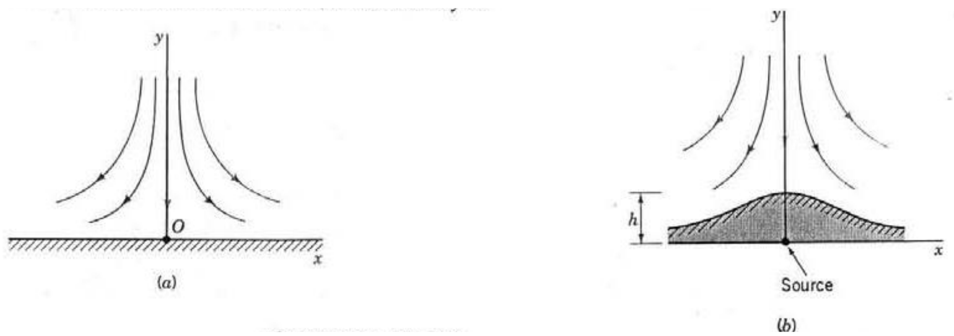
3. flow against a flat plate (Fig. a) can be described with the stream function $\psi = Axy$ where A is a constant. This type of flow is commonly called a "stagnation point" flow since it can be used to describe the flow in the vicinity of the stagnation point at O. By adding a source of strength m at O ($\psi = m\theta$), stagnation point flow against a flat plate with a "bump" is obtained as illustrated in Fig. b. Determine the bump height, h, as a function of the constant, A, and the source strength, m.

Hint : $\psi_a = Axy$ corresponds to $\psi = A(r\cos\theta)(r\sin\theta) = \frac{A}{2}r^2\sin 2\theta$ in Cylindrical Coordinates

$$\psi_b = \frac{A}{2}r^2\sin 2\theta + m\theta$$



$$\psi = \frac{A}{2}r^2\sin 2\theta + m\theta$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -Ar\sin 2\theta \qquad \textbf{(1)}$$

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = Ar\cos 2\theta + \frac{m}{r} \qquad \textbf{(1)}$$

For the bump, the stagnation point occurs at:

$$r = h, \qquad \theta = \frac{\pi}{2} \qquad \textbf{(1)}$$

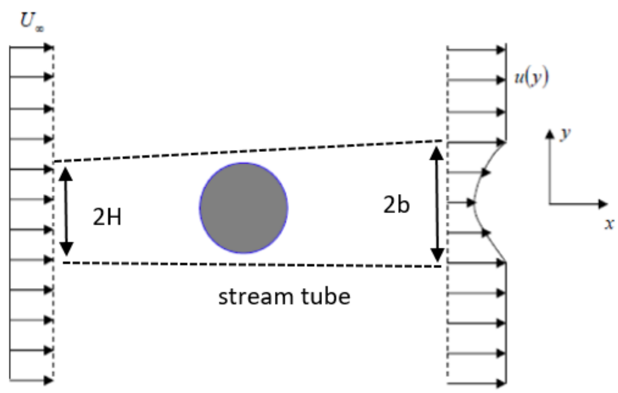
$$(v_\theta)_{stag} = -Ah\sin\pi = -Ah\,(0) = 0 \qquad \textbf{(2)}$$

$$(v_r)_{stag} = Ah\cos\pi + \frac{m}{h} = Ah\,(-1) + \frac{m}{h} = 0 \qquad \textbf{(2)}$$

$$Ah = \frac{m}{h} \quad \Rightarrow \quad h = \sqrt{\frac{m}{A}} \qquad \textbf{(1)}$$

5. Consider an experiment in which the drag on a two-dimensional body immersed in a steady incompressible flow can be determined from measurement of velocity distribution far upstream and downstream of the body as shown in Figure below. Velocity far upstream is the uniform flow U_∞, and that in the wake of the body is measured to be $u(y) = \frac{U_\infty}{2}\left(\frac{y^2}{b^2} + 1\right)$, which is less than U_∞ due to the drag of the body. Assume that there is a stream tube with inlet height of 2H and outlet height of 2b as shown in Figure below. (a) Determine the relationship between H and b using the continuity equation. (b) Find the drag per unit length of the body as a function of U_∞, b and ρ.

