ME:5160 (58:160) Intermediate Mechanics of Fluids
Fall 2023 – HW5 Solution

P3.60 Water at 20°C flows through the elbow in Fig. P3.60 and exits to the atmo-sphere. The pipe diameter is $D_1 = 10$ cm, while $D_2 = 3$ cm. At a weight flow rate of 150 N/s, the pressure $p_1 = 2.3$ atm (gage). Neglecting the weight of water and elbow, estimate the force on the flange bolts at section 1.

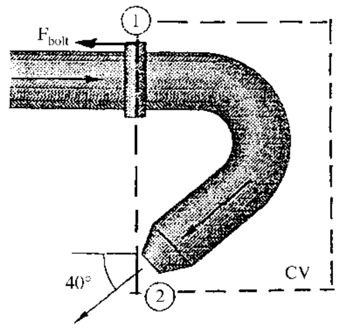


Fig. P3.60

Solution: First, from the weight flow, compute Q = (150 N/s)/(9790 N/m^3) = 0.0153 m^3/s. Then the velocities at (1) and (2) follow from the known areas:

$$V_1 = \frac{Q}{A_1} = \frac{0.0153}{(\pi/4)(0.1)^2} = 1.95\ \frac{m}{s}; \qquad V_2 = \frac{Q}{A_2} = \frac{0.0153}{(\pi/4)(0.03)^2} = 21.7\ \frac{m}{s}$$

The mass flow is $\rho A_1 V_1 = (998)(\pi/4)(0.1)^2(1.95) \approx 15.25$ kg/s. Then the balance of forces in the *x*-direction is:

$$\sum F_x = -F_{bolts} + p_1 A_1 = \dot{m}u_2 - \dot{m}u_1 = \dot{m}(-V_2 \cos 40° - V_1)$$

solve for $F_{bolts} = (2.3 \times 101350)\frac{\pi}{4}(0.1)^2 + 15.25(21.7 \cos 40° + 1.95) \approx \mathbf{2100\ N}$ *Ans.*

P3.88 The boat in Fig. P3.88 is jet-propelled by a pump which develops a volume flow rate Q and ejects water out the stern at velocity Vj. If the boat drag force is $F = kV^2$, where k is a constant, develop a formula for the steady forward speed V of the boat.

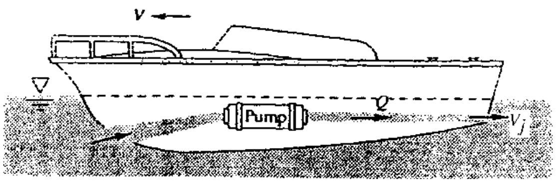


Fig. P3.88

Solution: Let the CV move to the left at boat speed V and enclose the boat and the pump's inlet and exit. Assume that the inlet forms an angle θ with the horizontal direction.

The inlet relative velocity is: $V_{r_i} = V_{inlet} + V$

The jet relative velocity is: $V_{r_o} = V_j + V$

Continuity gives:

$$A_{inlet}(V_{inlet}\cos\theta + V) = A_j\left(V_j + V\right) = \dot{m}_{pump}$$

Momentum balance in the x-direction gives:

$$\sum F_x = kV^2 = \dot{m}_{pump}\left(\underline{V_{r_o}}\cdot\underline{n_o} + \underline{V_{r_i}}\cdot\underline{n_i}\right)$$

$$\underline{V_{r_o}}\cdot\underline{n_o} = V_j + V \qquad \underline{V_{r_i}}\cdot\underline{n_i} = -(V_{inlet} + V\cos\theta)$$

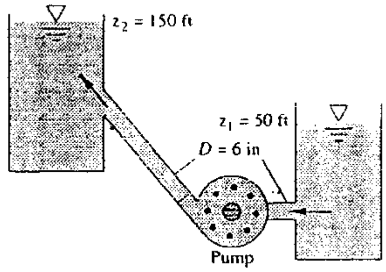
$$kV^2 = \dot{m}_{pump}\left(V_j + V - V_{inlet}\cos\theta - V\right) = \dot{m}_{pump}\left(V_j - V_{inlet}\cos\theta\right)$$

Formula for V:

$$V = \frac{1}{k}\sqrt{\dot{m}_{pump}\left(V_j - V_{inlet}\cos\theta\right)}$$

If $V_{inlet}\cos\theta \ll V_j \rightarrow\ V = \frac{1}{k}\sqrt{\dot{m}_{pump}V_j}$

P3.180 Water at 20°C is pumped at 1500 gal/ min from the lower to the upper reservoir, as in Fig. P3.180. Pipe friction losses are approximated by $h_f \approx 27V^2/(2g)$, where V is the average velocity in the pipe. If the pump is 75 percent efficient, what horse-power is needed to drive it?



