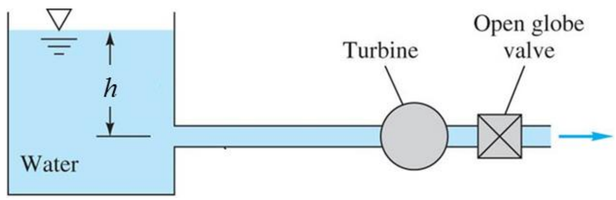
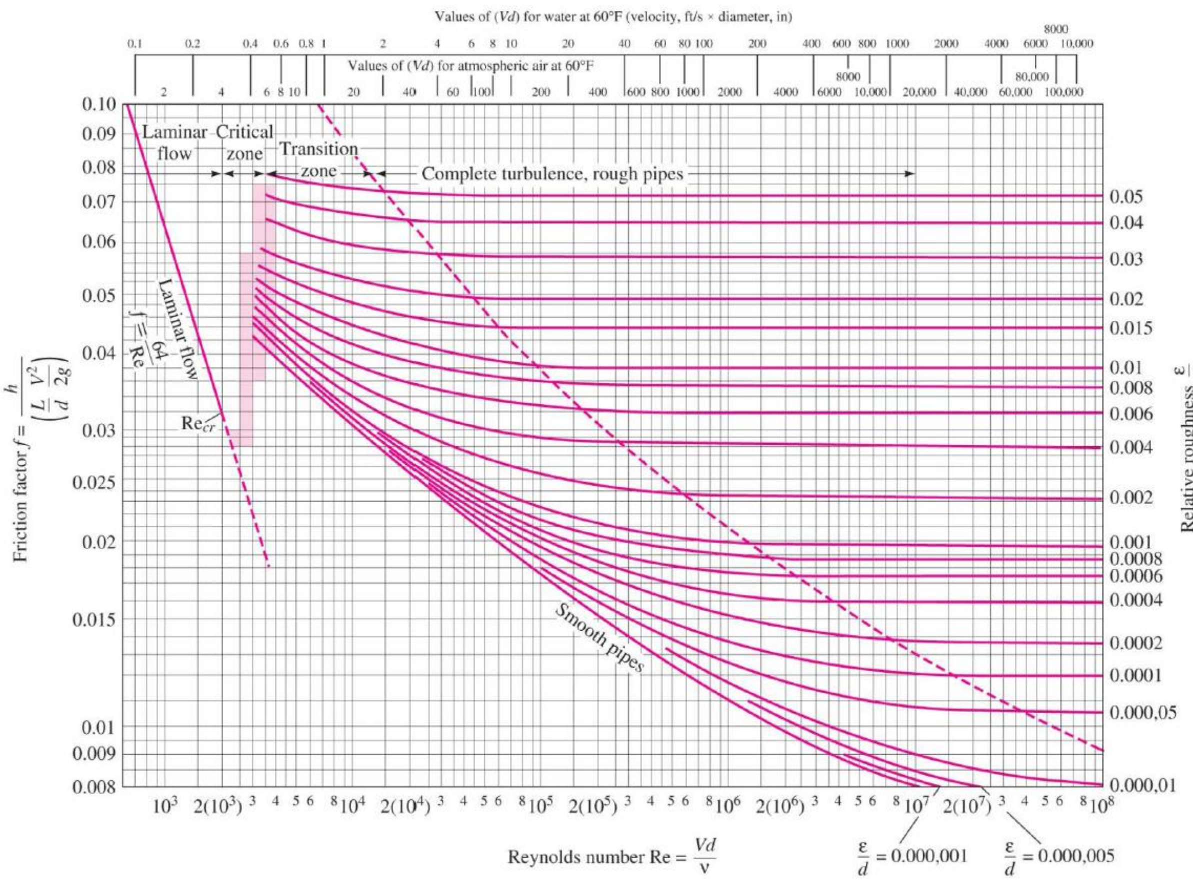
Work session 11/1/23

1. A tank of water with depth h is to be drained by a 5-cm-diameter exit pipe. Water density is 998 kg/m^3, water viscosity is 0.001 kg/ms. The pipe extends out for 15 m and a turbine and an open globe valve are located on the pipe. The head provided by the turbine is h_t = 10 m. (a) If the exit flow rate is Q = 0.04 m^3/s, calculate h assuming there are no minor losses, the turbine is 100% efficient, and the pipe is smooth. (b) Calculate Q if h is same as part (a) but there are minor losses (K = 0.5 for the sharp entrance and K = 6.9 for the open globe valve), the turbine has an efficiency of 80%, and the pipe is rough with ε = 0.3 mm. Use the value of f from part (a) as initial guess and stop at the end of the second iteration.