(Hint : Momentum Equation $\sum F_x = \int u\rho(\underline{V} \cdot \underline{n})dA$)



Solution 5:
a) Continuity:

$$2\rho H U_\infty = \rho \int_{-b}^{b} u(y)dy = \rho \int_{-b}^{b} \frac{U_\infty}{2}\left(\frac{y^2}{b^2} + 1\right) dy$$

$$2\rho H U_\infty = \rho \frac{U_\infty}{2} \int_{-b}^{b} \left(\frac{y^2}{b^2} + 1\right) dy = \rho \frac{U_\infty}{2} \left(\frac{y^3}{3b^2} + y\right)\Bigg]_{-b}^{b}$$ **(2)**

$$2H = \frac{1}{2}\left(\frac{b^3}{3b^2} + b + \frac{b^3}{3b^2} + b\right) = \frac{1}{2}\left(\frac{8}{3}b\right) = \frac{4}{3}b$$ **(1)**

$$H = \frac{2b}{3}$$ **(1)**

b) x-momentum:

$$\sum F_x = \int u\rho(\underline{V} \cdot \underline{n})dA$$

Drag per unit length: **(1)**

$$-F_D = -\rho U_\infty^2(2H) + \rho \int_{-b}^{b} u^2(y)dy$$

(2)

$$F_D = \rho U_\infty^2(2H) - \rho \int_{-b}^{b} \left[\frac{U_\infty}{2}\left(\frac{y^2}{b^2} + 1\right)\right]^2 dy = \rho U_\infty^2(2H) - \rho \frac{U_\infty^2}{4} \int_{-b}^{b} \left(\frac{y^2}{b^2} + 1\right)^2 dy$$

