good introduction.

As mentioned in the previous section, uncertainties associated with predictions from simulations can be divided into two broad categories: (1) numerical uncertainties, and (2) modeling uncertainties. The numerical uncertainty category includes uncertainties due to the numerical solution of the mathematical equations (discretization, artificial dissipation, iterative and grid non-convergence, local and global non-conservation of mass, momentum, energy, etc., internal and external boundary non-continuity, computer round-off, etc.) The modeling uncertainty category includes uncertainties due to assumptions and approximations in the mathematical representation of the physical process (geometry, mathematical equation, coordinate transformation approximations, free-surface boundary conditions, turbulence models, etc.) and also uncertainties due to the incorporation of previous experimental data into the model (such things as fluid property values and the "constants" in turbulence models). Examples of reported uncertainties associated with property data range from 0.25–0.5 percent for liquid oxygen density (Brown et al., 1994), to 2–5 percent for the thermal conductivity of air at atmospheric pressure (Coleman and Steele, 1989), to huge percentages for properties such as surface tension coefficient that are extremely sensitive to contaminants. A recent study (Beard and Landrum, 1996) utilizing laminar Navier-Stokes computations for hydrogen flow through a solar thermal thruster at temperatures up to 6100 R showed a ±2 percent variation in computed specific impulse due solely to the range of the available reaction rate data reported by different investigators.

The overall process leading to validation and simulation uncertainty estimation can be categorized as documentation, verification, and validation. Documentation involves detailed presentation of the mathematical equations and numerical method. Verification involves estimation of numerical uncertainty through parametric, convergence, and order-of-accuracy studies. Validation involves estimation of the difference (error) between the simulation's prediction and the truth, and this esti-
mate is impossible to make with any confidence without a benchmark. The benchmark can be an analytical solution (with an associated uncertainty) or, more likely and of primary interest here, an experimental value with its associated uncertainty. To paraphrase one reviewer, (verification)/(validation) can be viewed as addressing \((equations\ solved\ right?)/(right\ equations\ solved?)\).

Although not always available, documentation is relatively straightforward, whereas, in spite of the aforementioned efforts, specific implementation procedures for verification and validation are not yet established. Approaches for verification require procedures for the estimation of the numerical uncertainties. Stern et al. (1996) provided an example approach for their steady RANS CFD method with application to naval surface combatants. In this approach, estimates of uncertainties were provided for both integral and point quantities for iterative and grid nonconvergence and were combined using root sum square. Also, for conditions permitting, order-of-accuracy and Richardson extrapolation studies were conducted.

In the following section, a new approach to CFD validation is developed and discussed with regard to validation of a single CFD code, to validation of a comparison of multiple codes and/or models, and to validation of predictions of trends. Subsequently, in Section 4, example results of validations are presented both for a single CFD code and for a comparison of multiple turbulence models. The CFD code and verification procedures of Stern et al. (1996) are used for two applications for which numerical and experimental uncertainty analyses are available: marine-propulsor flow (Chen, 1996; Jessup, 1994) and two-dimensional turbulent flat-plate boundary-layer flow (Sreedhar and Stern, 1997; Longo, et al., 1998).

### 3 An Approach to CFD Code Validation

Consider the situation shown in Fig. 3. Using the example mentioned previously, the single-plane representation of \(r\) versus \(X\) might be a mean velocity component \(V\) vs. distance \((z)\) normal to a solid surface at a given time and position \((x, y, t)\) on that surface. Define the predicted \(r\)-value from the simulation \((r_c)\) as \(S\), the experimentally determined \(r\)-value of the \((x_i, r_i)\) data point as \(D\), and the comparison error, \(E\), as their difference:

\[
E = D - S \quad (1)
\]

The comparison error \(E\) is the resultant of all of the errors associated with the experimental data and the errors associated with the simulation. Here it is assumed that a correction has been made for any error whose value is known. Thus, the errors that are the subject of this discussion have unknown sign and magnitude, and the uncertainties are estimates of these errors.

If \(X_i, r_i\), and \(S\) share no common error sources, then the uncertainty \(U_E\) in the comparison error can be expressed as

\[
U_E^2 = \left(\frac{\partial E}{\partial D}\right)^2 U_D^2 + \left(\frac{\partial E}{\partial S}\right)^2 U_S^2 = U_D^2 + U_S^2 \quad (2)
\]

where \(U_D\) is the uncertainty in the data and \(U_S\) is the uncertainty in the simulation. The uncertainty \(U_E\) should bound the (true) absolute value of the comparison error \(E\) 95 times out of 100. The assumptions and approximations in deriving Eq. (2) are discussed in detail in Coleman and Steele (1995).