Solution: First evaluate the average velocity in the pipe and the friction head loss:

$$Q = \frac{1500}{448.8} = 3.34\ \frac{ft^3}{s},\quad \text{so}\ V = \frac{Q}{A} = \frac{3.34}{\pi(3/12)^2} = 17.0\ \frac{ft}{s}\quad \text{and}\quad h_f = 27\frac{(17.0)^2}{2(32.2)} \approx \mathbf{121\ ft}$$

Then apply the steady flow energy equation:

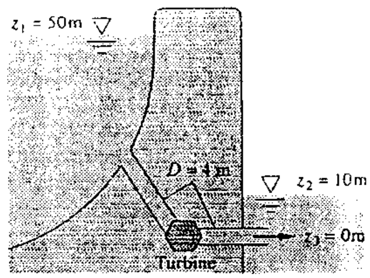
$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:}\quad 0 + 0 + 50 = 0 + 0 + 150 + 121 - h_p$$

$$\text{Thus}\ h_p = 221\ \text{ft},\quad \text{so}\ P_{pump} = \frac{\gamma Q h_p}{\eta} = \frac{(62.4)(3.34)(221)}{0.75}$$

$$= 61600\ \frac{ft\cdot lbf}{s} \approx \mathbf{112\ hp}\quad \textit{Ans.}$$

P3.184 The large turbine in Fig. P3.184 diverts the river flow under a dam as shown. System friction losses are $h_f = 3.5V^2/(2g)$, where V is the average velocity in the supply pipe. For what river flow rate in m^3/s will the power extracted be 25 MW? Which of the *two* possible solutions has a better "conversion efficiency"?



Solution: The flow rate is the unknown, with the turbine power known:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f + h_{turb}, \quad \text{or:} \quad 0 + 0 + 50 = 0 + 0 + 10 + h_f + h_{turb}$$

$$\text{where} \quad h_f = 3.5V_{pipe}^2/(2g) \quad \text{and} \quad h_p = P_p/(\gamma Q) \quad \text{and} \quad V_{pipe} = \frac{Q}{(\pi/4)D_{pipe}^2}$$

Introduce the given numerical data (e.g. Dpipe = 4 m, Ppump = 25E6 W) and solve:

$$Q^3 - 35410Q + 2.261E6 = 0, \quad \text{with roots } Q = \mathbf{+76.5}, \mathbf{+137.9}, \text{ and } -214.4 \text{ m}^3/\text{s}$$

The *negative* Q is nonsense. The large Q ($137.9 m^3/s$) gives large friction loss, hf ≈ 21.5 m. The smaller Q ($76.5 m^3/s$) gives hf ≈ 6.6 m, about right.
Select Qriver ≈ **76.5 m³/s**. *Ans.*

P4.2 Flow through the converging nozzle in Fig. P4.2 can be approximated by the one-dimensional velocity distribution

$$u \approx V_o\left(1+\frac{2x}{L}\right) \quad \upsilon \approx 0 \quad w \approx 0$$

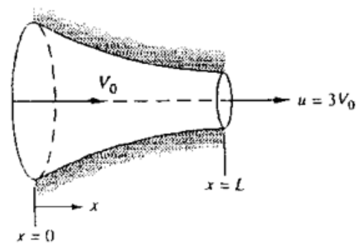


Fig. P4.2

(a) Find a general expression for the fluid acceleration in the nozzle. (b) For the specific case *V*o = 10 ft/s and *L* = 6 in, compute the acceleration, in *g*'s, at the entrance and at the exit.

Solution: Here we have only the single 'one-dimensional' convective acceleration:

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} = \left[V_o\left(1+\frac{2x}{L}\right)\right]\frac{2V_o}{L} = \mathbf{\frac{2V_o^2}{L}\left(1+\frac{2x}{L}\right)} \quad \textit{Ans.}\ \text{(a)}$$

$$For\ L = 6'' \ \ and\ \ V_o = 10\ \frac{ft}{s},\quad \frac{du}{dt} = \frac{2(10)^2}{6/12}\left(1+\frac{2x}{6/12}\right) = 400(1+4x),\ \ with\ x\ in\ feet$$

At x = 0, du/dt = **400** ft/s^2 (12 g's); at x = L = 0.5 ft, du/dt = **1200** ft/s^2 (37 g's). *Ans.* (b)

P4.27 A frictionless, incompressible steady-flow field is given by

$$\mathbf{V} = 2xy\mathbf{i} - y^2\mathbf{j}$$

in arbitrary units. Let the density be ρ_o = constant and neglect gravity. Find an expression for the pressure gradient in the *x* direction.

Solution: For this (gravity-free) velocity, the momentum equation is

$$\rho\left(u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}\right) = -\nabla p, \quad \text{or:} \quad \rho_o[(2xy)(2y\mathbf{i}) + (-y^2)(2x\mathbf{i} - 2y\mathbf{j})] = -\nabla p$$

$$\text{Solve for} \quad \nabla p = -\rho_o(2xy^2\mathbf{i} + 2y^3\mathbf{j}), \quad \text{or:} \quad \boldsymbol{\frac{\partial p}{\partial x} = -\rho_o 2xy^2} \quad \textit{Ans.}$$