ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2024 – HW1 Solution

P1.21 Aeronautical engineers measure the pitching moment M_o of a wing and then write it in the following form for use in other cases:

$$M_o = \beta V^2 AC\rho$$

where V is the wing velocity, A the wing area, C the wing chord length, and ρ the air density. What are the dimensions of the coefficient β?

Solution: Write out the dimensions of each term in the formula:

$$\{M_o\} = \{FL\} = \left\{\frac{ML^2}{T^2}\right\} = \{\beta V^2 AC\rho\} = \{\beta\}\left\{\frac{L^2}{T^2}\right\}\{L^2\}\{L\}\left\{\frac{M}{L^3}\right\} = \left\{\frac{ML^2}{T^2}\right\}$$

Thus $\{\beta\} = \{\text{unity}\}$ or *dimensionless*. It is proportional to the *moment coefficient* in aerodynamics.

P1.41 An aluminum cylinder weighing 30N, 6cm in diameter and 40cm long, is falling concentrically through a long vertical sleeve of diameter 6.04cm. The clearance is filled with SAE 50 oil at 20°C. Estimate the *terminal* (zero acceleration) fall velocity. Neglect air drag and assume a linear velocity distribution in the oil. [HINT: You are given diameters, not radii.]

Solution: From Table A.3 for SAE 50 oil, $\mu = 0.86$ kg/m-s. The clearance is the difference in *radii*: 3.02 – 3.0cm = 0.02cm = 0.0002m. At terminal velocity, the cylinder weight must balance the viscous drag on the cylinder surface:

$$W = \tau_{wall} A_{wall} = (\mu \frac{V}{C})(\pi DL) \;, \quad \text{where } C = \text{clearance} = r_{sleeve} - r_{cylinder}$$

$$\text{or:} \quad 30\,N = [0.86 \frac{kg}{m-s})(\frac{V}{0.0002\,m})]\; \pi (0.06\,m)(0.40\,m)$$

$$\text{Solve for } V = \mathbf{0.0925}\, m/s \qquad \textit{Ans.}$$