ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2023 – HW11 Solution

P7.6 For the laminar parabolic boundary-layer profile of Eq. (7.6), compute the shape factor "H" and compare with the exact Blasius-theory result, Eq. (7.31).

Solution: Given the profile approximation $u/U \approx 2\eta - \eta^2$, where $\eta = y/\delta$, compute

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy = \delta\int_0^1 (2\eta-\eta^2)(1-2\eta+\eta^2)\,d\eta = \frac{2}{15}\delta$$

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right) dy = \delta\int_0^1 (1-2\eta+\eta^2)\,d\eta = \frac{1}{3}\delta$$

Hence $H = \delta^*/\theta = (\delta/3)/(2\delta/15) \approx$ **2.50** (compared to *2.59* for Blasius solution)

P7.10 Repeat Prob. P7.9, using a trigonometric profile approximation:

$$\frac{u}{U} \approx \sin\left(\frac{\pi y}{2\delta}\right)$$

Does this profile satisfy the conditions of laminar flat plate flow?

Solution: Again carry out the integrations of Sec. 7.2:

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy = \left(\frac{4-\pi}{2\pi}\right)\delta \approx 0.1366\delta$$

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right) dy = \frac{\pi-2}{\pi}\delta \approx 0.3634\delta \qquad \textit{Ans.}$$

$$H = \frac{\delta^*}{\theta} = \frac{2\pi-4}{4-\pi} \approx 3.66$$

This is a good approximation. The velocity profile has $u = 0$ at the wall, $u = U$ at $y = \delta$, and, for $dp/dx = 0$, a flat plate, it has $\partial^2 u/\partial y^2 = 0$ at the wall, as required from momentum.

To find the skin friction and δ, we need to integrate the boundary layer integral relation:

$$\tau_w = \mu\frac{\partial u}{\partial y}\Big|_{y=0} = \frac{\pi}{2}\frac{\mu U}{\delta} = \rho U^2\frac{d\theta}{dx} = 0.1366\rho U^2\frac{d\delta}{dx}$$

Separate the variables and integrate to obtain

$$\delta\, d\delta = \frac{\pi}{2(0.1366)}\frac{dx}{\rho U}\ ;\ \text{Integrate}:\ \delta^2 = \frac{\pi}{(0.1366)}\frac{x}{\rho U} + const$$

Assuming that $\delta = 0$ at $x = 0$, the constant = 0. We obtain the final approximations:

$$\frac{\delta}{x} \approx \frac{4.8}{\sqrt{\mathrm{Re}_x}} \quad and \quad c_f = \frac{\theta}{x} \approx \frac{0.655}{\sqrt{\mathrm{Re}_x}} \qquad Ans.$$

Good accuracy! The sine wave is an excellent approximation to the Blasius profile.

P7.19 Air at 20°C and 1 atm flows at 50 ft/s past a thin flat plate whose area (bL) is 24 ft^2. If the total friction drag is 0.3 lbf, what are the length and width of the plate?

Solution: For air at 20°C and 1 atm, take $\rho = 0.00238$ slug/ft^3 and $\mu = 3.76$E-7 slug/ft-s. Low speed air, not too big a plate: Guess *laminar* flow and check this later. Use Eq. (7.27):

$$C_D = \frac{1.328}{\sqrt{\mathrm{Re}_L}}\ (one\ side)\quad hence\quad F = C_D\frac{\rho}{2}V^2\, 2bL\ ,\quad where\ \ bL = 24\,ft^2$$

$$Apply\ data:\ F = 0.3lbf = \frac{1.328\sqrt{3.76E-7}}{\sqrt{(0.00238)(50)L}}\left(\frac{0.00238}{2}\right)(50)^2\, 2bL$$

$$Solve:\ b\sqrt{L} = 21.36 = \frac{24}{L}\sqrt{L}\ ,\quad or:\ \sqrt{L} = 1.12 \qquad L = \mathbf{1.26}\ ft,\ b = \mathbf{19.0}\ ft\ \ Ans.$$

Check the Reynolds number: Re_L = (0.00238)(50)(1.26)/(3.76E-7) = 399,000. **Laminar**, OK.

P7.27 Air at 20°C and 1 atm flows at 3 m/s past a sharp flat plate 2 m wide and 1 m long. (*a*) What is the wall shear stress at the end of the plate? (*b*) What is the air velocity at a point 4.5 mm normal to the end of the plate? (*c*) What is the total friction drag on the plate?

