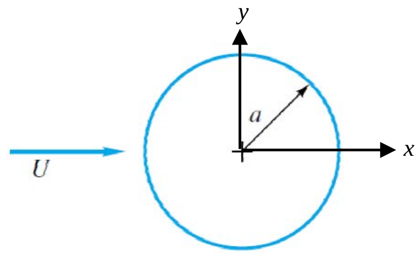
Name: ------------------ **Quiz: No. 14** **Time: 15 minutes**

ME:5160, Fall 2024

The exam is closed book and closed notes.

The velocity potential function for an ideal flow around a long cylinder centered at origin is given by $\phi = \left(\frac{B}{r} + Ar\right)\cos\theta$. The cylinder has a radius *a* and is placed in a uniform flow of velocity *U*. (a) Determine the constants *A* and *B* in terms of *a* and *U*, using boundary conditions at the surface and at (*x*, *y*) = (0, ∞). (b) Determine the location and magnitude of the maximum velocity on the surface.

Equations: $v_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$; $v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r}$; $r = \sqrt{x^2 + y^2}$; $\theta = \tan^{-1}\left(\frac{y}{x}\right)$



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(2) **ME:5160, Fall** 2024

Solution:

KNOWN: ϕ, U, a

(1) FIND: A, B, θ_{max}, V_{max}

ASSUMPTIONS: Irrotational flow

ANALYSIS:

(a)

$$v_r = \frac{\partial \phi}{\partial r} = \left(-\frac{B}{r^2} + A\right)\cos\theta \quad \textbf{(0.5)}$$

$$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = \frac{1}{r}\left(\frac{B}{r} + Ar\right)(-\sin\theta) = -\left(\frac{B}{r^2} + A\right)\sin\theta \quad \textbf{(0.5)}$$

B.C. at cylinder surface:

$$At \quad r = a, \qquad v_r = 0 \quad \textbf{(0.5)}$$

$$\left(-\frac{B}{a^2} + A\right)\cos\theta = 0 \quad \textbf{(0.5)}$$

$$\Rightarrow -\frac{B}{a^2} + A = 0 \quad \textbf{(0.5)}$$

B.C. above the cylinder far from surface:

$$At \quad \theta = \frac{\pi}{2} \ \ and \ \ r = \infty, \qquad v_\theta = U \quad \textbf{(0.5)}$$

$$-\left(\frac{B}{\infty} + A\right)\sin\left(\frac{\pi}{2}\right) = U \quad \textbf{(0.5)}$$

$$A = -U \quad \textbf{(0.5)}$$

$$\Rightarrow B = Aa^2 = -Ua^2 \quad \textbf{(0.5)}$$

(b)

Since $v_r = 0$ at the surface, the total velocity is $V = v_\theta$:

$$v_\theta(r = a) \overset{\textbf{(0.5)}}{=} -\left(\frac{B}{a^2} + A\right)\sin\theta \overset{\textbf{(0.5)}}{=} -\left(\frac{-Ua^2}{a^2} - U\right)\sin\theta = (2U)\sin\theta \quad \textbf{(0.5)}$$

The maximum velocity would be for where $\sin\theta = 1$:

$$\theta = 90°, \textbf{(0.5)} \qquad V_{max} = 2U \quad \textbf{(0.5)}$$