Solution 1

ANALYSIS:

Energy equation

between free-surface (1) and exit (2):

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_1=\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_2+h_f-h_p+h_t$$

$$p_1=p_2=p_{atm}$$

$$z_1-z_2=h$$

$$V_1=0;\quad h_p=0$$

$$h_f=\frac{V_2^2}{2g}\left(f\frac{L}{D}+\sum K\right)$$

Replace and find *h*:

$$h=\frac{V_2^2}{2g}\left(1+f\frac{L}{D}+\sum K\right)+h_t$$

Find velocity using the flow rate and then *Re*:

$$V_2=\frac{Q}{\frac{\pi}{4}D^2}=\frac{(0.04\ \mathrm{m^3/s})}{\frac{\pi}{4}(0.05\ \mathrm{m})^2}=20.4\ \mathrm{m/s}$$

$$Re=\frac{\rho V_2 D}{\mu}=\frac{(998\ \mathrm{kg/m^3})(20.4\ \mathrm{m/s})(0.05\ \mathrm{m})}{(0.001\ \mathrm{kg/ms})}=1.02\mathrm{E}6\quad (turb.)$$

(a)

Find the friction factor from the moody diagram using *Re*:

$$f_{smooth}\sim 0.011$$

$$h=\frac{V_2^2}{2g}\left(1+f_{smooth}\frac{L}{D}\right)+h_t$$

$$h=\frac{\left(20.4\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{(2)\left(9.81\,\frac{\mathrm{m}}{\mathrm{s^2}}\right)}\left[1+(0.011)\frac{(15\ \mathrm{m})}{(0.05\ \mathrm{m})}\right]+(10\ \mathrm{m})=101\ \mathrm{m}$$

(b)

Consider minor losses, turbine efficiency, and roughness of the pipe with *h* from part (a).

$$h = \frac{V_2^2}{2g}\left(1 + f_{rough}\frac{L}{D} + K_{ent} + K_{valve}\right) + \frac{h_t}{\eta}$$

Find an expression of V_2 as a function of f_{rough}:

$$V_2 = \sqrt{\frac{2g\left(h - \frac{h_t}{\eta}\right)}{1 + f_{rough}\frac{L}{D} + K_{ent} + K_{valve}}} = \sqrt{\frac{(2)(9.81)\left(101 - \frac{10}{0.8}\right)}{1 + f_{rough}\frac{(15)}{(0.05)} + 0.5 + 6.9}}$$

$$V_2 = \sqrt{\frac{1736.37}{300 f_{rough} + 8.4}}$$

Use f_{smooth} as initial guess to compute new velocity and *Re*:

$$V_2 = \sqrt{\frac{1736.37}{300(0.011) + 8.4}} = 12.18\,\frac{\mathrm{m}}{\mathrm{s}} \quad \rightarrow \quad Re = 6.08\mathrm{E}5$$

Find the friction factor from the Moody diagram using *Re* and relative roughness and iterate twice.

$$\frac{\varepsilon}{D} = \frac{(0.0003\ \mathrm{m})}{(0.05\ \mathrm{m})} = 0.006$$

<u>Iteration 1:</u>

$$f_{rough} \sim 0.032$$

$$V_2 = \sqrt{\frac{1736.37}{300(0.032) + 8.4}} = 9.82\ \frac{\mathrm{m}}{\mathrm{s}} \quad \rightarrow \quad Re = 4.90\mathrm{E}5$$

<u>Iteration 2:</u>

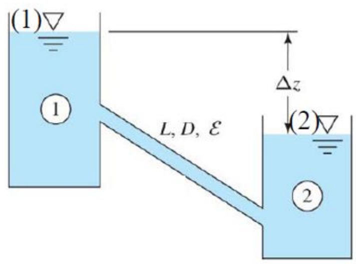
$$f_{rough} \sim 0.032 \text{ (converged)}$$

Compute flow rate:

$$Q = V_2\left(\frac{\pi}{4}D^2\right) = \left(9.82\,\frac{\mathrm{m}}{\mathrm{s}}\right)\frac{\pi}{4}(0.05\ \mathrm{m})^2 = 0.019\ \mathrm{m}^3/\mathrm{s}$$

2. The reservoirs in the Figure below contain water at 20°C (ρ = 998 kg/m3, μ = 0.001 kg/m.s.). Δz = 80 m, L = 185 m, and the pipe is of cast-iron (ε = 0.26 mm). If the pipe diameter is 30mm, calculate flow rate with the unit of {m^3/h}.