Recalling the discussion in Section 2, the simulation uncertainty \(U_S\) can be represented as

\[
U_S^2 = U_{SN}^2 + U_{SPD}^2 + U_{SM}^2 \quad (3)
\]

where \(U_{SN}\) is the simulation numerical solution uncertainty, \(U_{SPD}\) is the simulation modeling uncertainty arising from using previous experimental data, and \(U_{SM}\) is the simulation modeling uncertainty arising from modeling assumptions. Substituting Eq. (3) into Eq. (2) gives

\[
U_E^2 = U_D^2 + U_{SN}^2 + U_{SPD}^2 + U_{SM}^2 \quad (4)
\]

Ideally, we would like to postulate that if the absolute value of \(E\) is less than its uncertainty \(U_E\), then validation is achieved. In reality, the authors know of no approach that gives an estimate of \(U_{SM}\), so \(U_E\) cannot be estimated. That leaves a more stringent validation test as the practical alternative. If we define the validation uncertainty \(U_{SV}\) as the combination of all uncertainties that we know how to estimate (i.e., all but \(U_{SM}\)), then

\[
U_v^2 = U_D^2 + U_{SN}^2 + U_{SPD}^2 \quad (5)
\]

If \(|E|\) is less than the validation uncertainty \(U_v\), then the combination of all the errors in \(D, S\) is smaller than the estimated validation uncertainty and validation has been achieved at the \(U_v\) level. This quantity \(U_v\) is the key metric in the validation process. \(U_v\) is the validation "noise level" imposed by the uncertainties inherent in the data, the numerical solution, and the previous experimental data used in the simulation model—one cannot discriminate once \(|E|\) is less than this, i.e., as long as \(|E|\) is less than this one cannot evaluate the effectiveness of proposed model "improvements." Choice of the required level of \(U_v\) is associated with the degree of risk deemed acceptable in a program.

To estimate \(U_{SPD}\) for a case in which the simulation uses previous data \(d_i\) in \(m\) instances, one would need to evaluate

\[
U_{SPD}^2 = \sum_{i=1}^{m} \left(\frac{\partial X_i}{\partial k}\right)^2 (U_{k_i})^2 \quad (6)
\]

where the \(U_{k_i}\) would be estimated using established uncertainty analysis procedures (Coleman and Steele, 1989, 1995).

As discussed in Section 1, for the data point \((X_i, r_i)\), \(U_D\) should include both the experimental uncertainty in \(r_i\) and the additional uncertainties in \(r\), arising from experimental uncertainties in the measurements of the \(r\) independent variables \((X_i)\) in \(X_i\). The expression for \(U_D\) that should be used in the \(U_v\) calculation is then

\[
U_D^2 = U_d^2 + \sum_{j=1}^{n} \left(\frac{\partial X_j}{\partial X_j}\right)^2 (U_{X_j})^2 \quad (7)
\]

In some cases, the terms in the summation in Eq. (7) may be shown to be very small using an order-of-magnitude analysis and then neglected. This would occur in situations in which the \(U_{X_j}\)'s are of "reasonable" magnitude and gradients in \(r\) are small. In regions of the flow with high gradients (near a surface in a turbulent flow), however, these terms may be very significant.
There is also a very real possibility that measurements of different variables might share identical bias errors. This is easy to imagine for measurements of x, y, and z. Another possibility is D and S sharing an identical error source, for example if r is drag coefficient and the same density table (curves) is used both in data reduction in the experiment and in the simulation. In such cases, additional “correlated bias” terms must be included in Eq. (2). Approximation and inclusion of such terms are discussed in Coleman and Steele (1989, 1995), Coleman et al. (1995), and Brown et al. (1996) and will not be covered in further detail in this article.

Validation of a Single CFD Code. The validation uncertainty, \( U_\text{v} \), sets the level at which validation can be achieved. If the objectives of a program require that validation be accomplished “within ±2 percent,” then \( |E| \) must be less than \( U_\text{v} \) and \( U_\text{v} \) must be less than (roughly) 0.02 D. If \( U_\text{v} \) is greater than 0.02 D, the objectives of the program cannot be achieved until the sum of the terms on the right-hand-side of Eq. (5) is reduced to an acceptable level. When \( |E| \) is greater than \( U_\text{v} \), validation is not accomplished, and the magnitude and sign of \( E \) can be valuable in designing strategies to improve the mathematical modeling.