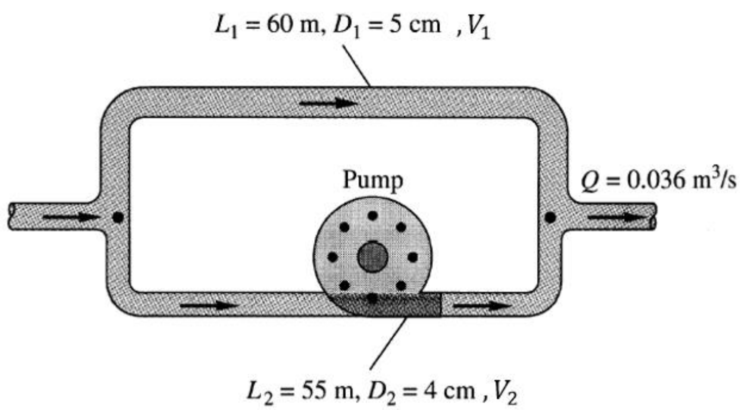
Calculating integral: **(1)**

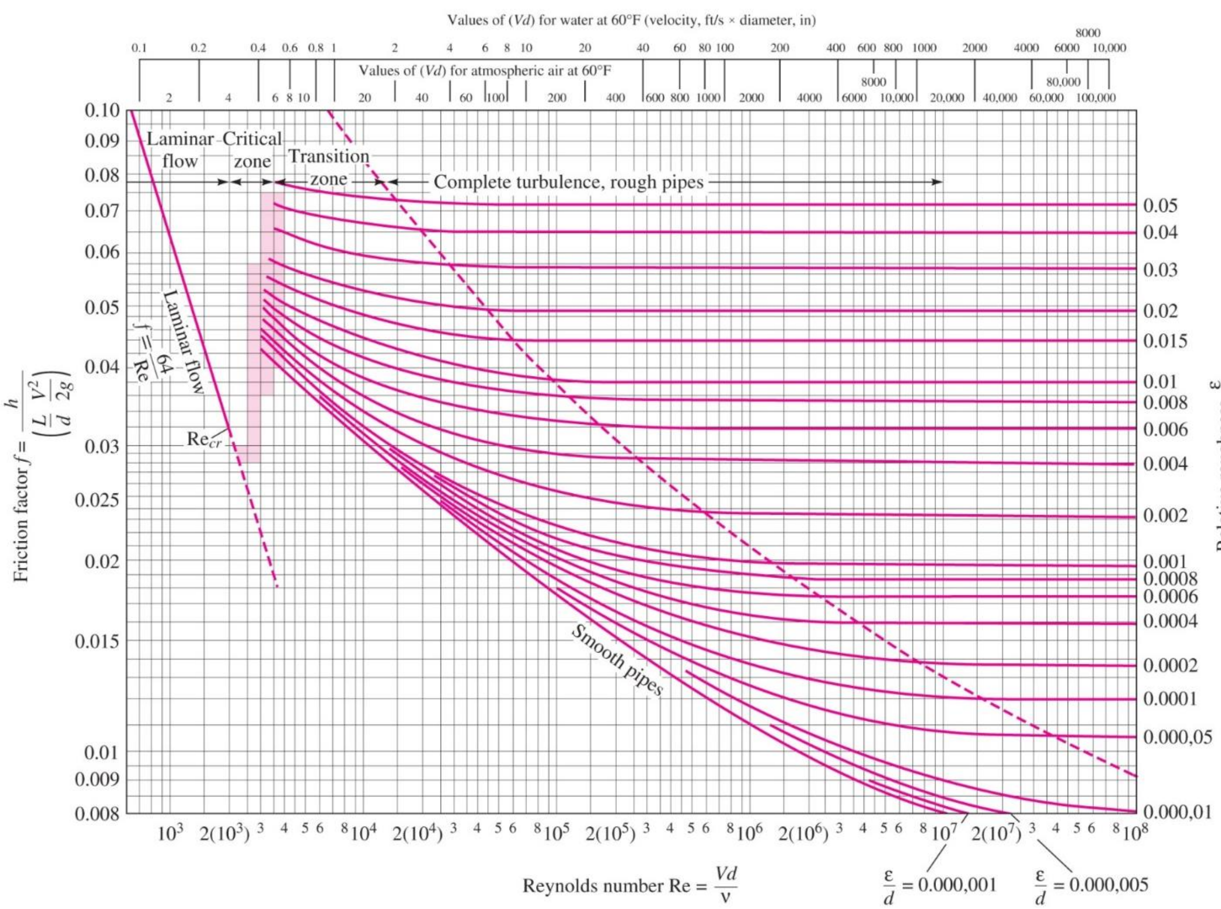
$$\int_{-b}^{b} \left(\frac{y^2}{b^2} + 1\right)^2 dy = \int_{-b}^{b} \left(\frac{y^4}{b^4} + \frac{2y^2}{b^2} + 1\right) dy = \frac{y^5}{5b^4} + \frac{2y^3}{3b^2} + y\Bigg]_{-b}^{b} = 2\left(\frac{b}{5} + \frac{2b}{3} + b\right) = \frac{56}{15}b$$ **(1)**

Entering into the momentum equation:

$$F_D = 2\rho H U_\infty^2 - \frac{1}{4}\rho U_\infty^2 \left(\frac{56}{15}b\right) = \rho U_\infty^2 \left(\frac{4}{3}b - \frac{14}{15}b\right) = \rho U_\infty^2 \frac{2b}{5}$$ **(1)**

The parallel galvanized-iron pipe system ($\epsilon = 0.15\ mm$) delivers water at 20^{0}C ($\rho = 998\ kg/m^3$ and $\mu = 0.001\ kg/m \cdot s$) with a total flow rate of 0.036 m^3/s. (a) Find out the relation between V_1 and V_2. If the pump is wide open and not running, with a loss coefficient of K=1.5,
(b) determine the velocity in each pipe(V_1 and V_2). Use $f_1 = f_2 = 0.02$ for your initial guess.
(Hint : $h_f = f\frac{L}{d}\frac{V^2}{2g}$)





a) Continuity:

$$Q_1 + Q_2 = \frac{\pi}{4} d_1^2 V_1 + \frac{\pi}{4} d_2^2 V_2 = Q_{total}; \quad V_2 = \frac{4}{\pi d_2^2} Q_{total} - \frac{d_1^2}{d_2^2} V_1 \qquad \textbf{(1)}$$

$$V_2 = \frac{4}{\pi 0.04^2} 0.036 - \frac{0.05^2}{0.04^2} V_1 \qquad \textbf{(1)}$$

$$V_2 = 28.65 - 1.56\, V_1$$

(b)

Same head loss for parallel pipes:

$$h_{f1} = h_{f2} + h_{m2} \qquad \textbf{(2)}$$

$$f_1 \frac{L_1}{d_1} \frac{V_1^2}{2g} - \frac{V_2^2}{2g}\left(f_2 \frac{L_2}{d_2} + K\right) = 0$$

$$\textbf{(1)}$$

$$f_1 \frac{60}{0.05} \frac{V_1^2}{2 \times 9.81} - \frac{V_2^2}{2 \times 9.81}\left(f_2 \frac{55}{0.04} + 1.5\right) = 0$$

$$61.16 f_1 V_1^2 - (28.65 - 1.56\, V_1)^2 (70.08 f_2 + 0.076) = 0 \qquad \textbf{(1)}$$

