

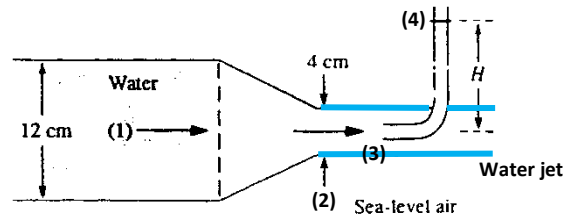
The exam is closed book and closed notes.

An open water jet exits from a nozzle into sea-level air, as shown, and strikes a stagnation tube ($V=0$). If the centerline pressure at section (1) is 110 kPa and losses are neglected, estimate (a) the mass flow in kg/s; and (b) the height H of the fluid in the tube.

Hint a): $V_1 A_1 = V_2 A_2$

Hint b): $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$

Hint c): Use Bernoulli from (1) to (2) and manometer equation from (3) to (4)



Solution:

Continuity:

$$V_1 A_1 = V_2 A_2 \quad +2$$

$$V_1 \left(\frac{\pi}{4} D_1^2 \right) = V_2 \left(\frac{\pi}{4} D_2^2 \right)$$

$$V_2 = V_1 \left(\frac{D_1}{D_2} \right)^2 \quad +1$$

Writing Bernoulli and continuity between pipe and jet yields jet velocity:

$$p_1 - p_a = \frac{\rho}{2} V_{jet}^2 \left[1 - \left(\frac{D_{jet}}{D_1} \right)^4 \right] = 110000 - 101350 = \frac{998}{2} V_{jet}^2 \left[1 - \left(\frac{4}{12} \right)^4 \right], \quad +3$$

$$\text{solve } V_{jet} = 4.19 \frac{\text{m}}{\text{s}} \quad +1$$

$$\text{Then the mass flow is } \dot{m} = \rho A_{jet} V_{jet} = 998 \frac{\pi}{4} (0.04)^2 (4.19) = 5.25 \frac{\text{kg}}{\text{s}} \quad \text{Ans. (a)} \quad +1$$

(b) Bernoulli between (2) and (3):

$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + z_3$$

$$z_2 = z_3, P_2 = P_a, V_3 = 0, V_2 = V_{jet}$$

$$\frac{P_a}{\rho g} + \frac{V_{jet}^2}{2g} = \frac{P_3}{\rho g} \rightarrow P_3 = P_a + \frac{\rho V_{jet}^2}{2}$$

$$P_3 = 101350 + \frac{998(4.19)^2}{2} = 110110 \text{ Pa} \quad +1$$

Bernoulli between (3) and (4):

$$\frac{P_3}{\rho g} + \frac{V_3^2}{2g} + z_3 = \frac{P_4}{\rho g} + \frac{V_4^2}{2g} + z_4$$
$$z_4 - z_3 = H, V_3 = 0, P_4 = P_a, V_4 = 0$$
$$H = \frac{P_3 - P_a}{\rho g} = \frac{110110 - 101350}{998 \cdot 9.81} = 0.894m \quad +1$$