# ME:5160 (58:160) Intermediate Mechanics of Fluids

# Fall 2023 – HW4 Solution

**P3.18** Gasoline enters Section 1 in Fig. P3.18 at 0.5 m$^3$/s. It leaves Section 2 at an average velocity of 12 m/s. What is the average velocity at Section 3? Is it in or out?

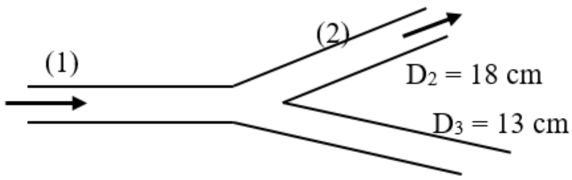



Fig. P3.18

**Solution**: Given $Q_1$ = 0.5 m$^3$/s, evaluate

$$Q_2 = \frac{\pi}{4} D_2^2 V_2 = \frac{\pi}{4}(0.18m)^2 (12\frac{m}{s}) = 0.305\, m^3/s$$

Then

$$Q_3 = Q_1 - Q_2 = 0.5 - 0.305 = +0.195\, m^3/s = (\pi/4)(0.13m)^2 V_3, \text{ solve } V_3 = \mathbf{14.7}\, m/s \ \ \textbf{\textit{out}} \ \ Ans.$$

**P3.22** The converging-diverging nozzle shown in Fig. P3.22 expands and accelerates dry air to supersonic speeds at the exit, where $p_2 = 8$ kPa and $T_2 = 240$ K. At the throat, $p_1 = 284$ kPa, $T_1 = 665$ K, and $V_1 = 517$ m/s. For steady compressible flow of an ideal gas, estimate (a) the mass flow in kg/h, (b) the velocity $V_2$, and (c) the Mach number $Ma_2$.

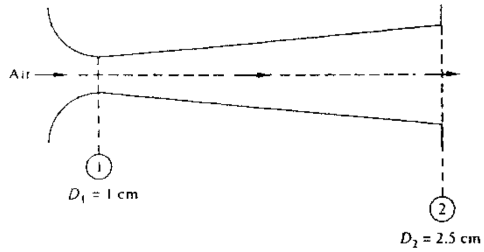



**Fig. P3.22**

**Solution:** The mass flow is given by the throat conditions:

$$\dot{m} = \rho_1 A_1 V_1 = \left[\frac{284000}{(287)(665)}\frac{\text{kg}}{\text{m}^3}\right]\frac{\pi}{4}(0.01\text{ m})^2\left(517\ \frac{\text{m}}{\text{s}}\right) = \mathbf{0.0604\ \frac{kg}{s}} \quad \textit{Ans.}\text{ (a)}$$

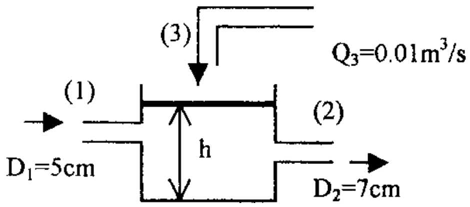
For steady flow, this must equal the mass flow at the exit:

$$0.0604\ \frac{\text{kg}}{\text{s}} = \rho_2 A_2 V_2 = \left[\frac{8000}{287(240)}\right]\frac{\pi}{4}(0.025)^2 V_2, \quad \text{or} \quad V_2 \approx \mathbf{1060\ \frac{m}{s}} \quad \textit{Ans.}\text{ (b)}$$

Recall from Eq. (1.39) that the speed of sound of an ideal gas = $(kRT)^{1/2}$. Then

$$\text{Mach number at exit:} \quad Ma = V_2/a_2 = \frac{1060}{[1.4(287)(240)]^{1/2}} \approx \mathbf{3.41} \quad \textit{Ans.}\text{ (c)}$$

**P3.14** The open tank in the figure contains water at 20°C. For incompressible flow, (a) derive an analytic expression for *dh/dt* in terms of ($Q_1$, $Q_2$, $Q_3$). (b) If *h* is constant, determine $V_2$ for the given data if $V_1 = 3$ m/s and $Q_3 = 0.01$ m³/s.




