ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2023 – HW8 Solution

***P5.60** The thrust F of a free propeller, either aircraft or marine, depends upon density ρ, the rotation rate n in r/s, the diameter D, and the forward velocity V. Viscous effects are slight and neglected here. Tests of a 25-cm-diameter model aircraft propeller, in a sea-level wind tunnel, yield the following thrust data at a velocity of 20 m/s:

Rotation rate, r/min	4800	6000	8000
Measured thrust, N	6.1	19	47

(*a*) Use this data to make a crude but effective dimensionless plot. (b) Use the dimensionless data to predict the thrust, in newtons, of a similar 1.6-m-diameter prototype propeller when rotating at 3800 r/min and flying at 225 mi/h at 4000 m standard altitude.

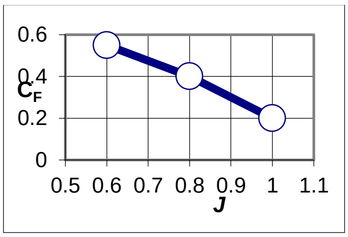
Solution: The given function is $F = \text{fcn}(\rho, n, D, V)$, and we note that $j = 3$. Hence we expect 2 pi groups. The writer chose (ρ, n, D) as repeating variables and found this:

$$C_F = fcn(J), \quad where \quad C_F = \frac{F}{\rho n^2 D^4} \quad and \quad J = \frac{V}{nD}$$

The quantity C_F is called the *thrust coefficient*, while J is called the *advance ratio*. Now use the data (at $\rho = 1.2255$ kg/m^3) to fill out a new table showing the two pi groups:

n, r/s	133.3	100.0	80.0
C_F	0.55	0.40	0.20
J	0.60	0.80	1.00

A crude but effective plot of this data is as follows. *Ans.*(*a*)



(*b*) At 4000 m altitude, from Table A.6, $\rho = 0.8191$ kg/m^3. Convert 225 mi/h = 101.6 m/s. Convert 3800 r/min = 63.3 r/s. Then find the prototype advance ratio:

$$J = (101.6 \text{ m/s})/[(63.3 \text{ r/s})(1.6 \text{ m})] = 1.00$$

Well, lucky us, that's our third data point! Therefore $C_{F,prototype} \approx 0.20$. And the thrust Is:

$$F_{prototype} = C_F \rho n^2 D^4 = (0.20)(0.8191\frac{kg}{m^3})(63.3\frac{r}{s})^2(1.6m)^4 \approx \mathbf{4300\ N} \quad Ans.(b)$$

***P5.76** A 2-ft-long model of a ship is tested in a freshwater tow tank. The measured drag may be split into "friction" drag (Reynolds scaling) and "wave" drag (Froude scaling). The model data are as follows:

Tow speed, ft/s:	0.8	1.6	2.4	3.2	4.0	4.8
Friction drag, lbf:	0.016	0.057	0.122	0.208	0.315	0.441
Wave drag, lbf:	0.002	0.021	0.083	0.253	0.509	0.697

The prototype ship is 150 ft long. Estimate its total drag when cruising at 15 kn in seawater at 20°C.

Solution: For fresh water at 20°C, take $\rho = 1.94$ slug/ft³, $\mu = 2.09\text{E-5}$ slug/ft·s. Then evaluate the Reynolds numbers and the Froude numbers and respective force coefficients:

V_m, ft/s:	0.8	1.6	2.4	3.2	4.0	4.8
$Re_m = V_m L_m/\nu$:	143000	297000	446000	594000	743000	892000
$C_{F,friction}$:	0.00322	0.00287	0.00273	0.00261	0.00254	0.00247
$Fr_m = V_m/\sqrt{(gL_m)}$:	0.099	0.199	0.299	0.399	0.498	0.598
$C_{F,\,wave}$:	0.00040	0.00106	0.00186	0.00318	0.00410	0.00390

