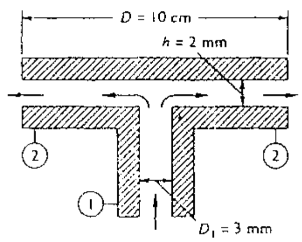
ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2024 – HW4 Solution

P3.20 Oil (SG-0.91) enters the thrust bearing at 250 N/hr and exits radially through the narrow clearance between thrust plates. Compute (a) the outlet volume flow in mL/s, and (b) the average outlet velocity in cm/s.



Solution: The specific weight of the oil is (0.91)(9790) = 8909 N/m^3. Then

$$Q_2 = Q_1 = \frac{250/3600 \text{ N/s}}{8909 \text{ N/m}^3} = 7.8\times10^{-6}\ \frac{\text{m}^3}{\text{s}} = \mathbf{7.8}\ \frac{\textbf{mL}}{\textbf{s}} \quad \textit{Ans.}\text{ (a)}$$

But also $Q_2 = V_2\,\pi(0.1\text{ m})(0.002\text{ m}) = 7.8\times10^{-6}$, solve for $V_2 = \mathbf{1.24}\ \frac{\textbf{cm}}{\textbf{s}}$ *Ans.* (b)

P3.22 The converging-diverging nozzle shown in Fig. P3.22 expands and accelerates dry air to supersonic speeds at the exit, where p_2 = 8 kPa and T_2 = 240 K. At the throat, p_1 = 284 kPa, T_1 = 665 K, and V_1 = 517 m/s. For steady compressible flow of an ideal gas, estimate (a) the mass flow in kg/h, (b) the velocity V_2, and (c) the Mach number Ma_2.

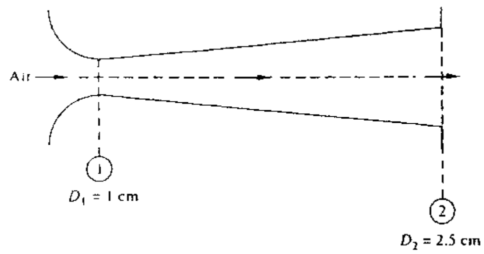


Fig. P3.22

Solution: The mass flow is given by the throat conditions:

$$\dot{m} = \rho_1 A_1 V_1 = \left[\frac{284000}{(287)(665)}\frac{\text{kg}}{\text{m}^3}\right]\frac{\pi}{4}(0.01\text{ m})^2\left(517\ \frac{\text{m}}{\text{s}}\right) = \mathbf{0.0604\ \frac{kg}{s}} \quad \textit{Ans.}\text{ (a)}$$

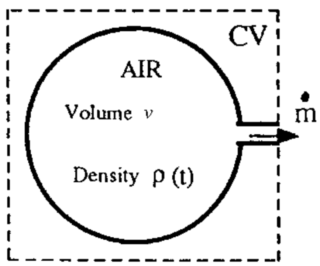
For steady flow, this must equal the mass flow at the exit:

$$0.0604\ \frac{\text{kg}}{\text{s}} = \rho_2 A_2 V_2 = \left[\frac{8000}{287(240)}\right]\frac{\pi}{4}(0.025)^2 V_2, \quad \text{or} \quad V_2 \approx \mathbf{1060\ \frac{m}{s}} \quad \textit{Ans.}\text{ (b)}$$

Recall from Eq. (1.39) that the speed of sound of an ideal gas = $(kRT)^{1/2}$. Then

$$\text{Mach number at exit:} \quad Ma = V_2/a_2 = \frac{1060}{[1.4(287)(240)]^{1/2}} \approx \mathbf{3.41} \quad \textit{Ans.}\text{ (c)}$$

P3.29 In elementary compressible-flow theory (Chap. 9), compressed air will exhaust from a small hole in a tank at the mass flow rate $\dot{m} \approx C\rho$, where ρ is the air density in the tank and C is a constant. If ρ_o is the initial density in a tank of volume v, derive a formula for the density change $\rho(t)$ after the hole is opened. Apply your formula to the following case: a spherical tank of diameter 50 cm, with initial pressure 300 kPa and temperature 100°C, and a hole whose initial exhaust rate is 0.01 kg/s. Find the time required for the tank density to drop by 50 percent.