As one reviewer pointed out, consideration of Eq. (5) shows that (a) the more uncertain the data, and/or (b) the more inaccurate the code (greater \( U_{\text{sd}} \) and \( U_{\text{spp}} \)), the easier it is to validate a code. That is true, since the greater the uncertainties in the data and the code predictions, the greater the “noise level” (\( U_\text{v} \)). If this value of \( U_\text{v} \) is greater than that designated as necessary in a research/design/development program, however, then the required level of validation could not be achieved without improvement in the quality of the data, the code, or both. Likewise, if \( U_{\text{sn}} \) and \( U_{\text{spp}} \) are not estimated but \( |E| \) is less than \( U_\text{v} \), then validation has been achieved but at an unknown level. Obviously, if no uncertainties are estimated, no statements about validation can be made within the concept of the validation process as considered in this article.

In general, validation of a code’s predictions of a number (\( N \)) of different variables is desired, and this means that in a particular validation effort there will be \( N \) different \( E \)'s and \( U_\text{v} \)'s and (perhaps) some successful validations and some unsuccessful. For each variable, a plot of the simulation prediction versus \( X \) compared with the \( \{X_i, r_i \} \) data points gives a traditional overview of the validation status, but the interpretation of the comparison is greatly affected by choice of the scale and the size of the symbols. A plot of \( \pm U_\text{v} \) and \( E \) versus \( X \) for each variable is particularly useful in drawing conclusions, as demonstrated in Section 4, and the interpretation of the comparison is more insensitive to scale and symbol size choices.

Comparison of Multiple Codes and/or Models in a Validation Effort. When a validation effort involves multiple codes and/or models, the procedure discussed above—comparison of the \( E \)'s and \( U_\text{v} \)'s for the \( N \) variables—should be performed for each code/model.

Since each code/model may have a different \( U_\text{v} \), some method to compare the different codes/models’ performance for each variable in the validation is useful. The range within which (95 times out of 100) the true value of \( E \) lies is \( E \pm U_\text{v} \). From Eq. (5), when \( U_{\text{sma}} \) is zero then \( U_\text{v} = U_\text{v} \), so that for that ideal condition the maximum absolute magnitude of the 95% confidence interval is given by \( |E| + U_\text{v} \). Comparison of the \( (|E| + U_\text{v}) \)'s for the different codes/models then shows which has the smallest range of likely error assuming all \( U_{\text{sma}} \)'s are zero. This allows appropriate comparisons of \( (\text{low } E)/(\text{high } U_\text{v}) \) with \( (\text{high } E/\text{low } U_\text{v}) \) codes/models.

Validation of Predictions of Trends. In some instances, the ability of a code or model to predict the trend of a variable may be the subject of a validation effort. An example would be the difference in drag for two ship configurations tested at the same Froude number. The procedure discussed above—comparison of \( |E| \) and \( U_\text{v} \) for the drag—should be performed for each configuration. The difference \( \delta \) in drag for the two configurations should then be considered as the variable that is the subject of the validation. As discussed in detail in Coleman et al. (1995), because of correlated bias uncertainty effects in the experimental data the magnitude of the uncertainty in \( \delta \) may be significantly less than the uncertainty in either of the two experimentally-determined drag values. This means that the value of \( U_\text{v} \) for \( \delta \) may be significantly less than the \( U_\text{v} \)'s for the drag values, allowing for a more stringent validation criterion for the difference than for the absolute magnitudes of the variables.

It is probable that in some instances the \( U_{\text{sma}} \) for the difference \( \delta \) will be less than the \( U_{\text{sma}} \)'s for the absolute magnitudes of the variables because of correlated systematic uncertainty effects in the modeling. This is an unexplored area at this time.

4 Results of Verification/Validation for a Single CFD Code and for a Comparison of Multiple Turbulence Models

Example results of verification/validation are presented both for a single CFD code and for a comparison of multiple turbulence models. In both cases, the magnitude of \( U_{\text{sma}} \) was assumed negligible relative to the other uncertainties. The CFD code and verification procedures of Stern et al. (1996) are used for two applications for which numerical and experimental uncertainty analyses are available: marine-propulsor flow (Chen, 1996; Jessup, 1994) and two-dimensional turbulent flat-plate boundary-layer flow (Sreedhar and Stern, 1997; Longo et al., 1998). The former represents a practical geometry and is used for the single CFD code validation example, whereas the latter represents an idealized geometry and is used for the comparison of turbulence models example.