**Solution:** For a control volume enclosing the tank,

$$\frac{d}{dt}\left(\int_{CV} \rho\, d\upsilon\right) + \rho(Q_2 - Q_1 - Q_3) = \rho\frac{\pi d^2}{4}\frac{dh}{dt} + \rho(Q_2 - Q_1 - Q_3),$$

$$solve \quad \mathbf{\frac{dh}{dt} = \frac{Q_1 + Q_3 - Q_2}{(\pi d^2/4)}} \quad \textit{Ans. (a)}$$

If *h* is constant, then

$$Q_2 = Q_1 + Q_3 = 0.01 + \frac{\pi}{4}(0.05)^2(3.0) = 0.0159 = \frac{\pi}{4}(0.07)^2 V_2,$$

$$solve \quad \mathbf{V_2 = 4.13\ m/s} \quad \textit{Ans. (b)}$$

**P3.54** For the pipe-flow reducing section of Fig. P3.54, D1 = 8 cm, D2 = 5 cm, and p2 = 1 atm. All fluids are at 20°C. If V1 = 5 m/s and the manometer reading is *h* = 58 cm, estimate the total horizontal force resisted by the flange bolts.

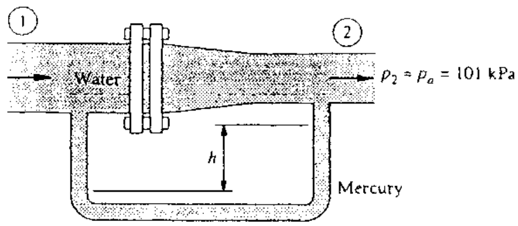



**Fig. P3.54**

**Solution:** Let the CV cut through the bolts and through section 2. For the given manometer reading, we may compute the upstream pressure:

$$p_1 - p_2 = (\gamma_{merc} - \gamma_{water})h = (132800 - 9790)(0.58\text{ m}) \approx 71300\text{ Pa (gage)}$$

Now apply conservation of mass to determine the exit velocity:

$$Q_1 = Q_2, \quad \text{or} \quad (5\text{ m/s})(\pi/4)(0.08\text{ m})^2 = V_2(\pi/4)(0.05)^2, \quad \text{solve for } V_2 \approx 12.8\text{ m/s}$$

Finally, write the balance of horizontal forces:

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 = \dot{m}(V_2 - V_1),$$

$$\text{or:} \quad F_{bolts} = (71300)\frac{\pi}{4}(0.08)^2 - (998)\frac{\pi}{4}(0.08)^2(5.0)[12.8 - 5.0] \approx \mathbf{163\ N} \quad \textit{Ans.}$$

**P3.61** A 20°C water jet strikes a vane on a tank with frictionless wheels, as shown. The jet turns and falls into the tank without spilling. If $\theta = 30°$, estimate the horizontal force F needed to hold the tank stationary.

**Solution:** The CV surrounds the tank and wheels and cuts through the jet, as shown. We should *assume that the splashing into the tank does not increase the x-momentum of the water in the tank*. Then we can write the CV horizontal force relation:

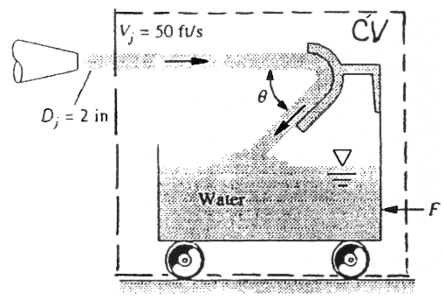



**Fig. P3.61**

$$\sum F_x = -F = \frac{d}{dt}\left(\int u\rho\, d\upsilon\right)_{tank} - \dot{m}_{in}u_{in} = 0 - \dot{m}V_{jet} \quad \text{independent of } \theta$$

$$\text{Thus} \quad F = \rho A_j V_j^2 = \left(1.94\ \frac{slug}{ft^3}\right)\frac{\pi}{4}\left(\frac{2}{12}\ ft\right)^2\left(50\ \frac{ft}{s}\right)^2 \approx \mathbf{106\ lbf} \quad \textit{Ans.}$$