ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2024 – HW9 Solution

P6.39 By analogy with laminar shear, $\tau = \mu\, du/dy$. T. V. Boussinesq in 1877 postulated that turbulent shear could also be related to the mean-velocity gradient $\tau_{turb} = \varepsilon\, du/dy$, where ε is called the *eddy viscosity* and is much larger than μ. If the logarithmic-overlap law, Eq. (6.28), is valid with $\tau \approx \tau_w$, show that $\varepsilon \approx \kappa\rho u^* y$.

Solution: Differentiate the log-law, Eq. (6.28), to find du/dy, then introduce the eddy viscosity into the turbulent stress relation

$$\text{If}\quad \frac{u}{u^*} = \frac{1}{\kappa}\ln\left(\frac{yu^*}{\nu}\right) + B, \quad \text{then}\quad \frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$\text{Then, if}\quad \tau \approx \tau_w \equiv \rho u^{*2} = \varepsilon\frac{du}{dy} = \varepsilon\frac{u^*}{\kappa y}, \quad \text{solve for}\ \ \varepsilon = \kappa\rho u^* y \quad \textit{Ans.}$$

Note that $\varepsilon/\mu = \kappa y^+$, which is much larger than unity in the overlap region.

P6.40 Theodore von Kármán in 1930 theorized that turbulent shear could be represented by $\tau_{turb} = \varepsilon\, du/dy$ where $\varepsilon = \rho\kappa^2 y^2 \left|du/dy\right|$ is called the *mixing-length eddy viscosity* and $\kappa \approx 0.41$ is Kármán's dimensionless *mixing-length constant* [2,3]. Assuming that $\tau_{turb} \approx \tau_w$ near the wall, show that this expression can be integrated to yield the logarithmic-overlap law, Eq. (6.28).

Solution: This is accomplished by straight substitution:

$$\tau_{turb} \approx \tau_w = \rho u^{*2} = \varepsilon\frac{du}{dy} = \left[\rho\kappa^2 y^2\left|\frac{du}{dy}\right|\right]\frac{du}{dy}, \quad \text{solve for}\quad \frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$\text{Integrate:}\quad \int du = \frac{u^*}{\kappa}\int\frac{dy}{y}, \quad \text{or:}\quad \mathbf{u = \frac{u^*}{\kappa}\ln(y) + constant} \quad \textit{Ans.}$$

To convert this to the exact form of Eq. (6.28) requires fitting to experimental data

P6.44 Mercury at 20°C flows through 4 meters of 7-mm-diameter glass tubing at an average velocity of 5 m/s. Estimate the head loss in meters and the pressure drop in kPa.

Solution: For mercury at 20°C, take ρ = 13550 kg/m^3 and μ = 0.00156 kg/m·s. Glass tubing is considered hydraulically "smooth," ε/d = 0. Compute the Reynolds number:

$$Re_d = \frac{\rho V d}{\mu} = \frac{13550(5)(0.007)}{0.00156} = 304{,}000; \quad \text{Moody chart smooth:} \;\; f \approx 0.0143$$

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = 0.0143\left(\frac{4.0}{0.007}\right)\frac{5^2}{2(9.81)} = \mathbf{10.4\ m} \quad \textit{Ans.}\text{ (a)}$$

$$\Delta p = \rho g h_f = (13550)(9.81)(10.4) = 1{,}380{,}000\ Pa = \mathbf{1380\ kPa} \quad \textit{Ans.}\text{ (b)}$$

P6.55 The reservoirs in Fig. P6.55 contain water at 20°C. If the pipe is smooth with L = 4500 m and d = 4 cm, what will the flow rate in m^3/h be for Δz = 100 m?

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. The energy equation from surface 1 to surface 2 gives

$$p_1 = p_2 \quad \text{and} \quad V_1 = V_2,$$
$$\text{thus} \quad h_f = z_1 - z_2 = 100\ \text{m}$$

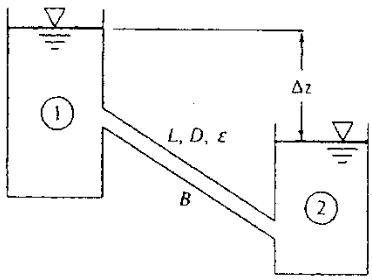

Fig. P6.55

$$\text{Then } 100\ \text{m} = f\left(\frac{4500}{0.04}\right)\frac{V^2}{2(9.81)}, \quad \text{or} \quad fV^2 \approx 0.01744$$

Iterate with an initial guess of $f \approx 0.02$, calculating V and Re and improving the guess:

$$V \approx \left(\frac{0.01744}{0.02}\right)^{1/2} \approx 0.934\ \frac{m}{s}, \quad Re \approx \frac{998(0.934)(0.04)}{0.001} \approx 37300, \quad f_{smooth} \approx 0.0224$$

$$V_{better} \approx \left(\frac{0.01744}{0.0224}\right)^{1/2} \approx 0.883\ \frac{m}{s}, \quad Re_{better} \approx 35300, \quad f_{better} \approx 0.0226, \text{ etc......}$$

Alternately, one could, of course, use Excel. The above process converges to

$f = 0.0227$, $Re = 35000$, $V = 0.877$ m/s, $Q \approx 0.0011\ m^3/s \approx$ **4.0 $\mathbf{m^3/h}$**. *Ans.*