For the marine-propulsor geometry, Jessup (1989) provided an extensive data set (including circumferential-average, phase-average, and detailed blade boundary-layer, wake, and tip-vortex velocities) for the relatively simple marine propulsor P4119. This propeller-shaft configuration, shown viewed from upstream in Fig. 4, was tested in a 24 in. water tunnel with measurements made using a three-component laser-Doppler velocimeter (LDV) system. Detailed uncertainty estimates have also been reported (Jessup, 1994). An initial validation effort was
reported by Stern et al. (1994). Documentation and verification are reported by Chen (1996) in conjunction with studies of design and off-design marine-propulsion performance.

For the two-dimensional turbulent flat-plate boundary-layer flow, Longo et al. (1996) provided data and uncertainty analysis in conjunction with their study of solid/free-surface juncture boundary layer and wake. A 1.2 m surface-piercing flat plate was tested in a 100 × 3 m towing tank with measurements made using a two-component LDV system configured to obtain three mean velocities and five Reynolds stresses for both boundary layer and wake planes and regions deep and very close to the free surface. Sreedhar and Stern (1997) provided verification for multiple turbulence models for this application in conjunction with the development of nonlinear eddy-viscosity turbulence models, including both wall and free-surface effects.

The CFD method solves the unsteady incompressible RANS and continuity equations using either noninterior cylindrical or inertial cartesian coordinates and the Baldwin-Lomax turbulence model. The RANS equations are solved using finite-analytic spatial and first-order (steady flow) or second-order (unsteady flow) backward difference time discretization. The pressure equation is derived from a discretized form of the continuity equation and solved using second-order-central finite differences. The overall solution procedure is based on the two-step pressure-implicit-split-operator (PISO) algorithm. For steady flow, subiteration convergence is not required and time serves as an iteration parameter.

**Verification Procedures.** The verification procedures follow the approach of Stern et al. (1996). This approach is based on the editorial policy statement of the ASME (ASME, 1993). The ten issues of the statement are divided as documentation (1, 7, 8), verification (2–6), and validation (9, 10). Verification is comprised of grid-, iterative-, and time-convergence (4, 5, 6), artificial dissipation (3), and order-of-accuracy (2) studies. These studies are implemented using a five-step procedure: (i) grid design and identification of important parameters; (ii) convergence studies; (iii) determination of the effects of explicit artificial dissipation, if used; (iv) estimation of overall uncertainties for integral and point variables; and (v) order of accuracy and Richardson extrapolation.

Step (i) is self-explanatory. For steady flow, step (ii) consists of obtaining estimates for iterative and grid convergence uncertainties ($U_{st}$ and $U_{sg}$, respectively) for integral and point variables.

The estimates for grid convergence uncertainty require a minimum of three grids and are based either on (a) the grid convergence metric

$$
\epsilon = \frac{(\phi_i - \phi_j)}{\phi_i}
$$

where $\phi$ represents either an integral or point variable with subscripts $i$ and $j$ corresponding to the finer and coarser grids, respectively, or (b) the grid-index density (Roache, 1997)

$$
GCI = \frac{3e}{(r^n - 1)}
$$

where $r$ is the grid refinement ratio and $p$ the order of accuracy such that for grid doubling and second-order methods $\epsilon = GCI$. For small grid refinement and/or order, the GCI is recommended since $\epsilon$ is arbitrarily small and inappropriate as a metric of grid convergence. Note that for small $\phi$ (including point variables with regions of small $\phi$), $\epsilon$ should be normalized by the range of $\phi$. Decreasing (increasing) $\epsilon/GCI$ indicates grid convergence (divergence) with uncertainty estimates based on $\epsilon$ for the finest grids. Oscillatory $\epsilon/GCI$ is indeterminate, with uncertainty estimated as roughly one-half the difference between the maximum and minimum values. For simple geometries and flows, negligible grid convergence uncertainty is attainable. For complex geometries and flows, convergence may be limited and oscillatory.

