ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2023 – HW6 Solution

P3.150 In Prob. 3.60 find the torque caused around flange 1 if the center point of exit 2 is 1.2 m directly below the flange center.

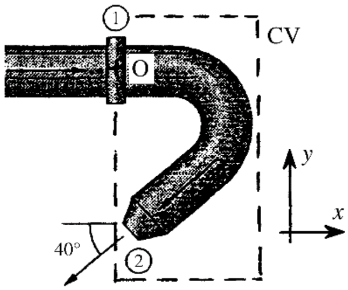


Fig. P3.60

Solution: The CV encloses the elbow and cuts through flange (1). Recall from Prob. 3.60 that $D_1 = 10$ cm, $D_2 = 3$ cm, weight flow = 150 N/s, whence $V_1 = 1.95$ m/s and $V_2 = 21.7$ m/s. Let "O" be in the center of flange (1). Then $\mathbf{r}_{02} = -1.2\mathbf{j}$ and $\mathbf{r}_{01} = 0$. The pressure at (1) passes through O, thus causes no torque. The moment relation is:

$$\sum M_O = \mathbf{T}_O = \dot{m}[(\mathbf{r}_{O2} \times \mathbf{V}_2) - (\mathbf{r}_{O1} \times \mathbf{V}_1)] = \left(\frac{150}{9.81}\ \frac{\text{kg}}{\text{s}}\right)[(-1.2\mathbf{j}) \times (-16.6\mathbf{i} - 13.9\mathbf{j})]$$

$$\text{or:} \quad \mathbf{T}_O = -305\,\mathbf{k}\ \text{N}\cdot\text{m} \quad \textit{Ans.}$$

***P4.79** Study the combined effect of the two viscous flows in Fig. 4.12. That is, find *u*(*y*) when the upper plate moves at speed *V* and there is also a constant pressure gradient (*dp/dx*). Is superposition possible? If so, explain why. Plot representative velocity profiles for (a) zero, (b) positive, and (c) negative pressure gradients for the same upper-wall speed *V*.

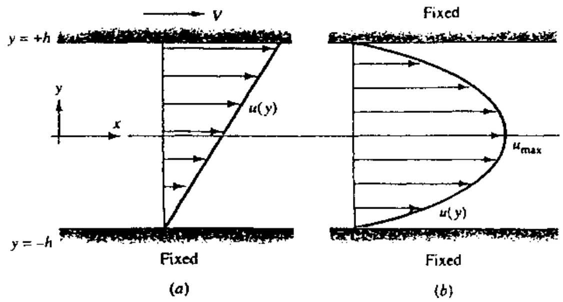


Fig. 4.16

Solution:

$$\overset{\text{Fully developed}}{\cancel{\frac{\partial u}{\partial x}}} + \overset{\text{Continuity}}{\cancel{\frac{\partial v}{\partial y}}} + \overset{\text{2D}}{\cancel{\frac{\partial w}{\partial z}}} = 0$$

$$\rho\left(\overset{\text{Steady}}{\cancel{\frac{\partial u}{\partial t}}} + u\overset{\text{Fully developed}}{\cancel{\frac{\partial u}{\partial x}}} + v\cancel{\frac{\partial u}{\partial y}} + \underset{\text{2D}}{\cancel{w}}\frac{\partial u}{\partial z}\right) = \underset{\text{Neglect gravity}}{\cancel{\rho g_x}} - \frac{\partial p}{\partial x} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \underset{\text{2D}}{\cancel{\frac{\partial^2 u}{\partial z^2}}}\right) =$$

$$\therefore 0 = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right)$$

$$\Rightarrow \mu\frac{d^2 u}{dy^2} = \frac{\partial p}{\partial x}$$

Integrate twice:

$$u = \left(\frac{1}{\mu}\frac{\partial p}{\partial x}\right)\frac{y^2}{2} + C_1 y + C_2$$

Boundary conditions:

