**Name: ------------------** **Quiz 6** **Time: 20 minutes**

**ME:5160** **Fall 2024**

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**The exam is closed book and closed notes.**

A large tank of liquid under pressure is drained through a smoothly contoured nozzle of area A. The mass flow rate $\dot{m}$ is thought to depend on the nozzle area A, the liquid density $\rho$, the difference in height between the liquid surface and the nozzle $h$, the change in pressure $\Delta p$, and the gravitational acceleration g. Taking $\rho$, $A$, and $g$ as repeating variables, find an expression for the mass flow rate $\dot{m}$ as a function of the other parameters in the problem in terms of dimensionless Pi groups.

| | | Dimensions | |
|---|---|---|---|
| **Quantity** | **Symbol** | ***MLT*Θ** | ***FLT*Θ** |
| Length | $L$ | $L$ | $L$ |
| Area | $A$ | $L^2$ | $L^2$ |
| Volume | $\mathcal{V}$ | $L^3$ | $L^3$ |
| Velocity | $V$ | $LT^{-1}$ | $LT^{-1}$ |
| Acceleration | $dV/dt$ | $LT^{-2}$ | $LT^{-2}$ |
| Speed of sound | $a$ | $LT^{-1}$ | $LT^{-1}$ |
| Volume flow | $Q$ | $L^3T^{-1}$ | $L^3T^{-1}$ |
| Mass flow | $\dot{m}$ | $MT^{-1}$ | $FTL^{-1}$ |
| Pressure, stress | $p, \sigma, \tau$ | $ML^{-1}T^{-2}$ | $FL^{-2}$ |
| Strain rate | $\dot{\epsilon}$ | $T^{-1}$ | $T^{-1}$ |
| Angle | $\theta$ | None | None |
| Angular velocity | $\omega, \Omega$ | $T^{-1}$ | $T^{-1}$ |
| Viscosity | $\mu$ | $ML^{-1}T^{-1}$ | $FTL^{-2}$ |
| Kinematic viscosity | $\nu$ | $L^2T^{-1}$ | $L^2T^{-1}$ |
| Surface tension | $\Upsilon$ | $MT^{-2}$ | $FL^{-1}$ |
| Force | $F$ | $MLT^{-2}$ | $F$ |
| Moment, torque | $M$ | $ML^2T^{-2}$ | $FL$ |
| Power | $P$ | $ML^2T^{-3}$ | $FLT^{-1}$ |
| Work, energy | $W, E$ | $ML^2T^{-2}$ | $FL$ |
| Density | $\rho$ | $ML^{-3}$ | $FT^2L^{-4}$ |
| Temperature | $T$ | $\Theta$ | $\Theta$ |
| Specific heat | $c_p, c_v$ | $L^2T^{-2}\Theta^{-1}$ | $L^2T^{-2}\Theta^{-1}$ |
| Specific weight | $\gamma$ | $ML^{-2}T^{-2}$ | $FL^{-3}$ |
| Thermal conductivity | $k$ | $MLT^{-3}\Theta^{-1}$ | $FT^{-1}\Theta^{-1}$ |
| Thermal expansion coefficient | $\beta$ | $\Theta^{-1}$ | $\Theta^{-1}$ |

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**Solution:**

Assumptions: the problem is only a function of the given dimensional variables.

$$\dot{m} = f(A, \rho, h, \Delta p, g)$$

$$n = 6$$

$$\dot{m} = \{MT^{-1}\};\ A = \{L^2\};\ \ \rho = \{ML^{-3}\};\ h = \{L\};\ \Delta p = \{ML^{-1}T^{-2}\};\ \ g = \{LT^{-2}\}$$ **(3)**

$$j = 3 \rightarrow \ k = n - j = 3$$ **(1)**

The repeating variables are $\rho$, $A$, and $g$; adding each remaining variable in turn, we find the Pi groups:

$$\Pi_1 = \rho^a A^b g^c \dot{m} = \{(ML^{-3})^a (L^2)^b (LT^{-2})^c (MT^{-1})\} = \{M^0 L^0 T^0 \Theta^0\}$$

$$a = -1;\ \ b = -5/4;\ \ c = -1/2$$

$$\Pi_1 = \frac{\dot{m}}{\rho A^{5/4} g^{1/2}}$$ **(2)**

$$\Pi_2 = \rho^a A^b g^c h = \{(ML^{-3})^a (L^2)^b (LT^{-2})^c (L)\} = \{M^0 L^0 T^0 \Theta^0\}$$

$$a = 0;\ \ b = -1/2;\ \ c = 0$$

$$\Pi_2 = \frac{h}{A^{1/2}}$$ **(2)**

$$\Pi_3 = \rho^a A^b g^c \Delta p = \{(ML^{-3})^a (L^2)^b (LT^{-2})^c (ML^{-1}T^{-2})\} = \{M^0 L^0 T^0 \Theta^0\}$$

$$a = -1;\ \ b = -1/2;\ \ c = -1$$

$$\Pi_3 = \frac{\Delta p}{\rho A^{1/2} g}$$ **(2)**

Thus the arrangement of the dimensionless variables is:

$$\frac{\dot{m}}{\rho A^{5/4} g^{1/2}} = f\left(\frac{h}{A^{1/2}}, \frac{\Delta p}{\rho A^{1/2} g}\right)$$