For seawater, take $\rho = 1.99$ slug/ft³, $\mu = 2.23\text{E-5}$ slug/ft·s. With L_p=150 ft and V_p =15 knots = 25.3 ft/s, evaluate

$$Re_{proto} = \frac{\rho_p V_p L_p}{\mu_p} = \frac{1.99(25.3)(150)}{2.23E\text{–}5} \approx 3.39E8; \quad Fr_p = \frac{25.3}{[32.2(150)]^{1/2}} \approx 0.364$$

$$\text{For} \quad Fr \approx 0.364, \quad \text{interpolate to } C_{F,wave} \approx 0.0027$$

Thus we can immediately estimate $F_{wave} \approx 0.0027(1.99)(25.3)^2(150)^2 \approx \underline{77000}$ lbf. However, as mentioned in Fig. 5.8 of the text, **Re_p is far outside the range of the friction force data**, therefore we must *extrapolate* as best we can. A power-law curve-fit is

$$C_{F,friction} \approx \frac{0.0178}{Re^{0.144}}, \quad \text{hence } C_{F,proto} \approx \frac{0.0178}{(3.39E8)^{0.144}} \approx 0.00105$$

Thus $F_{friction} = 0.00105(1.99)(25.3)^2(150)^2 \approx \underline{30000}$ lbf. **F_{total} ≈ 107000 lbf**. *Ans.*

$$\frac{Q_m}{Q_p} = \frac{V_m}{V_p}\frac{A_m}{A_p} = \sqrt{\frac{L_m}{L_p}}\left(\frac{L_m}{L_p}\right)^2 = \left(\frac{L_m}{L_p}\right)^{5/2} \quad \text{or} \quad \alpha^{5/2}$$

$$Fig. 5/9: \alpha = 1:65; \quad \therefore\ Q_{model} = \left(10100\frac{ft^3}{s}\right)\left(\frac{1}{65}\right)^{5/2} = 0.30\frac{ft^3}{s} \quad Ans.$$

P6.4 For flow of SAE 30W oil through a 5-cm-diameter pipe, from Fig. A.1, for what flow rate in m^3/h would we expect transition to turbulence at (a) 20°C and (b) 100°C?

Solution: For SAE 30W oil take $\rho = 891\ \text{kg/m}^3$ and take $\mu = 0.29$ kg/m·s at 20°C (Table A.3) and 0.01 kg/m-s at 100°C (Fig A.1). Write the critical Reynolds number in terms of flow rate Q:

$$\text{(a)}\ \ \text{Re}_{crit} = 2300 = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4(891\ kg/m^3)Q}{\pi(0.29\ kg/m{\cdot}s)(0.05\ m)},$$

$$\text{solve} \quad Q = 0.0293\ \frac{m^3}{s} = \mathbf{106}\ \frac{\mathbf{m^3}}{\mathbf{h}} \quad \textit{Ans.}\ \text{(a)}$$

$$\text{(b)}\ \ \text{Re}_{crit} = 2300 = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4(891\ kg/m^3)Q}{\pi(0.010\ kg/m{\cdot}s)(0.05\ m)},$$

$$\text{solve} \quad Q = 0.00101\ \frac{m^3}{s} = \mathbf{3.6}\ \frac{\mathbf{m^3}}{\mathbf{h}} \quad \textit{Ans.}\ \text{(b)}$$

P6.24 Two tanks of water at 20°C are connected by a capillary tube 4 mm in diameter and 3.5 m long. The surface of tank 1 is 30 cm higher than the surface of tank 2. (a) Estimate the flow rate in m^3/h. Is the flow laminar? (b) For what tube diameter will Re_d be 500?