$$\text{at } y = -h\text{: } u(-h) = 0$$

$$\text{at } y = +h\text{: } u(+h) = V$$

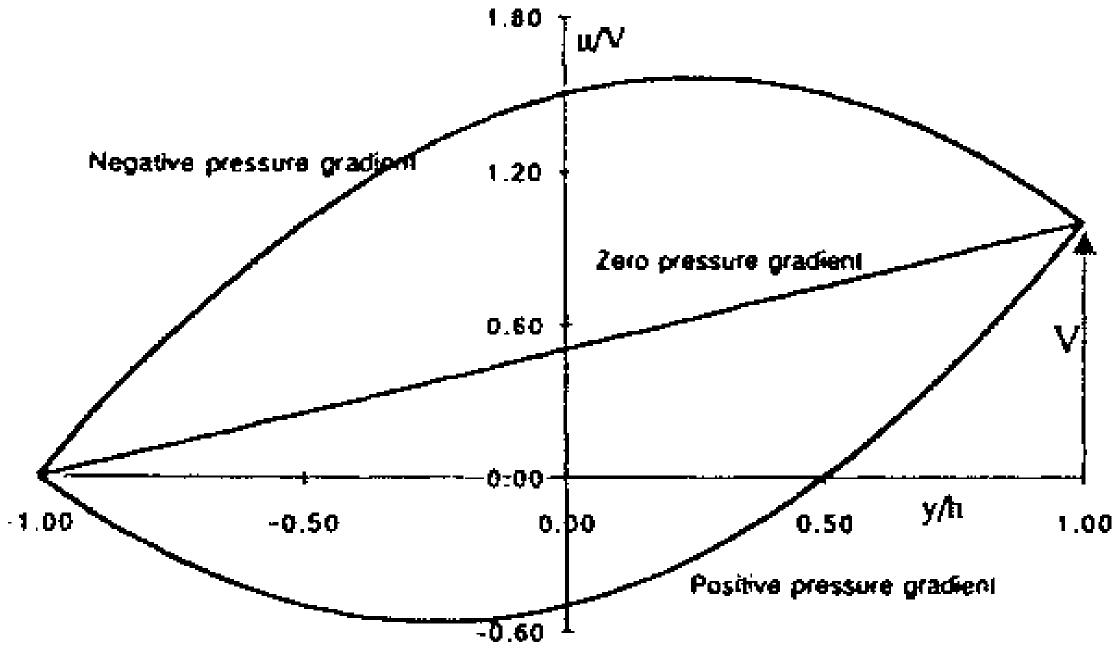
$$\therefore \quad C_1 = \frac{V}{2h}, \quad C_2 = \frac{V}{2} - \left(\frac{1}{\mu}\frac{\partial p}{\partial x}\right)\frac{h^2}{2}$$

Combined solution is

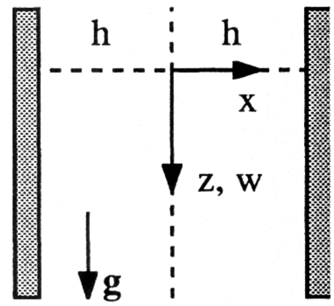
$$\therefore \quad u = \left(\frac{1}{\mu}\frac{\partial p}{\partial x}\right)\frac{y^2}{2} + \frac{V}{2h}y + \frac{V}{2} - \left(\frac{1}{\mu}\frac{\partial p}{\partial x}\right)\frac{h^2}{2}$$

$$u = \frac{V}{2}\left(1+\frac{y}{h}\right) + \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\left(1-\frac{y^2}{h^2}\right)$$

The superposition is quite valid because the convective acceleration is zero, hence what remains is linear: $\nabla p = \mu\nabla^2\mathbf{V}$. Three representative velocity profiles are plotted at right for various (dp/dx).



***P4.37** A viscous liquid of constant density and viscosity falls due to gravity between two parallel plates a distance 2h apart, as in the figure. The flow is fully developed, that is, w = w(x) only. There are no pressure gradients, only gravity. Set up and solve the Navier-Stokes equation for the velocity profile w(x).



Solution: Only the z-component of Navier-Stokes is relevant

$$\rho\frac{dw}{dt} = 0 = \rho g + \mu\frac{d^2w}{dx^2},\ \ or:\ w'' = -\frac{\rho g}{\mu}, \qquad w(-h) = w(+h) = 0$$

The solution is very similar to Eqs. (4.142) to (4.143) of the text:

$$w = \frac{\rho \mathbf{g}}{2\mu}(\mathbf{h}^2 - \mathbf{x}^2) \quad \textit{Ans.}$$

P4.88 The viscous oil in Fig. P4.88 is set into steady motion by a concentric inner cylinder moving axially at velocity *U* inside a fixed outer cylinder. Assuming constant pressure and density and a purely axial fluid motion, solve Eqs. (4.38) for the fluid velocity distribution $v_z(r)$. What are the proper boundary conditions?