Hint: Start with an initial guess of f_0 = 0.038 and perform 2 iterations, assuming the solution is converged at f_2.

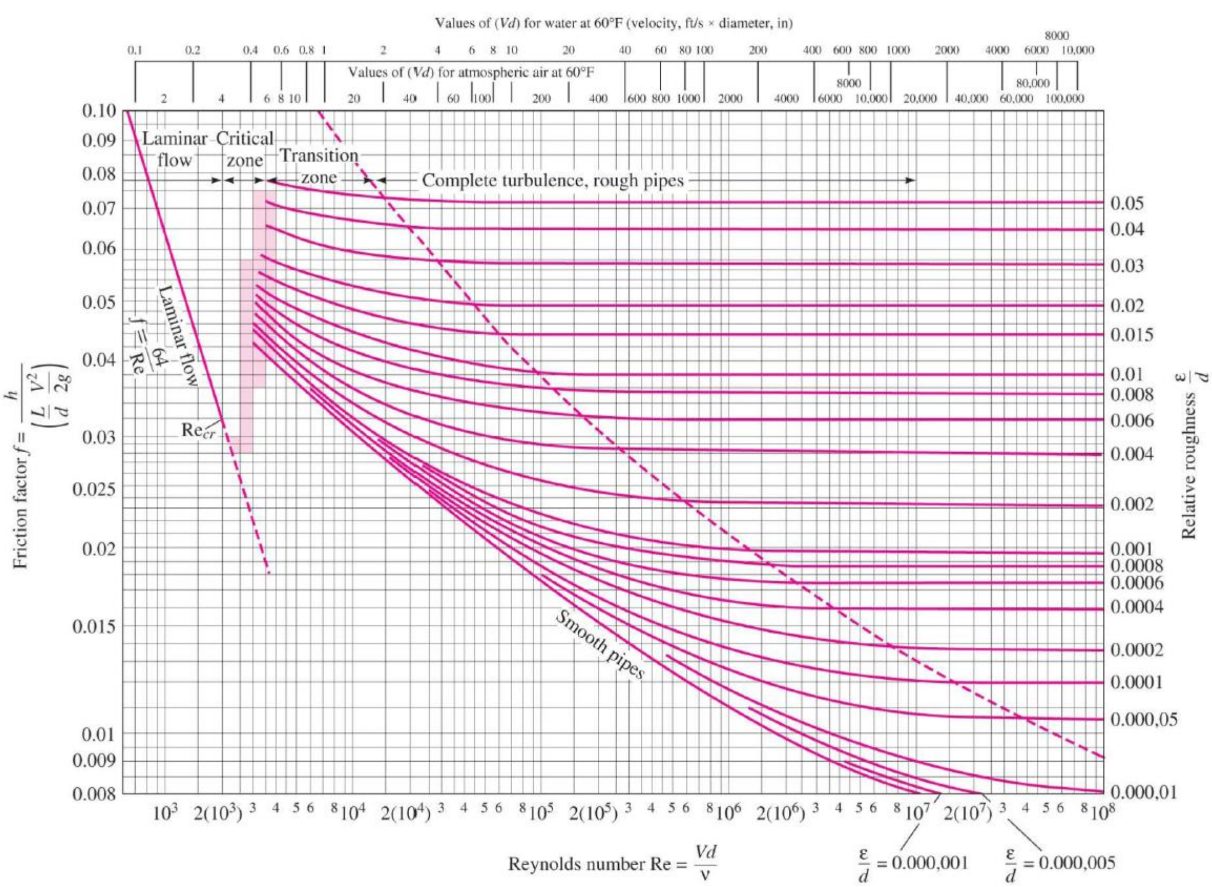


Energy Equation

$$\left(\frac{P}{\rho g}+\frac{V^2}{2g}+z\right)_1=\left(\frac{P}{\rho g}+\frac{V^2}{2g}+z\right)_2+h_f$$

