P3.46 When a jet strikes an inclined plate, it breaks into two jets of equal velocity V but unequal fluxes αQ at (2) and $(1-\alpha)Q$ at (3), as shown. Find α, assuming that the tangential force on the plate is zero. Why doesn't the result depend upon the properties of the jet flow?

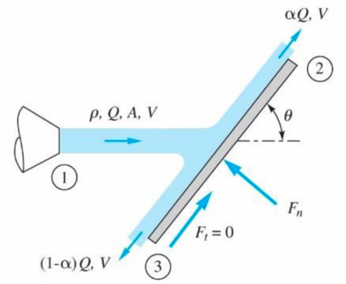

Fig. P3.46

$$0 = \frac{d}{dt}\int_{cv} \rho \, d\forall + \int_{cs} \rho \underline{V}_R \cdot \underline{n} \, dA$$

$$0 = \int_{cs} \rho \underline{V} \cdot \underline{n} \, dA = \Sigma \rho \underline{V}_i \cdot \underline{n} A$$

continuity : $\underset{\dot m_1}{-\rho Q} + \underset{\dot m_2}{\rho \alpha Q} + \underset{\dot m_3}{\rho(1-\alpha) Q} = 0$

$V^2/2g + P/\gamma + z = \text{constant}$

$$-h_1 V + h_2 V + h_3 V = 0 \quad h_1 = h_2 + h_3$$

$$\text{or } h_2 = h_1 - h_3$$

$\frac{V_1^2}{2g} = \frac{V_2^2}{2g} = \frac{V_3^2}{2g}$

Bernoulli

momentum : $\sum_{\text{on } cv} \underline{F} = \frac{d}{dt}\int_{cv} \underline{V} \rho \, d\forall + \int_{cs} \underline{V} \rho \underline{V}_R \cdot \underline{n} \, dA$

$$= \Sigma(\dot m_i \underline{V}_i)_{out} - \Sigma(\dot m_i \underline{V}_i)_{in}$$

$$\Sigma F_t = F_t = 0 = -\dot m_1 V \cos\theta + \dot m_2 V - \dot m_3 V$$

$\dot m$ cancels

$\therefore$ does not

depend ρ

$$0 = -\dot m \cos\theta + \alpha \dot m - (1-\alpha)\dot m$$

$$\rho V h_1 \qquad \rho V h_2 \quad - \rho V h_3 \quad \div \rho V$$

$$0 = -\cos\theta + \alpha - (1-\alpha)$$

$$0 = -\cos\theta + 2\alpha - 1 \Rightarrow \alpha = \tfrac{1}{2}(1+\cos\theta)$$

$$1-\alpha = \tfrac{1}{2}(1-\cos\theta)$$

$$\Sigma F_n = -F_n = -\dot m_1 V \sin\theta$$

$$F_n = \dot m V \sin\theta$$

From F_z equation: $0 = -h_1 \cos\theta + h_2 - h_3$

$= -h_1 \cos\theta + h_1 - h_3 - h_3 \Rightarrow h_3 = \frac{h_1}{2}(1-\cos\theta)$

$h_2 = h_1 - \frac{h_1}{2}(1-\cos\theta)$

$= \frac{h_1}{2}(1+\cos\theta)$

$h_2^2 = \frac{h_1^2}{4}(1+\cos\theta)^2$

$h_3^2 = \frac{h_1^2}{4}(1-\cos\theta)^2$

$(1-\cos\theta)(1-\cos\theta)$

$1-2\cos\theta+\cos^2\theta$

$h_2^2 - h_3^2 = \frac{h_1^2}{4}\left[(1+\cos\theta)^2 - (1-\cos\theta)^2\right]$

$\cancel{1} + 2\cos\theta + \cancel{\cos^2\theta} \cancel{-1} + 2\cos\theta \cancel{-\cos^2\theta}$

$4\cos\theta$

$= h_1^2 \cos\theta$