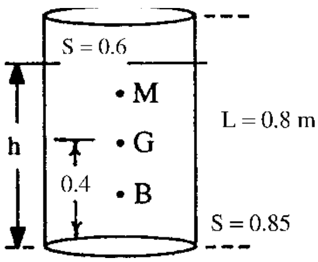
ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2024 – HW3 Solution

P2.130 Consider a wooden cylinder (SG = 0.6) 1 m in diameter and 0.8 m long. Would this cylinder be stable if placed to float with its axis vertical in oil (SG = 0.85)?

Solution: A vertical force balance gives

$$0.85\pi R^2 h = 0.6\pi R^2 (0.8\text{ m}),$$
$$\text{or:}\quad h = 0.565\text{ m}$$



The point B is at h/2 = 0.282 m above the bottom. Use Eq. (2.52) to predict the meta-center location:

$$MB = I_o/\upsilon_{sub} = [\pi(0.5)^4/4]/[\pi(0.5)^2(0.565)] = 0.111\text{ m} = MG + GB$$

Now GB = 0.4 m − 0.282 m = 0.118 m, hence **MG** = 0.111 − 0.118 = **−0.007 m**. This float position is thus **slightly unstable**. The cylinder would turn over. *Ans.*

P2.142 The tank of water in Fig. P2.142 is 12 cm wide into the paper. If the tank is accelerated to the right in rigid-body motion at 6 m/s², compute (a) the water depth at AB, and (b) the water force on panel AB

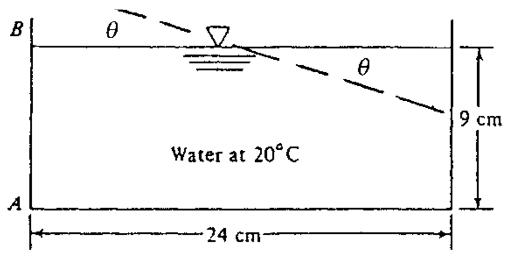


Fig. P2.142

Solution: *From Eq. (2.55),*

$$\tan\theta = a_x/g = \frac{6.0}{9.81} = 0.612, \quad \text{or} \quad \theta \approx 31.45^\circ$$

Then surface point B on the left rises an additional $\Delta z = 12 \tan\theta \approx 7.34$ cm,

$$\text{or:} \quad \text{water depth AB} = 9 + 7.34 \approx \mathbf{16.3\ cm} \quad \textit{Ans. (a)}$$

The water pressure on AB varies linearly due to gravity only, thus the water force is

$$F_{AB} = p_{CG}A_{AB} = (9790)\left(\frac{0.163}{2}\ \text{m}\right)(0.163\ \text{m})(0.12\ \text{m}) \approx \mathbf{15.7\ N} \quad \textit{Ans. (b)}$$

P2.154 A very tall 10-cm-diameter vase contains 1178 cm^3 of water. When spun steadily to achieve rigid-body rotation, a 4-cm-diameter dry spot appears at the bottom of the vase. What is the rotation rate, r/min, for this condition?

Solution: It is interesting that the answer has nothing to do with the water *density*. The value of 1178 cubic centimeters was chosen to make the rest depth a nice number:

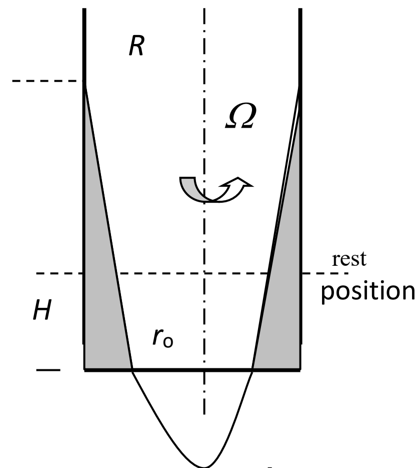


Fig. P2.154

$$\upsilon = 1178 cm^3 = \pi(5cm)^2 H \text{ , solve } H = 15.0cm$$

One way would be to integrate and find the volume of the shaded liquid in Fig. P2.154 in terms of vase radius *R* and dry-spot radius r_o. That would yield the following formula:

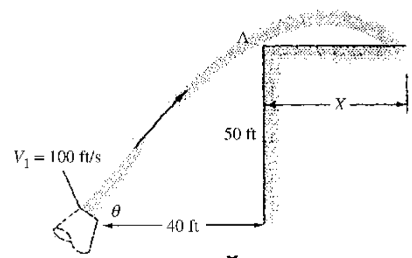
$$d\upsilon = \pi(R^2 - r_o^2)\,dz \text{ , but } z = \Omega^2 r^2 / 2g \text{ , hence } dz = (\Omega^2 r / g)\,dr$$

$$\text{Thus } \upsilon = \int_{r_o}^{R} \pi(R^2 - r_o^2)(\Omega^2 r / g)\,dr = \frac{\pi\Omega^2}{g}\int_{r_o}^{R}(R^2 r - r^3)\,dr = \frac{\pi\Omega^2}{g}\left(\frac{R^2 r^2}{2} - \frac{r^4}{4}\right)\Big|_{r_o}^{R}$$

$$\text{Finally: } \upsilon = \frac{\pi\Omega^2}{g}\left(\frac{R^4}{4} - \frac{R^2 r_o^2}{2} + \frac{r_o^4}{4}\right) = 0.001178\,m^3$$

$$\text{Solve for } R = 0.05m,\ r_o = 0.02m \ :\ \Omega^2 = 3336,\ \Omega = 57.8\frac{rad}{s} = \mathbf{552\frac{r}{min}} \quad Ans.$$

P3.115 A free liquid jet, as in Fig. P3.115, has constant ambient pressure and small losses; hence from Bernoulli's equation $z + V^2/(2g)$ is constant along the jet. For the fire nozzle in the figure, what are (a) the minimum and (b) the maximum values of θ for which the water jet will clear the corner of the building? For which case will the jet velocity be higher when it strikes the roof of the building?



