

8.31 A Rankine half-body is formed as shown in Fig. P8.31. For the conditions shown, compute (a) the source strength  $m$  in  $\text{m}^2/\text{s}$ ; (b) the distance  $a$ ; (c) the distance  $h$ ; and (d) the total velocity at point A.

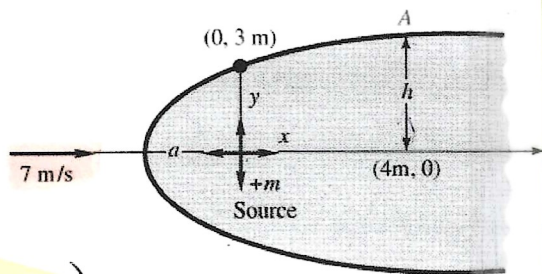


Fig. P8.31

$\psi = \text{uniform stream} + \text{source}$

$$\psi(r, \theta) = \underbrace{U r \sin \theta}_\psi + m \theta$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\theta = \tan^{-1} y/x$$

$$\psi(r, 0) = \text{constant}$$

$$\psi(r, \pi) = \pm m \pi \quad \begin{array}{l} + \text{ upper} \\ - \text{ lower} \end{array}$$

$$\psi(r, \theta) = U r \sin \theta + m \theta = m \pi$$

(a)  $r_s = m(\pi - \theta) / U \sin \theta$

$$y \Big|_{\substack{x=0 \\ \theta=\pi/2}} = 3 = \pi m / 2U \Rightarrow m = 13.4 \text{ m}^2/\text{s} \quad U = 7 \text{ m/s}$$

(b)

$$u = \frac{\partial \psi}{\partial x} = U + \frac{m}{r} \cos \theta$$

$$\frac{\partial}{\partial x} (\tan^{-1} y/x) = \frac{1}{1 + (y/x)^2} \cdot \frac{-y}{x^2}$$

$$u(\pi) = 0 = U - \frac{m}{a} \Rightarrow m = Ua$$

$$= \frac{x}{x^2 + y^2} \cdot x$$

$$a = m/U = 1.91 \text{ m}$$

$$= \frac{r \cos \theta}{r^2} = \frac{\cos \theta}{r}$$

(c)

$$r_s = m(\pi - \theta) / U \sin \theta = 4 / \cos \theta \quad @ \quad x = 4$$

$$\tan x = \frac{13.4(\pi - x)}{28}$$

$$\frac{13.4(\pi - \theta)}{7 \sin \theta} = \frac{4}{\cos \theta}$$

$$\cos \theta = \frac{4}{r_s}$$

using series expansion  $= x + \frac{x^3}{3} + \dots$

$$13.4/28 = \tan \theta / (\pi - \theta)$$

$$x^3 + x(4.436) - 4.51 = 0$$

$$x + \frac{x^3}{3} = 4 - 3x$$

$$x = .8688 \text{ rad} = 49.8^\circ$$

$$r_s = 4 / \cos \theta = 5.95 \text{ m}$$

$$x^3 + 3x = 39 - 35x$$

Solution named  $47.8^\circ$

$$h = r_s \sin \theta = 4.41 \text{ m}$$

$$x^3 + x(3+35) - 39 = 0$$

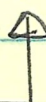
(d)

$$v = -\psi_x = \frac{m}{r} \sin \theta = -m \frac{\partial}{\partial x} (\tan^{-1} y/x)$$

$$V = [u^2 + v^2]^{1/2} = \left[ \left( U + \frac{m}{r} \cos \theta \right)^2 + \left( \frac{m}{r} \sin \theta \right)^2 \right]^{1/2} = 8.7 \text{ m/s}$$

$$U^2 + \frac{m^2}{r^2} \cos^2 \theta + \frac{2Um}{r} \cos \theta + \frac{m^2}{r^2} \sin^2 \theta$$

$$= U^2 + \frac{m^2}{r^2} + \frac{2Um}{r} \cos \theta$$



$$(49 + 5.1 + 20.4)^{1/2} = 8.6$$