Solution: For a control volume enclosing the tank and the exit jet, we obtain

$$0 = \frac{d}{dt}\left(\int \rho dv\right) + \dot{m}_{out}, \quad or:\ v\frac{d\rho}{dt} = -\dot{m}_{out} = -C\rho$$

$$\text{or:} \quad \int_{\rho_o}^{\rho} \frac{d\rho}{\rho} = -\frac{C}{v}\int_0^t dt, \quad \text{or:} \quad \frac{\rho}{\rho_o} \approx \mathbf{exp}\left[-\frac{\mathbf{C}}{\boldsymbol{v}}\mathbf{t}\right] \quad \textit{Ans.}$$

Now apply this formula to the given data. If $p_0 = 300$ kPa and $T_0 = 100°C = 373°K$, then $\rho_0 = p/RT = (300{,}000)/[287(373)] \approx 2.80\ kg/m^3$. This establishes the constant "C":

$$\dot{m}_o = C\rho_o = 0.01\ \frac{kg}{s} = C\left(2.80\ \frac{kg}{m^3}\right), \quad \text{or} \quad C \approx 0.00357\ \frac{m^3}{s} \text{ for this hole.}$$

The tank volume is $v = (\pi/6)D^3 = (\pi/6)(0.5\ m)^3 \approx 0.0654\ m^3$. Then we require

$$\rho/\rho_o = 0.5 = \exp\left[-\frac{0.00357}{0.0654}t\right] \quad \text{if} \quad t \approx 13\,s \quad \textit{Ans.}$$

P3.58 The water tank in Fig. P3.58 stands on a frictionless cart and feeds a jet of diameter 4 cm and velocity 8 m/s, which is deflected 60° by a vane. Compute the tension in the supporting cable.

Solution: The CV should surround the tank and wheels and cut through the cable and the exit water jet. Then the horizontal force balance is

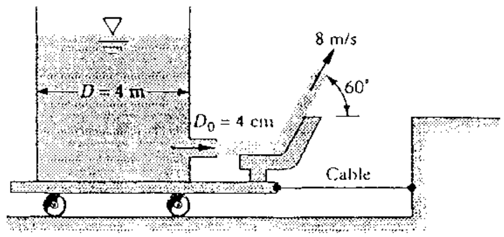
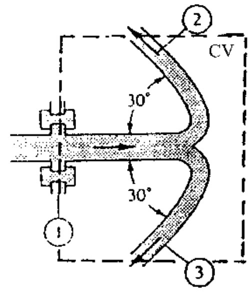


Fig. P3.58

$$\sum F_x = T_{cable} = \dot{m}_{out}u_{out} = (\rho A V_j)V_j\cos\theta = 998\left(\frac{\pi}{4}\right)(0.04)^2(8)^2\cos 60° = \mathbf{40\ N} \quad \textit{Ans.}$$

P3.62 Water at 20°C exits to the standard sea-level atmosphere through the split nozzle in Fig. P3.62. Duct areas are
$A_1 = 0.02$ m^2 and $A_2 = A_3 = 0.008$ m^2. If
$p_1 = 135$ kPa (absolute) and the flow rate is $Q_2 = Q_3 = 275$ m^3/h, compute the force on the flange bolts at section 1



Solution: With the known flow rates, we can compute the various velocities:

$$V_2 = V_3 = \frac{275/3600 \text{ m}^3/\text{s}}{0.008 \text{ m}^2} = 9.55 \ \frac{\text{m}}{\text{s}}; \quad V_1 = \frac{550/3600}{0.02} = 7.64 \ \frac{\text{m}}{\text{s}}$$

The CV encloses the split nozzle and cuts through the flange. The balance of forces is

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 = \rho Q_2(-V_2 \cos 30°) + \rho Q_3(-V_3 \cos 30°) - \rho Q_1(+V_1),$$

or: $$F_{bolts} = 2(998)\left(\frac{275}{3600}\right)(9.55 \cos 30°) + 998\left(\frac{550}{3600}\right)(7.64) + (135000 - 101350)(0.02)$$

$$= 1261 + 1165 + 673 \sim \mathbf{3100N} \ Ans.$$