ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2023 – HW3 Solution

P2.129 The iceberg of Prob. P2.128 may become unstable if its width decreases. Suppose that the height is L and the depth into the paper is L but the width decreases to H < L. Again with S = 0.88 for the iceberg, determine the ratio H/L for which the iceberg becomes unstable.

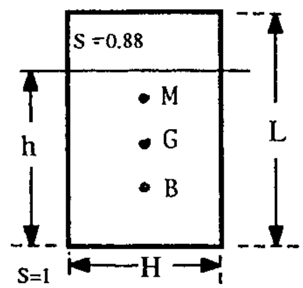


Fig. P2.129

Solution: As in Prob. P2.128, the submerged distance h = SL = 0.88L, with G at L/2 above the bottom and B at h/2 above the bottom. From Eq. (2.52), the distance MB is

$$MB = \frac{I_o}{\upsilon_{sub}} = \frac{LH^3/12}{HL(SL)} = \frac{H^2}{12SL} = MG + GB = MG + \left(\frac{L}{2} - \frac{SL}{2}\right)$$

Then neutral stability occurs when MG = 0, or

$$\frac{H^2}{12SL} = \frac{L}{2}(1-S), \quad \text{or} \quad \frac{H}{L} = [6S(1-S)]^{1/2} = [6(0.88)(1-0.88)]^{1/2} = \mathbf{0.796} \quad \textit{Ans.}$$

P2.142 The tank of water in Fig. P2.142 is 12 cm wide into the paper. If the tank is accelerated to the right in rigid-body motion at 6 m/s², compute (a) the water depth at AB, and (b) the water force on panel AB

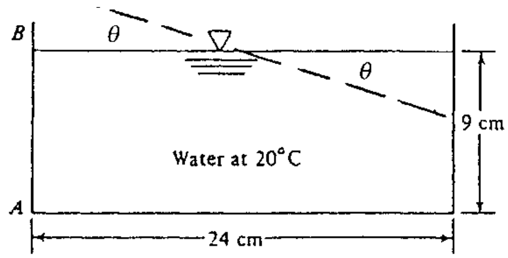


Fig. P2.142

Solution: From Eq. (2.55),

$$\tan\theta = a_x/g = \frac{6.0}{9.81} = 0.612, \quad \text{or} \quad \theta \approx 31.45°$$

Then surface point B on the left rises an additional $\Delta z = 12 \tan\theta \approx 7.34$ cm,

$$\text{or:} \quad \text{water depth AB} = 9 + 7.34 \approx \mathbf{16.3\ cm} \quad \textit{Ans.}\ \text{(a)}$$

The water pressure on AB varies linearly due to gravity only, thus the water force is

$$F_{AB} = p_{CG}A_{AB} = (9790)\left(\frac{0.163}{2}\text{ m}\right)(0.163\text{ m})(0.12\text{ m}) \approx \mathbf{15.7\ N} \quad \textit{Ans.}\ \text{(b)}$$

P2.153 A cylindrical container, 14 inches in diameter, is used to make a mold for forming salad bowls. The bowls are to be 8 inches deep. The cylinder is half-filled with molten plastic, μ = 1.6 kg/(m-s), rotated steadily about the central axis, then cooled while rotating. What is the appropriate rotation rate, in r/min?

Solution: The molten plastic viscosity is a red herring, ignore. The appropriate final rotating surface shape is a paraboloid of radius 7 inches and depth 8 inches. Thus, from Fig. 2.23,

$$h = 8\ in = \frac{8}{12}\ ft = \frac{\Omega^2 R^2}{2g} = \frac{\Omega^2 (7/12\ ft)^2}{2(32.2\ ft/s^2)}$$

$$\text{Solve for} \quad \Omega = 11.2\frac{rad}{s} \times \frac{60}{2\pi} = \mathbf{107}\ \frac{r}{min} \qquad Ans.$$

P3.115 A free liquid jet, as in Fig. P3.115, has constant ambient pressure and small losses; hence from Bernoulli's equation $z + V^2/(2g)$ is constant along the jet. For the fire nozzle in the figure, what are (a) the minimum and (b) the maximum values of θ for which the water jet will clear the corner of the building? For which case will the jet velocity be higher when it strikes the roof of the building?

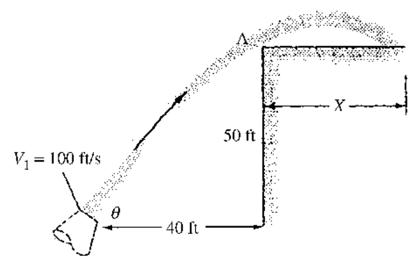


Fig. P3.115

Solution: The two extreme cases are when the jet just touches the corner A of the building. For these two cases, Bernoulli's equation requires that

$$V_1^2 + 2gz_1 = (100)^2 + 2g(0) = V_A^2 + 2gz_A = V_A^2 + 2(32.2)(50), \quad \text{or:} \quad V_A = 82.3\ \frac{\text{ft}}{\text{s}}$$

The jet moves like a frictionless particle as in elementary particle dynamics:

$$\text{Vertical motion:} \quad z = (V_1 \sin\theta)t - \frac{1}{2}gt^2; \quad \text{Horizontal motion:} \quad x = (V_1 \cos\theta)t$$