Reynolds Number:

$$Re_1 = \frac{\rho V_1 D_1}{\mu} = \frac{998 \times 0.05}{0.001} V_1 = 49900\, V_1 \qquad \textbf{(0.5)}$$

$$Re_2 = \frac{\rho V_1 D_1}{\mu} = \frac{998 \times 0.04}{0.001} V_2 = 39920 V_2 \qquad \textbf{(0.5)}$$

Relative roughness:

$$\frac{\epsilon}{D_1} = \frac{0.15}{50} = 0.003 \qquad \textbf{(0.5)}$$

$$\frac{\epsilon}{D_2} = \frac{0.15}{40} = 0.00375 \qquad \textbf{(0.5)}$$

Guessing $f_1 = f_2 = 0.02$

$$f_1 = 0.02, f_2 = 0.02 \rightarrow V_1 = 11.59 \rightarrow V_2 = 10.54 \rightarrow Re_1 = 57800, Re_2 = 421000 \qquad \textbf{(1)}$$

$$f_1 = 0.0264, f_2 = 0.0282 \rightarrow V_1 = 11.69 \rightarrow V_2 = 10.37$$

$$\therefore V_1 = 11.69\, m/s \qquad \textbf{(1)}$$

Water (ρ=998 kg/m^3, μ=0.001 kg/m·s) flows from a container and "bubbles up" a distance h above the outlet pipe, as shown in the Figure below. The pipe diameter is D = 0.013m, ε = 0.15mm, K_{entrance} = 0.3 for the rounded entrance, $H_1 = 1.14\ m$, $L_1 = 0.46\ m$, $L_2 = 0.81\ m$, $H_2 = 0.05\ m$, and $h = 0.08\ m$. (a) Find the velocity V in the pipe. (b) Find the loss coefficient in the valve.

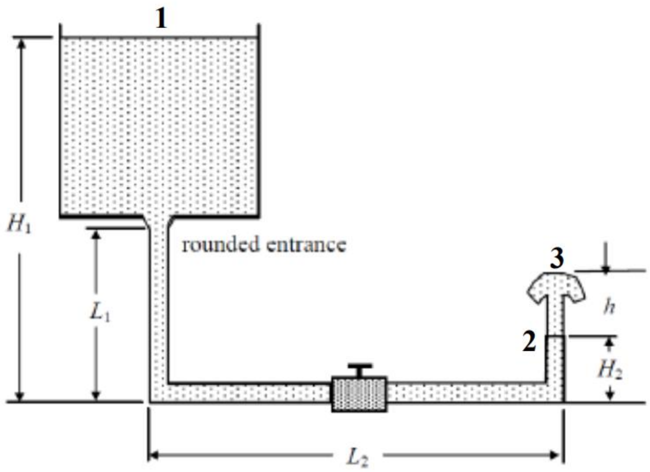
Hint

Energy equation between 1 and 2

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_1=\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_2+\sum h_l$$

Energy equation between 2 and 3

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_3$$



(a) Energy equation between 2 and 3

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_3$$

$$\left(0+\frac{V^2}{2g}+H_2\right)=[0+0+(H_2+h)] \qquad \textbf{(1)}$$

$$h=\frac{V^2}{2g} \quad \rightarrow \quad V=\sqrt{2gh}=\sqrt{2(9.81)(0.08)}=1.25\ m/s \qquad \textbf{(1)}$$

(b) Energy equation between 1 and 2

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_1=\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_2+\sum h_l$$

$$(0+0+H_1)=\left(0+\frac{V^2}{2g}+H_2\right)+\sum h_l \qquad \textbf{(1.5)}$$

$$\sum h_l=H_1-H_2-\frac{V^2}{2g}=(1.14)-(0.05)-\frac{(1.25)^2}{2(9.81)}=1.01\ m \qquad \textbf{(2.5)}$$