Pipe flow

$$h_f=f\frac{L}{D}\frac{V^2}{2g}=\frac{8fLQ^2}{\pi^2 gD^5};\quad Re_D=\frac{\rho VD}{\mu}=\frac{4\rho Q}{\mu\pi D}$$



Solution 2

KNOWN: Q, Δz, L
FIND: D
ASSUMPTIONS: the pipe flow is turbulent and α ≈1; no minor losses
ANALYSIS:

The energy equation between points (1) and (2) at the free surface yields:

$$\left(\frac{P}{\rho g}+\frac{V^2}{2g}+z\right)_1=\left(\frac{P}{\rho g}+\frac{V^2}{2g}+z\right)_2+h_f$$

$$\frac{P_{atm}}{\rho g}+\frac{0^2}{2g}+z_1=\frac{P_{atm}}{\rho g}+\frac{0^2}{2g}+z_2+h_f \qquad \boxed{+1.0}$$

$$\Delta z=h_f$$

$$\Delta z=\frac{8fLQ^2}{\pi^2 gD^5} \qquad \boxed{+1.0}$$

Rearrange and find D as a function of f:

$$\Delta z=\frac{8fLQ^2}{\pi^2 gD^5}\rightarrow Q^2=\frac{\Delta z\,\pi^2 gD^5}{8fL}\rightarrow Q=\sqrt{\frac{\Delta z\,\pi^2 gD^5}{8L}}\sqrt{\frac{1}{f}} \qquad \boxed{+1.5}$$

Replace numerical values:

$$Q=\sqrt{\frac{(80)\,\pi^2(9.81)(0.03)^5}{8(185)}}\sqrt{\frac{1}{f}}=\frac{3.56617\times10^{-4}}{\sqrt{f}} \qquad \boxed{+1.5}$$

Initial guess: $f_0=0.038$

$$Q=\frac{3.56617\times10^{-4}}{\sqrt{0.038}}=1.829\times10^{-3}\ m^3/s$$

$$Re_D = \frac{4\rho Q}{\mu\pi D} = \frac{4(998)(1.829 \times 10^{-3})}{(0.001)\pi(0.03)} = 77469.9$$

$$\frac{\varepsilon}{D} = \frac{0.26 \times 10^{-3}}{0.03} = 0.0086667$$

From Moody chart; $f_1 = 0.037$ +2.0

One more iteration

$$Q = \frac{3.56617 \times 10^{-4}}{\sqrt{0.037}} = 1.854 \times 10^{-3}\ m^3/s$$

$$Re_D = \frac{4\rho Q}{\mu\pi D} = \frac{4(998)(1.854 \times 10^{-3})}{(0.001)\pi(0.03)} = 78528.8$$

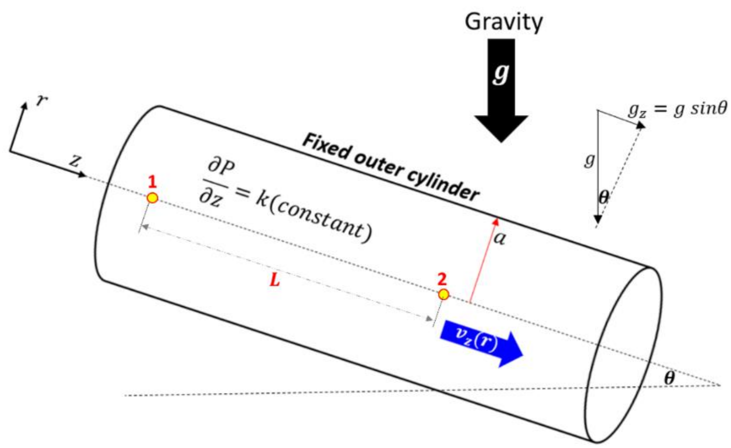
$$\frac{\varepsilon}{D} = \frac{0.26 \times 10^{-3}}{0.03} = 0.0086667$$

From Moody chart; $f_2 = 0.0365$ +2.0

$$Q = \frac{3.56617 \times 10^{-4}}{\sqrt{0.0365}} = 1.8666 \times 10^{-3}\ m^3/s$$

$$\therefore 1.8666 \times 10^{-3} \times 3600 = 6.72 m^3/h$$ +1.0

3. The viscous oil in below Figure is set into steady motion by a constant pressure gradient $\frac{\partial P}{\partial z}$ and gravity. The radius of pipe is a. Assuming fully developed flow, constant density, circumferentially symmetric flow, and a purely axial fluid motion. (a) Simplify the governing equation with these given conditions. (b) Apply appropriate boundary condition and derive the fluid velocity distribution of $v_z(r)$. (c) Calculate wall shear stress at pipe wall. (d) Calculate head loss between point **1** and **2**, and express it with ***L***.



