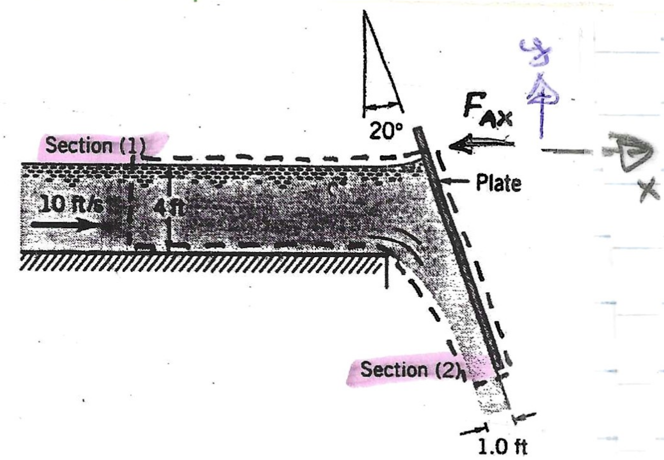
5.48 Water flows from a two-dimensional open channel and is diverted by an inclined plate as illustrated in Fig. P5.48. When the velocity at section (1) is 10 ft/s, what horizontal force (per unit width) is required to hold the plate in position? At section (1) the pressure distribution is hydrostatic, and the fluid acts as a free jet at section (2). Neglect friction.



■ FIGURE P5.48

Momentum: $\sum F_x = \int_{cs} \rho u \underline{V} \cdot \underline{n}\, dA = \sum_{cs} \rho u \underline{V} \cdot \underline{n} A$

$$-F_{Ax} + P_1 A_1 = -V_1 \rho V_1 A_1 + V_2 \sin 20^\circ \rho V_2 A_2$$

$$\bar{P}A = \frac{\gamma h_1}{2} A_1$$

Continuity: $V_1 A_1 = V_2 A_2$ $\qquad \underline{V_1} = 10\,ft/s\,\hat{i}$

$$V_2 = \frac{V_1 h_1}{h_2} \qquad \underline{V_2} = V_2 \sin\theta\,\hat{i} - V_2 \cos\theta\,\hat{j}$$

$$-V_1^2 \rho h_1 (1) + \left(\frac{V_1 h_1}{h_2}\right)^2 \sin 20^\circ \rho h_2 (1) = -F_{Ax} + \frac{1}{2}\gamma h_1^2 (1)$$

$$F_{Ax} = \left[\frac{1}{2}\gamma h_1^2 + V_1^2 \rho h_1 - \left(\frac{V_1 h_1}{h_2}\right)^2 \sin 20^\circ \rho h_2\right] \times (1)$$

$$= \frac{1}{2}\left(62.4 \frac{lb}{ft^3}\right)(4ft)^2 (1ft) + \left(10 \frac{ft}{s}\right)^2 \left(1.94 \frac{slugs}{ft^3}\right)\left(\frac{1\,lb\,s^2}{slug\,ft}\right)(4ft)(1ft)$$

$$- \left(\frac{10 \frac{ft}{s} \times 4ft}{1ft}\right)^2 \sin 20^\circ (\;\;)(1ft)(1ft)$$

$= 213\ lb$

$lbm = slug \frac{ft}{s^2}$

$slug = lbm\, s^2$