ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2022 – HW9 Solution

P6.39 By analogy with laminar shear, $\tau = \mu\, du/dy$. T. V. Boussinesq in 1877 postulated that turbulent shear could also be related to the mean-velocity gradient $\tau_{turb} = \varepsilon\, du/dy$, where ε is called the *eddy viscosity* and is much larger than μ. If the logarithmic-overlap law, Eq. (6.28), is valid with $\tau \approx \tau_w$, show that $\varepsilon \approx \kappa \rho u^* y$.

Solution: Differentiate the log-law, Eq. (6.28), to find du/dy, then introduce the eddy viscosity into the turbulent stress relation

$$\textit{If}\quad \frac{u}{u^*} = \frac{1}{\kappa}\ln\left(\frac{yu^*}{\nu}\right) + B, \quad \text{then} \quad \frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$\text{Then, if} \quad \tau \approx \tau_w \equiv \rho u^{*2} = \varepsilon\frac{du}{dy} = \varepsilon\frac{u^*}{\kappa y}, \quad \text{solve for} \;\; \varepsilon = \kappa \rho u^* y \quad \textit{Ans.}$$

Note that $\varepsilon/\mu = \kappa y^+$, which is much larger than unity in the overlap region.

P6.40 Theodore von Kármán in 1930 theorized that turbulent shear could be represented by $\tau_{turb} = \varepsilon\, du/dy$ where $\varepsilon = \rho\kappa^2 y^2 |du/dy|$ is called the *mixing-length eddy viscosity* and $\kappa \approx 0.41$ is Kármán's dimensionless *mixing-length constant* [2,3]. Assuming that $\tau_{turb} \approx \tau_w$ near the wall, show that this expression can be integrated to yield the logarithmic-overlap law, Eq. (6.28).

Solution: This is accomplished by straight substitution:

$$\tau_{turb} \approx \tau_w = \rho u^{*2} = \varepsilon\frac{du}{dy} = \left[\rho\kappa^2 y^2\left|\frac{du}{dy}\right|\right]\frac{du}{dy}, \quad \text{solve for} \quad \frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$\text{Integrate:} \quad \int du = \frac{u^*}{\kappa}\int\frac{dy}{y}, \quad \text{or:} \quad \mathbf{u = \frac{u^*}{\kappa}\ln(y) + constant} \quad \textit{Ans.}$$

To convert this to the exact form of Eq. (6.28) requires fitting to experimental data

P6.45 Oil, SG = 0.88 and $\nu = 4E{-}5\ m^2/s$, flows at 400 gal/min through a 6-inch asphalted cast-iron pipe. The pipe is 0.5 miles long (2640 ft) and slopes upward at 8° in the flow direction. Compute the head loss in feet and the pressure change.

Solution: First convert 400 gal/min = 0.891 ft^3/s and $\nu = 0.000431\ ft^2/s$. For asphalted cast-iron, $\varepsilon = 0.0004$ ft, hence $\varepsilon/d = 0.0004/0.5 = 0.0008$. Compute V, Red, and f:

$$V = \frac{0.891}{\pi(0.25)^2} = 4.54\ \frac{ft}{s};\quad Re_d = \frac{4.54(0.5)}{0.000431} = 5271;\quad \text{calculate}\quad f_{Moody} = 0.0377$$

$$\text{then}\quad h_f = f\frac{L}{d}\frac{V^2}{2g} = 0.0377\left(\frac{2640}{0.5}\right)\frac{(4.54)^2}{2(32.2)} = \mathbf{63.8\ ft}\quad \textit{Ans.}\ \text{(a)}$$

If the pipe slopes upward at 8°, the pressure drop must balance both friction and gravity:

$$\Delta p = \rho g(h_f + \Delta z) = 0.88(62.4)[63.8 + 2640\sin 8°] = \mathbf{23700}\ \frac{\mathbf{lbf}}{\mathbf{ft^2}}\quad \textit{Ans.}\ \text{(b)}$$

P6.61 What level h must be maintained in Fig. P6.61 to deliver a flow rate of 0.015 ft^3/s through the $\frac{1}{2}$-in commercial-steel pipe?

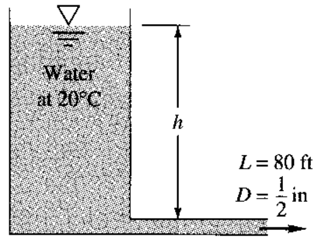

Fig. P6.61

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For commercial steel, take $\varepsilon \approx 0.00015$ ft, or $\varepsilon/d = 0.00015/(0.5/12) \approx 0.0036$. Compute

$$V = \frac{Q}{A} = \frac{0.015}{(\pi/4)(0.5/12)^2} = 11.0\ \frac{ft}{s};$$

$$Re = \frac{\rho V d}{\mu} = \frac{1.94(11.0)(0.5/12)}{2.09E{-}5} \approx 42500 \quad \varepsilon/d = 0.0036, \quad f_{Moody} \approx 0.0301$$

The energy equation, with $p_1 = p_2$ and $V_1 \approx 0$, yields an expression for surface elevation:

$$h = h_f + \frac{V^2}{2g} = \frac{V^2}{2g}\left(1 + f\frac{L}{d}\right) = \frac{(11.0)^2}{2(32.2)}\left[1 + 0.0301\left(\frac{80}{0.5/12}\right)\right] \approx \mathbf{111\ ft} \quad \textit{Ans.}$$