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. (a) Both tank surfaces are at atmospheric pressure and have negligible velocity. The energy equation, when neglecting minor losses, reduces to:

$$\Delta z = 0.3\ m = h_f = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.001\ kg/m{\cdot}s)(3.5\ m)Q}{\pi(998\ kg/m^3)(9.81\ m/s^2)(0.004\ m)^4}$$

$$\text{Solve for} \quad Q = 5.3E\text{–}6\ \frac{m^3}{s} = \mathbf{0.019\ \frac{m^3}{h}} \quad \textit{Ans.}\text{ (a)}$$

Check $\quad Re_d = 4\rho Q/(\pi\mu d) = 4(998)(5.3\text{E–}6)/[\pi(0.001)(0.004)]$

$\mathbf{Re_d = 1675}$ ***laminar.*** *Ans.* (a)

(b) If $Re_d = 500 = 4\rho Q/(\pi\mu d)$ and $\Delta z = h_f$, we can solve for both Q and d:

$$Re_d = 500 = \frac{4(998\ kg/m^3)Q}{\pi(0.001\ kg/m{\cdot}s)d}, \quad \textit{or} \quad Q = 0.000394d$$

$$h_f = 0.3\ m = \frac{128(0.001\ kg/m{\cdot}s)(3.5\ m)Q}{\pi(998\ kg/m^3)(9.81\ m/s^2)d^4}, \quad or \quad Q = 20600d^4$$

Combine these two to solve for $\quad Q = 1.05E\text{–}6\ m^3/s \quad$ *and* $\quad$ **d = 2.67 mm** $\quad$ *Ans.* (b)

P6.29 SAE 30W oil at 20°C flows through a straight pipe 25 m long, with diameter 4 cm. The average velocity is 2 m/s. (*a*) Is the flow laminar? Calculate (*b*) the pressure drop; and (*c*) the power required. (*d*) If the pipe diameter is doubled, for the same average velocity, by what percent does the required power increase?

Solution: For SAE 30W oil at 20°C, Table A.3, ρ = 891 kg/m^3, and μ = 0.29 kg/m-s. (a) We have enough information to calculate the Reynolds number:

$$\mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{(891)(2.0)(0.04)}{0.29} = \mathbf{246} < 2300 \qquad \textbf{Yes, laminar flow} \quad Ans.(a)$$

(*b*, c) The pressure drop and power follow from the laminar formulas of Eq. (6.12):

$$\Delta p = \frac{32\mu L V}{D^2} = \frac{32(0.29)(25)(2.0)}{(0.04)^2} = \mathbf{290{,}000}\ Pa \qquad Ans.(b)$$

$$Q = \frac{\pi}{4}D^2 V = \frac{\pi}{4}(0.04)^2(2.0) = 0.00251\ \frac{m^3}{s}$$

$$Power = Q\,\Delta p = (0.00251\frac{m^3}{2})(290{,}000\,Pa) = \mathbf{729}\ W \quad Ans.(c)$$

(*d*) If *D* doubles to 8 cm and *V* remains the same at 2.0 m/s, the new pressure drop will be 72,500 Pa, and the new flow rate will be Q = 0.01005 m^3/s, hence the new power will be

$$P = Q\,\Delta p = (0.01005)(72{,}500) = \mathbf{729}\ \mathrm{W} \qquad \textit{\textbf{Zero percent change}}!$$

This is because D^2 cancels in the product $P = Q\,\Delta p = 8\pi\mu L V^2$. *Ans*. (*d*)
NOTE: The flow is still laminar, Re_D = 492.

C5.4 The Taco Inc. Model 4013 centrifugal pump has an impeller of diameter D = 12.95 in. When pumping 20°C water at Ω = 1160 rev/min, the measured flow rate Q and pressure rise Δp are given by the manufacturer as follows:

Q (gal/min)	~	200	300	400	500	600	700
Δp (psi)	~	36	35	34	32	29	23

(a) Assuming that Δp = fcn(ρ, Q, D, Ω), use the Pi theorem to rewrite this function in terms of dimensionless parameters and then plot the given data in dimensionless form. (b) It is desired to use the same pump, running at 900 rev/min, to pump 20°C gasoline at 400 gal/min. According to your dimensionless correlation, what pressure rise Δp is expected, in lbf/in^2?