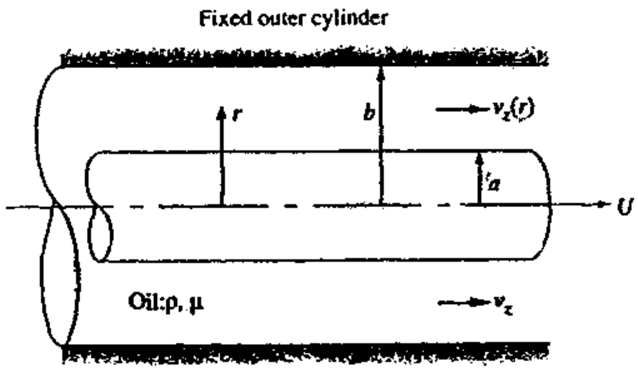


Fig. P4.88

Solution: If vz = fcn(r) only, the *z*-momentum equation (Appendix E) reduces to:

$$\rho\frac{dv_z}{dt} = -\frac{\partial p}{\partial z} + \rho g_z + \mu\nabla^2 v_z, \quad \text{or:} \quad 0 = 0 + 0 + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)$$

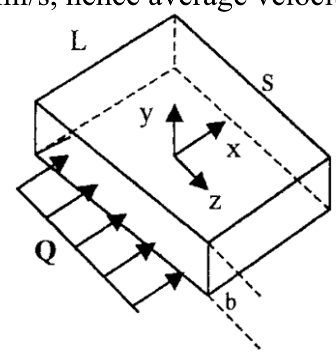
The solution is $v_z = C_1 \ln(r) + C_2$, subject to $v_z(a) = U$ and $v_z(b) = 0$

Solve for $C_1 = U/\ln(a/b)$ and $C_2 = -C_1 \ln(b)$

The final solution is: $\mathbf{v_z = U\dfrac{\ln(r/b)}{\ln(a/b)}}$ *Ans.*

C4.1 In a certain medical application, water at room temperature and pressure flows through a rectangular channel of length L = 10 cm, width s = 1 cm, and gap thickness b = 0.3 mm. The volume flow is sinusoidal, with amplitude Q_o = 0.5 ml/s and frequency f = 20 Hz, that is, $Q = Q_o\sin(2\pi f\, t)$.
(a) Calculate the maximum Reynolds number Re = Vb/ν, based on maximum average velocity and gap thickness. Channel flow remains *laminar* for Re < 2000, otherwise it will be *turbulent*. Is this flow laminar or turbulent?
(b) Assume quasi-steady flow, that is, solve as if the flow were steady at any given Q(t). Find an expression for streamwise velocity u as a function of y, μ, dp/dx, and b, where dp/dx is the pressure gradient required to drive the flow through the channel at flow rate Q. Also estimate the maximum magnitude of velocity component u.
(c) Find an analytic expression for flow rate Q(t) as a function of dp/dx.
(d) Estimate the wall shear stress τ_w as a function of Q, f, μ, b, s, and time t.
(e) Finally, use the given numbers to estimate the wall shear amplitude, τ_{wo}, in Pa.

Solution: (a) Maximum flow rate is the amplitude, Q_o = 0.5 ml/s, hence average velocity V = Q/A:



$$V = \frac{Q}{bs} = \frac{0.5E\text{–}6\ m^3/s}{(0.0003\,m)(0.01\,m)} = 0.167\,m/s$$

$$\mathrm{Re}_{\max} = \frac{Vb}{\nu} = \frac{(0.167)(0.0003)}{(0.001/998)} = \mathbf{50\ (laminar)} \quad Ans.\ (a)$$

(b, c) The quasi-steady analysis is just like Eqs. (4.137-4.139) of the text, with "h" = b/2:

$$\boldsymbol{u = \frac{-1}{2\mu}\frac{dp}{dx}\left(\frac{b^2}{4} - y^2\right), \quad u_{\max} = \frac{-1}{2\mu}\frac{dp}{dx}\frac{b^2}{4}, \quad Q_{\max} = \frac{2}{3}u_{\max}bs = \frac{-sb^3}{12\mu}\frac{dp}{dx}} \quad Ans.\ (b, c)$$

$$\text{(d) Wall shear:} \quad \tau_{wall} = \mu\left|\frac{du}{dy}\right|_{wall} = \boldsymbol{\frac{b}{2}\frac{dp}{dx} = \frac{6\mu Q}{sb^2} = \frac{6\mu Q_o}{sb^2}\sin(2\pi f\,t)} \quad Ans.\ (d)$$

(e) For our given numerical values, the amplitude of wall shear stress is:

$$\tau_{wo} = \frac{6\mu Q_o}{sb^2} = \frac{6(0.001)(0.5E\text{–}6)}{(0.01)(0.0003)^2} = \boldsymbol{3.3\ Pa} \quad Ans.\ (e)$$