Major and minor losses

$$\sum h_l = \left(f\frac{\sum L}{D} + K_{entrance} + K_{valve}\right)\frac{V^2}{2g}$$

$$Re_D = \frac{\rho V D}{\mu} = \frac{(998)(1.25)(0.013)}{(0.001)} = 16217 \quad Turbulent$$

$$\frac{\varepsilon}{D} = \frac{(0.00015)}{(0.013)} = 0.0115$$

$$f_{Moody} = 0.043$$ **(1)**

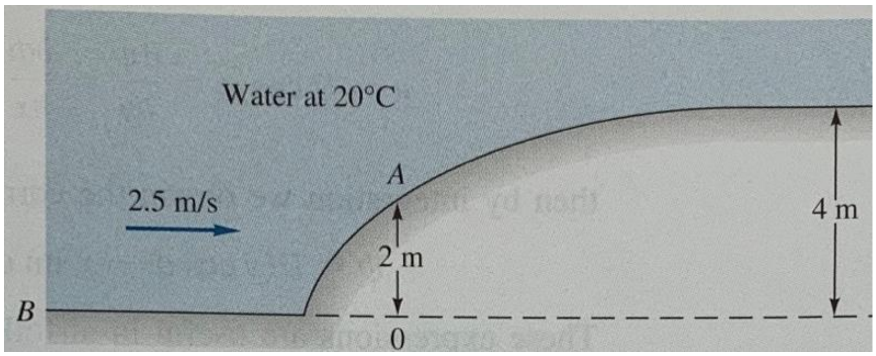
$$\sum h_l = \left(f\frac{(L_1 + L_2 + H_2)}{D} + K_{entrance} + K_{valve}\right)\frac{V^2}{2g}$$

$$(1.01) = \left((0.043)\frac{(0.46 + 0.81 + 0.05)}{(0.013)} + (0.3) + K_{valve}\right)\frac{(1.25)^2}{2(9.81)}$$ **(2)**

Solve for K_{valve}:

$$K_{valve} = 8.02$$ **(1)**

5. The bottom of a river has a 4 *m* high bump that approximates a Rankine half-body, as shown in the figure. The pressure at point B on bottom is 130 *kPa*, and the river velocity is 2.5 *m/s*. Use inviscid theory to estimate the water **(a)** velocity and **(b)** pressure at point A on the bump, which is 2 *m* above point B. ($\sin^2\theta + \cos^2\theta = 1; \rho = 998\ kg/m^3; \gamma = 9790\ N/m^3$)



Hint: Rankine half body equations:

$\Psi = Ur\sin\theta + m\theta;\ m = Ua$

$v_r = \frac{1}{r}\frac{\partial\Psi}{\partial\theta};\ v_\theta = -\frac{\partial\Psi}{\partial r}$

$r = \frac{m(\pi-\theta)}{U\sin\theta}$ (on surface)

Bump downstream height = πa

a) Velocity

$$\theta = \frac{\pi}{2} = 90^0$$

$$a = \frac{4}{\pi} = 1.27$$

$$r = \frac{m(\pi-\theta)}{U\sin\theta} = \frac{a(\pi-\theta)}{\sin\theta} = \frac{\pi}{2}a$$ +1

$$v_r = \frac{1}{r}\frac{\partial\Psi}{\partial\theta} = \frac{1}{r}(U\,\mathrm{r}\cos\theta + m) = U\cos\theta + \frac{m}{r}$$

$$v_\theta = -\frac{\partial\Psi}{\partial r} = -U\sin\theta$$

$$V^2 = U^2\sin^2\theta + U^2\cos^2\theta + \frac{m^2}{r^2} + \frac{2Um}{r}\cos\theta = U^2 + \frac{U^2a^2}{r^2} + \frac{2U^2a}{r}\cos\theta$$

$$V^2 = U^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$$

$$V_A^2 = U^2\left(1 + \frac{a^2}{\pi^2 a^2/4} + \frac{2a}{\pi a/2}\cos\frac{\pi}{2}\right) = 1.405U^2$$

$$V_A = 1.185U = 1.185 \times 2.5 = 2.96\, m/s$$

b) Bernoulli

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$p_A = p_B + \frac{\gamma}{2g}(V_B^2 - V_A^2) + \gamma(z_B - z_A)$$

$$p_A = \frac{130000}{9790} + \frac{9790}{2 \times 9.81}(2.5^2 - 2.96^2) + 9790(-2) = 109200\ Pa$$

+3