ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2024 – HW7 Solution

P4.94 A long solid cylinder rotates steadily in a very viscous fluid, as in Fig. P4.94. Assuming laminar flow, solve the Navier-Stokes equation in polar coordinates to determine the resulting velocity distribution. The fluid is at rest far from the cylinder. [HINT: the cylinder does not induce any radial motion.]

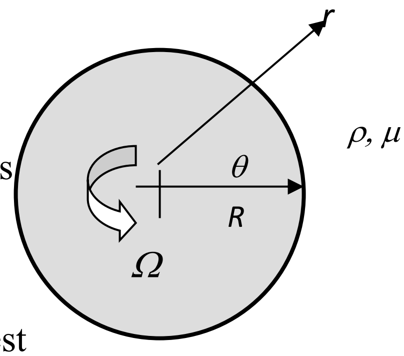


Fig. P4.94

Solution: We already have the useful hint that $v_r = 0$. Continuity then tells us that $(1/r)\partial v_\theta/\partial\theta = 0$, hence v_θ does not vary with θ. Navier-Stokes then yields the flow. From Eq. D.6, the tangential momentum relation, with $\partial p/\partial\theta = 0$ and $v_\theta = f(r)$, we obtain Eq. (4.143):

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_\theta}{dr}\right) = \frac{v_\theta}{r^2}, \quad \text{Solution:} \quad v_\theta = C_1 r + \frac{C_2}{r}$$

$$\text{As } r \to \infty,\ v_\theta \to 0, \quad \text{hence } C_1 = 0$$

$$\text{At } r = R,\ v_\theta = \Omega R = \frac{C_2}{R};\ C_2 = \Omega R^2;\ \text{Finally,}\quad v_\theta = \frac{\Omega R^2}{r} \qquad \textit{Ans.}$$

Rotating a cylinder in a large expanse of fluid sets up (eventually) a *potential vortex flow*.

P5.30 When a large tank of high-pressure ideal gas discharges through a nozzle, the maximum exit mass flow $\dot{m}$ is a function of tank pressure p_o and temperature T_o, gas constant R, specific heat c_p, and nozzle diameter D. Rewrite this as a dimensionless function. Check to see if you can use (p_o, T_o , R, D) as repeating variables.

Solution: Using Table 5.1, write out the dimensions of the six variables:

$\dot{m}$	p_o	T_o	R	D	c_p
$\{MT^{-1}\}$	$\{ML^{-1}T^{-2}\}$	$\{\Theta\}$	$\{L^2T^{-2}\Theta^{-1}\}$	$\{L\}$	$\{L^2T^{-2}\Theta^{-1}\}$

By inspection, we see that (p_o, T_o , R, D) are indeed good repeating variables.
There are two pi groups:

$$\Pi_1 = p_o^a T_o^b R^c c_p^d \dot{m} \quad \text{yields} \quad \Pi_1 = \frac{\dot{m}\sqrt{RT_o}}{p_o D^2}$$

$$\Pi_2 = p_o^a T_o^b R^c c_p^d c_p^1 \quad \text{yields} \quad \Pi_1 = \frac{c_p}{R}$$

$$\text{Thus} \quad \frac{\dot{m}\sqrt{RT_o}}{p_o D^2} = fcn\left(\frac{c_p}{R}\right) \qquad \textit{Ans.}$$

The group $(c_p/R) = k/(k\text{-}1)$, where $k = c_p/c_v$. We usually write the right hand side as $fcn(k)$.

P5.62 For the system of Prob. P5.22, assume that a small model wind turbine of diameter 90 cm, rotating at 1200 r/min, delivers 280 watts when subjected to a wind of 12 m/s. The data is to be used for a prototype of diameter 50 m and winds of 8 m/s. For dynamic similarity, estimate (*a*) the rotation rate, and (*b*) the power delivered by the prototype. Assume sea level air density.

Solution: If you worked Prob. P5.22, you would arrive at two Pi groups, like this:

$$\frac{P}{\rho D^2 V^3} = fcn(\frac{\omega D}{V})$$

Enter the model data to compute these two groups. Take ρ_{air} = 1.22 kg/m³.

$$\frac{P}{\rho D^2 V^3} = \frac{280 N\cdot m/s}{(1.22 kg/m^3)(0.9m)^2(12m/s)^3} = 0.164\ ; \quad \frac{\omega D}{V} = \frac{(20r/s)(0.9m)}{(12m/s)} = 1.5$$

Then, for the prototype,

$$\frac{\omega D}{V} = 1.5 = \frac{\omega(50m)}{8m/s}\ , \quad or: \omega = 0.24 r/s = \mathbf{14.4\ r/min} \quad Ans.(a)$$

$$P = 0.164\rho D^2 V^3 = 0.164(1.22\frac{kg}{m^3})(50m)^2(8\frac{m}{s})^3 = 256{,}000W = \mathbf{256kW} \quad Ans.(b)$$

P5.68 For the rotating-cylinder function of Prob. P5.20, if *L* >> *D*, the problem can be reduced to only two groups, $F/(rU^2LD)$ versus (W*D*/*U*). Here are experimental data for a cylinder 30 cm in diameter and 2 m long, rotating in sea-level air, with *U* = 25 m/s.

