

P3.46 When a jet strikes an inclined plate, it breaks into two jets of equal velocity V but unequal fluxes αQ at (2) and $(1 - \alpha)Q$ at (3), as shown. Find α , assuming that the tangential force on the plate is zero. Why doesn't the result depend upon the properties of the jet flow?

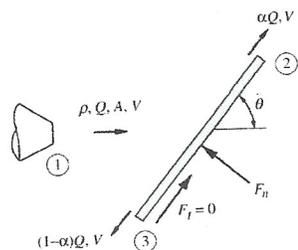


Fig. P3.46

$$0 = \frac{d}{dt} \int_{CV} \rho \, dV + \int_{CS} \rho \mathbf{v} \cdot \mathbf{n} \, dA$$

$$0 = \int_{CS} \rho \mathbf{v} \cdot \mathbf{n} \, dA = \sum (\rho \mathbf{v} \cdot \mathbf{n}) A$$

Continuity: $-\rho Q + \rho \alpha Q + \rho (1 - \alpha) Q = 0$

$\frac{v^2}{2g} + \frac{p}{\gamma} + z = \text{const}$

$$-h_1 V + h_2 V + h_3 V = 0 \quad h_1 = h_2 + h_3$$

$$\text{or } h_2 = h_1 - h_3$$

$$\frac{v_1^2}{2g} = \frac{v_2^2}{2g} = \frac{v_3^2}{2g}$$

Bernoulli

Momentum: $\sum \mathbf{F} = \frac{d}{dt} \int_{CV} \rho \mathbf{v} \, dV + \int_{CS} \rho \mathbf{v} \mathbf{v} \cdot \mathbf{n} \, dA$

$$= \sum (\dot{m}_i \mathbf{v}_i)_{out} - \sum (\dot{m}_i \mathbf{v}_i)_{in}$$

$$\sum F_x = F_x = 0 = -\dot{m}_1 V \cos \theta + \dot{m}_2 V - \dot{m}_3 V$$

in cancels

$$0 = -\dot{m} \cos \theta + \alpha \dot{m} - (1 - \alpha) \dot{m}$$

rho does not

$$\text{or } -\cos \theta + \alpha - (1 - \alpha) = 0$$

depend on

$$0 = -\cos \theta + \alpha - (1 - \alpha)$$

$$0 = -\cos \theta + 2\alpha - 1 \Rightarrow \alpha = \frac{1}{2}(1 + \cos \theta)$$

$$1 - \alpha = \frac{1}{2}(1 - \cos \theta)$$

$$\sum F_n = -F_n = -\dot{m}_1 V \sin \theta$$

$$F_n = \dot{m} V \sin \theta$$

From F_z equation: $0 = -h_1 \cos\theta + h_2 - h_3$

$$= -h_1 \cos\theta + h_1 - h_3 - h_3 \Rightarrow h_3 = \frac{h_1}{2}(1 - \cos\theta)$$

$$h_2 = h_1 - \frac{h_1}{2}(1 - \cos\theta)$$

$$h_2^2 = \frac{h_1^2}{4} (1 + \cos\theta)^2$$

$$= \frac{h_1^2}{4} (1 + \cos\theta)^2$$

$$h_3^2 = \frac{h_1^2}{4} (1 - \cos\theta)^2$$

$$(1 - \cos\theta)(1 - \cos\theta)$$

$$1 - 2\cos\theta + \cos^2\theta$$

$$h_2^2 - h_3^2 = \frac{h_1^2}{4} \left[(1 + \cos\theta)^2 - (1 - \cos\theta)^2 \right]$$

$$+ 2\cos\theta + \cos^2\theta - (1 - 2\cos\theta + \cos^2\theta)$$

$$+ \cos\theta$$

$$= h_1^2 \cos\theta$$