Solution: For at 20°C and 1 atm, take ρ = 1.2 kg/m3 and μ = 1.8E-5 kg/m-s. Check the Reynolds number to see if the flow is laminar or turbulent:

$$\mathrm{Re}_L = \frac{\rho UL}{\mu} = \frac{(1.2)(3.0)(1.0)}{1.8\mathrm{E}-5} = 200{,}000 \qquad \textbf{Laminar}$$

We can proceed with our laminar-flow formulas:

$$c_{f,x=L} = \frac{0.664}{\sqrt{\mathrm{Re}_L}} = \frac{0.664}{\sqrt{200000}} = 0.00148;\ \tau_w = c_f\frac{\rho}{2}U^2 = (0.00148)(\frac{1.2}{2})(3)^2 = \mathbf{0.0080}\,Pa\ \ Ans.(a)$$

$$\text{At } y = 4.5\,mm,\ \text{the Blasius } \eta = y\sqrt{\frac{U}{\nu x}} = (0.0045m)\sqrt{\frac{3.0}{(1.5E-5)(1.0)}} = 2.01$$

$$\text{Table 7.1: at } \eta = 2.0,\ \text{read } \frac{u}{U} \approx 0.63,\quad \text{hence } u = (0.63)(3.0) \approx \mathbf{1.89}\frac{m}{s}\ \ Ans.(b)$$

Finally, compute the drag for both sides of the plate, $A = 2bL$:

$$C_D = \frac{1.328}{\sqrt{200{,}000}} = 0.00297,$$

$$\text{or}: \quad F = C_D\frac{\rho}{2}U^2(2bL) = (0.00297)(\frac{1.2}{2})(3.0)^2[2(2.0)(1.0)] = \mathbf{0.064}\ N \quad Ans.(c)$$

NOTE: For part (*b*), we never had to compute the boundary layer thickness, $\delta \approx$ 11.2 mm.

P7.33 An alternate analysis of turbulent flat-plate flow was given by Prandtl in 1927, using a wall shear-stress formula from pipe flow

$$\tau_w = 0.0225\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4}$$

Show that this formula can be combined with Eqs. (7.32) and (7.40) to derive the following relations for turbulent flat-plate flow.

$$\frac{\delta}{x} = \frac{0.37}{\mathrm{Re}_x^{1/5}} \qquad c_f = \frac{0.0577}{\mathrm{Re}_x^{1/5}} \qquad C_D = \frac{0.072}{\mathrm{Re}_L^{1/5}}$$

These formulas are limited to Re_x between 5×10^5 and 10^7.

Solution: Use Prandtl's correlation for the left hand side of Eq. (7.32) in the text:

$$\tau_w \approx 0.0225\rho U^2(\nu/U\delta)^{1/4} = \rho U^2\frac{d\theta}{dx} \approx \rho U^2\frac{d}{dx}\left(\frac{7}{72}\delta\right), \quad \text{cancel } \rho U^2 \text{ and rearrange:}$$

$$\delta^{1/4}d\delta = 0.2314(\nu/U)^{1/4}dx, \quad \text{Integrate: } \frac{4}{5}\delta^{5/4} = 0.2314(\nu/U)^{1/4}x$$

Take the $(5/4)^{th}$ root of both sides and rearrange for the final thickness result:

$$\delta \approx 0.37(\nu/U)^{1/5}x^{4/5}, \quad \text{or: } \boldsymbol{\frac{\delta}{x} \approx \frac{0.37}{\mathrm{Re}_x^{1/5}}} \quad \textit{Ans. (a)}$$

$$\text{Substitute } \delta(x) \text{ into } \tau_w\text{: } C_f \approx \frac{2(0.0225)}{(0.37)^{1/4}}\left(\frac{\nu}{Ux}\right)^{1/5}, \quad \text{or } \boldsymbol{C_f \approx \frac{0.0577}{\mathrm{Re}_x^{1/5}}} \quad \textit{Ans. (b)}$$

$$\text{Finally, } C_D = \int_0^1 C_f\, d\left(\frac{x}{L}\right) = \frac{5}{4}C_f(\text{at } x = L) \approx \boldsymbol{\frac{0.072}{\mathrm{Re}_L^{1/5}}} \quad \textit{Ans. (c)}$$