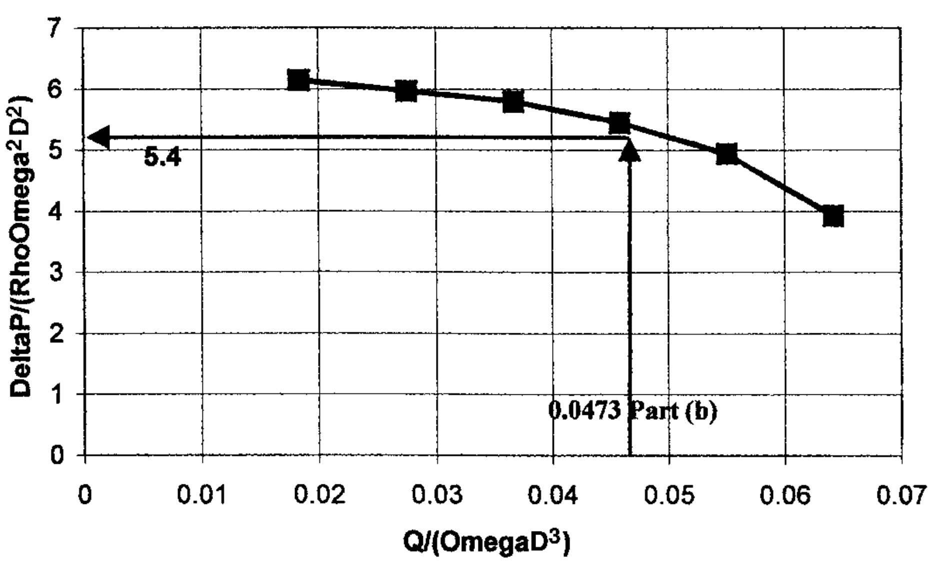
Solution: There are $n = 5$ variables and $j = 3$ dimensions (M, L, T), hence we expect $n - j = 5 - 3 = 2$ Pi groups. The author selects (ρ, D, Ω) as repeating variables, whence

$$\Pi_1 = \frac{\Delta p}{\rho\Omega^2 D^2};\quad \Pi_2 = \frac{Q}{\Omega D^3},\quad or:\quad \frac{\Delta p}{\rho\Omega^2 D^2} = fcn\left(\frac{Q}{\Omega D^3}\right)\quad \textit{Ans. (a)}$$

Convert the data to this form, using Ω = 19.33 rev/s, D = 1.079 ft, ρ = 1.94 slug/ft^3, and use Δp in lbf/ft^2, not psi, and Q in ft^3/s, not gal/min:

Q (gal/min)	~	200	300	400	500	600	700
$\Delta p/(\rho\Omega^2 D^2)$:		6.14	5.97	5.80	5.46	4.95	3.92
$Q/(\Omega D^3)$:		0.0183	0.0275	0.0367	0.0458	0.0550	0.0642

The dimensionless plot of Π1 versus Π2 is shown below.



(b) The dimensionless chart above is valid for the new conditions, also. Convert 400 gal/min to 0.891 ft^3/s and Ω = 900 rev/min to 15 rev/s. Then evaluate Π_2:

$$\Pi_2 = \frac{Q}{\Omega D^3} = \frac{0.891}{15(1.079)^3} = \mathbf{0.0473}$$

This value is entered in the chart above, from which we see that the corresponding value of Π_1 is about **5.4**. For gasoline (Table A-3), ρ = 1.32 $slug/ft^3$. Then this new running condition with gasoline corresponds to

$$\Pi_2 = 5.4 = \frac{\Delta p}{\rho\Omega^2 D^2} = \frac{\Delta p}{1.32(15)^2(1.079)^2}, \quad \textit{solve for } \ \Delta p = 1870 \ \frac{lbf}{ft^2} = \mathbf{13} \ \frac{\mathbf{lbf}}{\mathbf{in^2}} \quad \textit{Ans.}\ \text{(b)}$$