The estimates for iterative convergence uncertainty are based on evaluation of the iteration records for both integral and point variables. The level of iterative convergence is determined by the number of orders of magnitude reduction and magnitude in the residuals

$$
\zeta = \phi^n - \phi^{n-1}
$$

where $n$ is the iteration number and $\phi$ can either be the solution variables or equation imbalances obtained by back substitution. Average ($L_2$ norms) or maximum values are used. For simple geometries and flows, sixteen orders of magnitude reduction of $\zeta$ to machine zero is possible such that the iterative convergence uncertainty is negligible. However, for practical geometries and flows, only a few orders of magnitude reduction in $\zeta$ to about

**Table 1** Marine-propulsion flow: performance coefficients

<table>
<thead>
<tr>
<th>D</th>
<th>S</th>
<th>$E_1$</th>
<th>$U_1$</th>
<th>$U_2$</th>
<th>$U_3$</th>
<th>$U_{10}$</th>
<th>$U_{50}$</th>
<th>$U_{90}$</th>
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<tr>
<td>0.146</td>
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<td>2.0</td>
<td>2.7</td>
<td>1.0</td>
<td>2.5</td>
</tr>
</tbody>
</table>

% D.

**Table 2** Marine-propulsion flow: circumferential-average mean-velocity components (radial-magnitude average)

<table>
<thead>
<tr>
<th>$U_1$</th>
<th>$U_2$</th>
<th>$U_3$</th>
<th>$U_4$</th>
<th>$U_5$</th>
<th>$U_6$</th>
<th>$U_7$</th>
<th>$U_8$</th>
<th>$U_9$</th>
<th>$U_{10}$</th>
</tr>
</thead>
<tbody>
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<td>1.2</td>
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<td>1.2</td>
<td>1.2</td>
<td>0.02</td>
<td>0.8</td>
<td>0.4</td>
<td>0.7</td>
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</tr>
<tr>
<td>2.6</td>
<td>1.3</td>
<td>1.2</td>
<td>1.2</td>
<td>0.01</td>
<td>0.7</td>
<td>0.2</td>
<td>0.7</td>
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</tr>
<tr>
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<td>1.2</td>
<td>0.02</td>
<td>0.6</td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
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<td>3.1</td>
<td>0.02</td>
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<td>0.5</td>
<td>0.8</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
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<td>3.1</td>
<td>3.1</td>
<td>0.01</td>
<td>0.7</td>
<td>0.3</td>
<td>0.6</td>
<td>0.2</td>
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</tr>
<tr>
<td>2.7</td>
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<td>0.02</td>
<td>0.9</td>
<td>0.5</td>
<td>0.7</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

1 upstream.
2 downstream.
% $U_{10}$.
Table 3 Two-dimensional turbulent flat-plate boundary-layer flow: streamwise mean velocity (average magnitude across the boundary layer)

| Turbulence model   | $|E| \%$ | $U_1 \%$ | $U_0 \%$ | $U_2 \%$ | $\frac{\partial r}{\partial x}$ | $U_{SN} \%$ | $U_{SP} \%$ | $U_{SI} \%$ | $U_{SP}/U_D$ | $|E| + U_1 \%$ |
|--------------------|----------|----------|----------|----------|-------------------------------|----------|----------|----------|---------------|---------------|
| Baldwin Lomax (1978) | 1.30 | 2.52 | 1.60 | 1.60 | 0.09 | 1.95 | 0.00 | 1.95 | 1.22 | 3.82 |
| Chen and Patel (1988) | 2.07 | 2.40 | 1.60 | 1.60 | 0.09 | 1.79 | 0.00 | 1.79 | 1.12 | 4.47 |
| Myong and Kassagi (1990) | 2.30 | 2.67 | 1.60 | 1.60 | 0.09 | 2.14 | 0.00 | 2.14 | 1.34 | 4.97 |

% $U_r$
The validation comparisons for the radial profiles of the circumferential-average velocity components are shown in Fig. 5(a) for the upstream plane and Fig. 5(b) for the downstream plane. In this presentation, $R$ is the outer radius of the propeller, the outer radius of the shaft is at $r/R = 0.2$, and the three velocity components and $E$ and $U_v$ are normalized by $U_0$. The plots of $U$, $V$, and $W$ versus $r/R$ show the comparisons as they have traditionally been made in past validation efforts. The plots of $E$ and $±U_v$ present the new validation view introduced in this article, and it is immediately obvious where validation has been achieved and where it has not. In this specific case presented, validation is achieved for some variables in some regions of the flow but not for all variables in all regions of the flow field. The largest errors are for the regions of the flow corresponding to the shaft/blade juncture and the blade tip regions, which is confirmed by the validation results using the complete data set (Chen and Stern, 1997).