The equations of motion of an incompressible Newtonian fluid with constant density and viscosity in cylindrical coordinates (*r, θ, z*) with velocity components (*vr, vθ, vz*):

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

r-momentum:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = \rho g_r - \frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]$$

θ-momentum:

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = \rho g_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]$$

z-momentum:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

Boundary condition Hint

- At the pipe wall, the velocity is zero
- At the pipe center, the velocity gradient should be zero

Wall shear stress Hint

- $\tau_{wall} = \mu\frac{\partial v_z}{\partial y}\Big|_{y=0} = -\mu\frac{\partial v_z}{\partial r}\Big|_{r=a}$, $y = a - r$ where a: *Radius of pipe*

Flow rate Hint

- $Q = \int_0^{2\pi}\int_0^a v_z\, dr d\theta$

Head loss Hint – Use energy equation

- $\frac{p_1}{\rho g} + \frac{\alpha_1}{2g}V_1 + z_1 = \frac{p_2}{\rho g} + \frac{\alpha_2}{2g}V_2 + z_2 + h_L,\qquad \frac{\partial P}{\partial z} = \frac{P_2 - P_1}{L}$

3. Solution:

ASSUMPTIONS:

1. Steady flow ($\frac{\partial}{\partial t}$=0)
2. Incompressible flow (ρ=constant)
3. Purely axial flow (vr=vθ=0)
4. Circumferentially symmetric flow, so properties do not vary with θ ($\frac{\partial}{\partial \theta}$=0)
5. Constant pressure gradient ($\partial p/\,\partial$z =k)

(a)

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

$$0(3) + 0(3) + \frac{\partial v_z}{\partial z} = 0 \qquad \boxed{+1}$$

z-momentum:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

$$\rho(0(1) + 0(3) + 0(3{,}4) + 0(continuity)) = \rho g sin\theta - k(5) + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + 0(4) + 0(continuity)\right] \qquad \boxed{+1}$$

$$\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = k - \rho g sin\theta \qquad \boxed{+1}$$

(b) Integrate

$$\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = \frac{k - \rho g sin\theta}{\mu}r$$

$$r\frac{\partial v_z}{\partial r} = \frac{k - \rho g sin\theta}{2\mu}r^2 + C_1$$

$$\frac{\partial v_z}{\partial r} = \frac{k - \rho g sin\theta}{2\mu}r + \frac{C_1}{r}$$

$$\therefore v_z(r) = \frac{k - \rho g sin\theta}{4\mu}r^2 + C_1\ln(r) + C_2 \qquad \boxed{+1}$$

Apply two boundary conditions

$$v_z(a) = 0 \;\rightarrow\; \frac{k-\rho g sin\theta}{4\mu}a^2 + C_1\ln(a) + C_2 = 0 \qquad \boxed{+1}$$

$$\left.\frac{\partial v_z}{\partial r}\right|_{r=0} = 0 \;\rightarrow\; \frac{k-\rho g sin\theta}{2\mu}(0) + \frac{C_1}{(0)} = 0 \qquad \boxed{+1}$$

$$\therefore C_1 = 0$$

$$\therefore C_2 = -\frac{k-\rho g sin\theta}{4\mu}a^2$$

Hence,

$$\therefore v_z(r) = \frac{k-\rho g sin\theta}{4\mu}r^2 - \frac{k-\rho g sin\theta}{4\mu}a^2 = \frac{k-\rho g sin\theta}{4\mu}(r^2 - a^2) \qquad \boxed{+0.5}$$

$$= \frac{1}{4\mu}\left(\frac{\partial P}{\partial z} - \rho g sin\theta\right)(r^2 - a^2)$$

(c) Wall shear stress at the pipe wall

$$\tau_{wall} = -\mu\left.\frac{\partial v_z}{\partial r}\right|_{r=a}$$

$$\mu\frac{\partial v_z}{\partial r} = -\frac{k-\rho g sin\theta}{2}r \qquad \boxed{+0.5}$$