Solution: The two extreme cases are when the jet just touches the corner A of the building. For these two cases, Bernoulli's equation requires that

$$V_1^2 + 2gz_1 = (100)^2 + 2g(0) = V_A^2 + 2gz_A = V_A^2 + 2(32.2)(50), \quad \text{or:} \quad V_A = 82.3\ \frac{ft}{s}$$

The jet moves like a frictionless particle as in elementary particle dynamics:

$$\text{Vertical motion:} \quad z = (V_1 \sin\theta)\,t - \frac{1}{2}gt^2; \quad \text{Horizontal motion:} \quad x = (V_1 \cos\theta)\,t$$

Eliminate "t" between these two and apply the result to point A:

$$z_A = 50 = x_A \tan\theta - \frac{gx_A^2}{2V_1^2\cos^2\theta} = 40\tan\theta - \frac{(32.2)(40)^2}{2(100)^2\cos^2\theta}; \quad \text{clean up and rearrange:}$$

$$\tan\theta = 1.25 + 0.0644\sec^2\theta, \quad \text{solve for } \theta = \mathbf{85.94°} \quad \textit{Ans.}\text{ (a)} \quad \text{and} \quad \mathbf{55.40°} \quad \textit{Ans.}\text{ (b)}$$

Path (b) is shown in the figure, where the jet just grazes the corner A and goes over the top of the roof. Path (a) goes nearly straight up, to z = 155 ft, then falls down to pt. A. In both cases, the velocity when the jet strikes point A is the same, 82.3 ft/s.

P3.124 A necked-down section in a pipe flow, called a *venturi*, develops a low throat pressure which can aspirate fluid upward from a reservoir, as in Fig. P3.124. Using Bernoulli's equation with no losses, derive an expression for the velocity V_1 which is just sufficient to bring reservoir fluid into the throat.

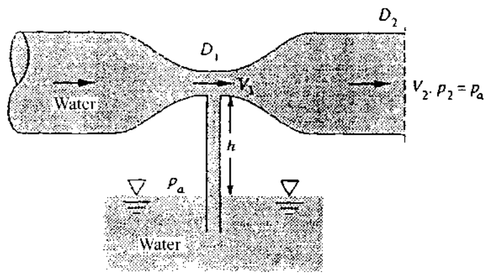


Fig. P3.124

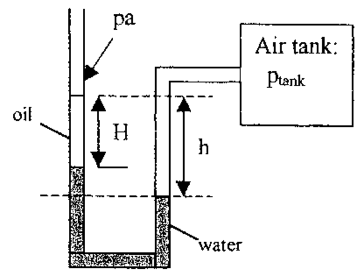
Solution: Water will begin to aspirate into the throat when $p_a - p_1 = \rho gh$. Hence:

$$\text{Volume flow:}\quad V_1 = V_2(D_2/D_1)^2;\quad \text{Bernoulli } (\Delta z = 0):\quad p_1 + \frac{1}{2}\rho V_1^2 \approx p_{atm} + \frac{1}{2}\rho V_2^2$$

$$\text{Solve for}\quad p_a - p_1 = \frac{\rho}{2}(\alpha^4 - 1)V_2^2 \ge \rho gh,\quad \alpha = \frac{D_2}{D_1},\quad \text{or:}\quad \mathbf{V_2 \ge \sqrt{\frac{2gh}{\alpha^4 - 1}}}\quad \textit{Ans.}$$

$$\textit{Similarly,}\quad \mathbf{V_{1,\,min}} = \alpha^2 V_{2,\,min} = \sqrt{\frac{\mathbf{2gh}}{\mathbf{1 - (D_1/D_2)^4}}}\quad \textit{Ans.}$$

C2.2 A prankster has added oil, of specific gravity SGo, to the left leg of the manometer at right. Nevertheless, the U-tube is still to be used to measure the pressure in the air tank. (a) Find an expression for *h* as a function of *H* and other parameters in the problem.
(b) Find the special case of your result when ptank = pa. (c) Suppose H = 5 cm, pa = 101.2 kPa, SGo = 0.85, and ptank is 1.82 kPa higher than pa. Calculate *h* in cm, ignoring surface tension and air density effects.



Solution: Equate pressures at level *i* in the tube (the right hand water level):

$$p_i = p_a + \rho g H + \rho_w g(h - H) = p_{tank},$$

$$\rho = SG_o \rho_w \quad \text{(ignore the column of air in the right leg)}$$

$$\text{Solve for: } \quad \boldsymbol{h = \frac{p_{tk} - p_a}{\rho_w g} + H(1 - SG_o)} \quad \textit{Ans.} \text{ (a)}$$

If ptank = pa, then

$$\boldsymbol{h = H(1 - SG_0)} \quad \textit{Ans.} \text{ (b)}$$

(c) For the particular numerical values given above, the answer to (a) becomes

$$h = \frac{1820\ Pa}{998(9.81)} + 0.05(1 - 0.85) = 0.186 + 0.0075 = 0.193\ m = \mathbf{19.3\ cm} \quad \textit{Ans.} \text{ (c)}$$

Note that this result is not affected by the actual value of atmospheric pressure.