**Verification/Validation for Comparison of Multiple Turbulence Models.** Variables chosen to illustrate the verification and validation are the average magnitude across the boundary layer of the streamwise mean velocity and the profile of the streamwise mean velocity. The comparisons are for three turbulence models of increasing complexity, i.e., the Baldwin and Lomax (1978) algebraic model (BL), the Chen and Patel (1988) $k$-$e$ and near-wall model (CP), and the Myong and Kasagi (1990) nonlinear $k$-$e$ model (MK). In the presentations, the velocity, comparison error, and validation uncertainty are normalized by the edge velocity $U_e$ and the independent variable $Y$ is normalized by the boundary-layer thickness $\delta$.

The results for the average velocity are shown in Table 3. As for the previous example, the uncertainty in grid convergence dominates the iteration convergence uncertainty, resulting in numerical uncertainties $U_{sw}$ for the three models ranging from about 1.8–2.1 percent. The data uncertainty $U_{d}$ is 1.6 percent leading to validation uncertainties $U_{v}$ of 2.5–2.7 percent. Here again, the contribution to $U_{v}$ from uncertainties in the measurement locations (where $r_i = U_i/U_e$ and $X_i = Y_i/\delta$) is negligible. All three models are validated at the level $U_{v} = 2.5–2.7$ percent. The estimates for $|E| + U_{v}$ range for increasing model complexity from 3.8–5 percent.

The validation comparisons for the streamwise mean velocity are shown in Figs. 6(a–c). The simulation prediction and data comparisons $U/U_e$ vs. $Y/\delta$ [Fig. 6(a)] for the CP and MK models show relatively large underprediction in the mid-region of the boundary-layer, whereas the BL model...
shows relatively small underprediction in the near-wall region. The plots of $E$ and $\pm U_e$ show that the BL model is validated across the entire boundary layer, whereas the CP and MK models failure is confined to the mid-region of the boundary layer. The plots of the $|E| + U_e$ comparisons for the three models show that the BL model maximum values are less than 4 percent $U_e$, except for the near-wall region where the values increase to 7 percent $U_e$ at the wall, whereas the CP and MK models have large values (8 percent $U_e$) at the wall and in the outer part and relatively small values (3 percent $U_e$) for the near-wall region.

5 Summary and Conclusions

A new approach to CFD validation is developed that gives proper consideration to both the experimental and simulation uncertainties. The comparison error $E$ is defined as the difference between the data $D$ (benchmark) and simulation prediction value $S$ and thus includes the errors associated with the experimental data and the errors associated with the simulation. The validation uncertainty is defined as the combination of the uncertainties in the experimental data and the portion of the uncertainties in the CFD prediction that can be estimated. Estimates for $U_{\text{on}}$, the simulation numerical uncertainty, are obtained through verification procedures involving parametric, convergence, and order-of-accuracy studies. The verification procedures are discussed in detail. $U_D$ includes contributions from the independent and dependent variable uncertainties and is obtained using established uncertainty analysis procedures.

The validation uncertainty $U_v$ sets the level at which the validation can be achieved. The criterion for validation is that $|E|$ must be less than $U_v$. The acceptable level of validation is set by program objectives. If $|E|$ is greater than $U_v$, then validation is not accomplished and the magnitude and sign of $E$ can be valuable in designing strategies to improve the mathematical modeling. When a validation effort involves multiple codes and/or models, additionally the comparisons should include the quantity $|E| + U_v$. Validation of the prediction of trends involves reduction in uncertainties, at least for the experiments, through inclusion of correlated bias errors.

Example results of verification/validation are presented both for a single CFD code and for a comparison of multiple turbulence models. A RANS CFD code is used for two applications for which numerical and experimental uncertainty analyses are available: marine-propulsor flow and two-dimensional turbulent flat-plate boundary-layer flow. The former represents a practical geometry and is used for the single CFD code validation example, whereas the latter represents an idealized geometry and is used for the comparison of turbulence models example. The results demonstrate the usefulness of the proposed validation strategy.

The authors recommend the adoption of this approach for CFD code validation. It will be useful both in guiding future developments in CFD through validation studies and in the transition of CFD codes to design through establishment of credibility and ultimately certification once procedures for the latter are established. Realization of the full potential of the approach requires refinements through applications using various CFD methods and data sets (benchmarks), especially with regard to verification procedures. The authors also recommend that general verification procedures be established similar to those used here, but generalized to encompass broad categories of simulation methods and perhaps additional error sources in those methods.

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