Apply

$$r = a$$

$$\therefore \tau_{wall} = -\frac{(k-\rho g sin\theta)a}{2} = -\frac{a}{2}\left(\frac{\partial P}{\partial z} - \rho g sin\theta\right) \qquad \boxed{+1}$$

(d) Head loss between Point 1 and 2.

$$\frac{p_1}{\rho g} + \frac{\alpha_1}{2g} V_1 + z_1 = \frac{p_2}{\rho g} + \frac{\alpha_2}{2g} V_2 + z_2 + h_L$$

Fully developed flow and $\alpha_1 = \alpha_2$. So dynamic pressure terms in both side are cancelled.

$$\frac{p_1}{\rho g} + z_1 = \frac{p_2}{\rho g} + z_2 + h_L$$

$$h_L = \frac{p_1 - p_2}{\rho g} + z_1 - z_2$$

+1

Express $p_1 - p_2$ using pressure gradient

$$\frac{\partial P}{\partial z} = k = \frac{p_2 - p_1}{L}$$

$$p_1 - p_2 = -Lk$$

$$\therefore h_L = -\frac{L}{\rho g}\frac{\partial P}{\partial z} + L sin\theta$$

+1

Alternatively,

$$h_L = \frac{2\tau_w}{\rho g}\frac{L}{a} = -\frac{2L}{\rho g a}\frac{a}{2}\left(\frac{\partial P}{\partial z} - \rho g sin\theta\right) = -\frac{L}{\rho g}\frac{\partial P}{\partial z} + L sin\theta$$

4.

As wind blows over a chimney, vortices are shedding in the wake as shown in the Figure below. The dimensional shedding frequency *f* depends on chimney diameter *D*, chimney length *L*, wind velocity *V*, and air kinematic viscosity *v*. (a) Find dimensionless *f* which depends on dimensionless groups. If a 1/10th scale model were to be tested in a wind tunnel and full dynamic similarity was required: (b) what air velocity would be necessary in the wind tunnel compared to the wind velocity experienced by the full-scale chimney?; (c) what shedding frequency would be observed in the wind tunnel compared to the shedding frequency generated by the full-scale chimney?

Solution 4:

ASSUMPTIONS: the problem is only a function of the above dimensional variables

ANALYSIS:

(a)

$$f = fcn(D, L, V, \nu\,) \;; \quad \text{n= = 5}$$

$$f = \{T^{-1}\} \quad D = \{L\} \quad L = \{L\} \quad V = \{LT^{-1}\} \quad \nu = \{L^2T^{-1}\}\,; \quad \text{j = 2}$$

$$\therefore k = n - j = 3 \qquad \textbf{(0.5)}$$

$$repeating\ variables = D, V$$

$$\pi_1 = fD^{a_1}V^{b_1} = \{(T^{-1})(L)^{a_1}(LT^{-1})^{b_1}\} = \{L^0T^0\}$$

$$a_1 = 1;\ \ b_1 = -1$$

$$\pi_1 = \frac{fD}{V} \qquad \textbf{(1.5)}$$

$$\pi_2 = LD^{a_2}V^{b_2} = \{(L)(L)^{a_2}(LT^{-1})^{b_2}\} = \{L^0T^0\}$$

$$a_2 = -1;\ \ b_2 = 0$$

$$\pi_2 = \frac{L}{D} \qquad \textbf{(1.5)}$$

$$\pi_3 = \nu D^{a_3}V^{b_3} = \{(L^2T^{-1})(L)^{a_3}(LT^{-1})^{b_3}\} = \{L^0T^0\}$$

$$a_3 = -1;\ \ b_3 = -1$$

$$\pi_3 = \frac{\nu}{VD} \qquad \textbf{(1.5)}$$

$$\frac{fD}{V} = fcn\left(\frac{L}{D}, \frac{\nu}{VD}\right)$$

(b)

(2)

$$\frac{\nu_m}{V_m D_m} = \frac{\nu_p}{V_p D_p}, \qquad \frac{V_m}{V_p} = \frac{\nu_m}{\nu_p}\frac{D_p}{D_m}$$

$$\frac{V_m}{V_p} = (1)(10) = 10$$

(c)

$$\frac{f_m D_m}{V_m} = \frac{f_p D_p}{V_p}, \qquad \frac{f_m}{f_p} = \frac{D_p}{D_m}\frac{V_m}{V_p}$$

$$\frac{f_m}{f_p} = (10)(10) = 100$$

(2)