Eliminate "t" between these two and apply the result to point A:

$$z_A = 50 = x_A \tan\theta - \frac{gx_A^2}{2V_1^2\cos^2\theta} = 40\tan\theta - \frac{(32.2)(40)^2}{2(100)^2\cos^2\theta}; \quad \text{clean up and rearrange:}$$

$\tan\theta = 1.25 + 0.0644\sec^2\theta$, solve for $\theta =$ **85.94°** *Ans.* (a) and **55.40°** *Ans.* (b)

Path (b) is shown in the figure, where the jet just grazes the corner A and goes over the top of the roof. Path (a) goes nearly straight up, to z = 155 ft, then falls down to pt. A. In both cases, the velocity when the jet strikes point A is the same, 82.3 ft/s.

P4.69 A steady, two-dimensional flow has the following polar-coordinate velocity potential:

$$\phi = Cr\cos\theta + K\ln r$$

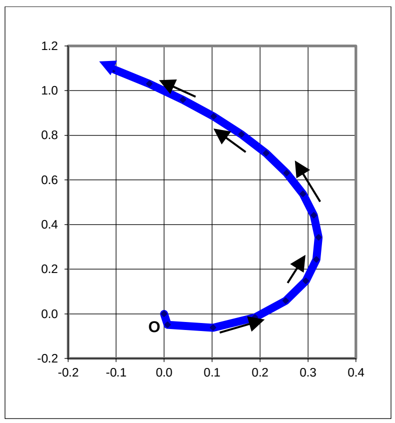
where C and K are constants. Determine the stream function $\psi(r, \theta)$ for this flow. For extra credit, let C be a velocity scale U, let $K = UL$, and sketch what the flow might represent.

Solution: Write out the ψ and ϕ expressions for polar-coordinate velocities:

$$v_r = \frac{\partial\phi}{\partial r} = C\cos\theta + \frac{K}{r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}, \quad \text{hence } \psi = Cr\sin\theta + K\theta + f(r)$$

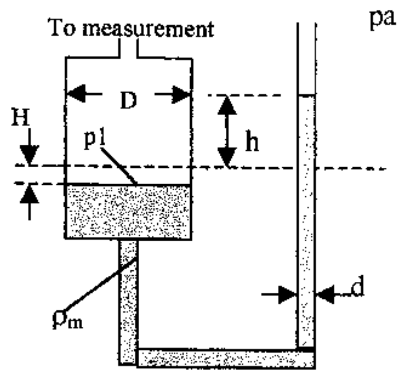
$$v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -C\sin\theta + K(0) = -\frac{\partial\psi}{\partial r}, \quad \textit{hence } \boxed{\psi = Cr\sin\theta + K\theta + constant} \quad \textit{Ans.}$$

Extra credit: Plot a typical streamline for $C = U$ and $K = UL$:



All the streamlines are logarithmic spirals coming out from the origin in every direction.

C2.1 Some manometers are constructed as in the figure at right, with one large reservoir and one small tube open to the atmosphere. We can then neglect movement of the reservoir level. If the reservoir is not large, its level will move, as in the figure. Tube height *h* is measured from the zero-pressure level, as shown.
(a) Let the reservoir pressure be high, as in the Figure, so its level goes down. Write an exact Expression for p_{1gage} as a function of h, d, D, and gravity *g*. (b) Write an approximate expression for p_{1gage}, neglecting the movement of the reservoir. (c) Suppose h = 26 cm, p_a = 101 kPa, and ρ_m = 820 kg/m³. Estimate the ratio (D/d) required to keep the error in (b) less than 1.0% and also < 0.1%. Neglect surface tension.



Solution: Let *H* be the downward movement of the reservoir. If we neglect air density, the pressure difference is $p_1 - p_a = \rho_m g(h + H)$. But volumes of liquid must balance:

$$\frac{\pi}{4}D^2 H = \frac{\pi}{4}d^2 h, \quad \text{or:} \quad H = (d/D)^2 h$$

Then the pressure difference (exact except for air density) becomes

$$\boldsymbol{p_1 - p_a = p_{1gage} = \rho_m g h(1 + d^2/D^2)} \quad \textit{Ans.} \text{ (a)}$$

If we ignore the displacement *H*, then $\boldsymbol{p_{1gage} \approx \rho m g h}$ *Ans.* (b)

(c) For the given numerical values, h = 26 cm and ρ_m = 820 kg/m³ are irrelevant, all that matters is the ratio d/D. That is,

$$Error\ E = \frac{\Delta p_{exact} - \Delta p_{approx}}{\Delta p_{exact}} = \frac{(d/D)^2}{1 + (d/D)^2}, \quad or: \quad D/d = \sqrt{(1-E)/E}$$

For E = 1% or 0.01, D/d = [(1 – 0.01)/0.01]$^{1/2}$ ≥ **9.95** *Ans.* (c-1%)
For E = 0.1% or 0.001, D/d = [(1 – 0.001)/0.001]$^{1/2}$ ≥ **31.6** *Ans.* (c-0.1%)