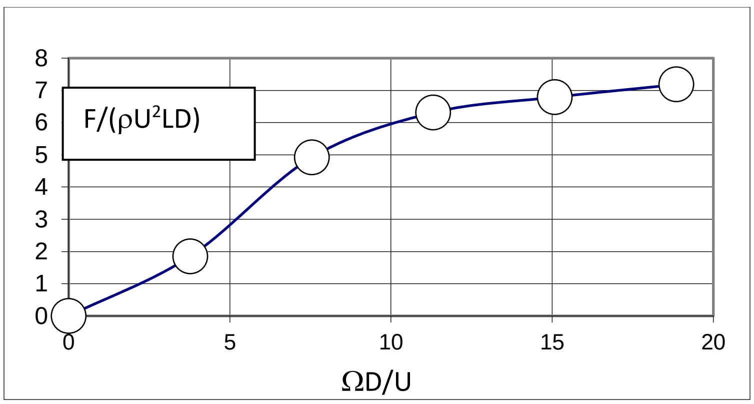
W, rev/min	0	3000	6000	9000	12000	15000
F, N	0	850	2260	2900	3120	3300

(*a*) Reduce this data to the two dimensionless groups and make a plot. (*b*) Use this plot to predict the lift of a cylinder with *D* = 5 cm, *L* = 80 cm, rotating at 3800 rev/min in water at *U* = 4 m/s.

Solution: (*a*) In converting the data, the writer suggests using W in rad/s, not rev/min. For sea-level air, *r* = 1.2255 kg/m^3. Take, for example, the first data point, W = 3000 rpm x (2p/60) = 314 rad/s, and *F* = 850 N.

$$\Pi_1 = \frac{F}{\rho U^2 LD} = \frac{850}{(1.2255)(25)^2(2.0m)(0.3m)} = 1.85 \; ; \; \Pi_2 = \frac{\Omega D}{U} = \frac{(314)(0.3)}{25} = 3.77$$

Do this for the other four data points, and plot as follows. *Ans*.(*a*)



(*b*) For water, take *r* = 998 kg/m^3. The new data are *D* = 5 cm, *L* = 80 cm, 3800 rev/min in water at *U* = 4 m/s. Convert 3800 rev/min = 398 rad/s. Compute the rotation Pi group:

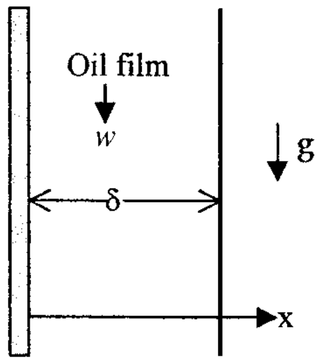
$$\Pi_2 = \frac{\Omega D}{U} = \frac{(398 rad/s)(0.05m)}{4 m/s} = 4.97$$

Read the chart for P_1. The writer reads $P_1 \approx 2.8$. Thus we estimate the water lift force:

$$F = \Pi_1 \rho U^2 LD = (2.8)(998)(4)^2(0.8m)(0.05m) \approx 1788 N \approx \mathbf{1800} N \quad Ans.(b)$$

C5.3 Reconsider the fully-developed drain-ing vertical oil-film problem (see Fig. P4.80) as an exercise in dimensional analysis. Let the vertical velocity be a function only of distance from the plate, fluid properties, gravity, and film thickness. That is, $w = \text{fcn}(x, \rho, \mu, g, \delta)$.

(a) Use the Pi theorem to rewrite this function in terms of dimensionless parameters.
(b) Verify that the exact solution from Prob. 4.80 is consistent with your result in part (a).



Solution: There are $n = 6$ variables and $j = 3$ dimensions (M, L, T), hence we expect only $n - j = 6 - 3 = 3$ Pi groups. The author selects (ρ, g, δ) as repeating variables, whence

$$\Pi_1 = \frac{w}{\sqrt{g\delta}}; \quad \Pi_2 = \frac{\mu}{\rho\sqrt{g\delta^3}}; \quad \Pi_3 = \frac{x}{\delta}$$

Thus the expected function is

$$\frac{w}{\sqrt{g\delta}} = fcn\left(\frac{\mu}{\rho\sqrt{g\delta^3}}, \frac{x}{\delta}\right) \quad \textit{Ans. (a)}$$

(b) The exact solution from Problem 4.80 can be written in just this form:

$$w = \frac{\rho g x}{2\mu}(x - 2\delta), \quad or: \quad \frac{w}{\sqrt{g\delta}}\,\frac{\mu}{\rho\sqrt{g\delta^3}} = \frac{1}{2}\frac{x}{\delta}\left(\frac{x}{\delta} - 2\right)$$

Π_1 Π_2 Π_3

Yes, the two forms of dimensionless function are the same. *Ans.* (b)