ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2023 – HW2 Solution

P1.70 Derive an expression for the capillary-height change h, as shown, for a fluid of surface tension Y and contact angle θ between two parallel plates W apart. Evaluate h for water at 20C if W=0.5 mm

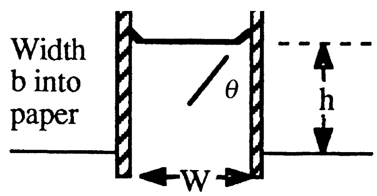


Fig. P1.70

Solution: With b the width of the plates into the paper, the capillary forces on each wall together balance the weight of water held above the reservoir free surface:

$$\rho g W h b = 2(Y b \cos\theta), \quad \text{or:} \quad h \approx \frac{\mathbf{2Y\cos\theta}}{\boldsymbol{\rho g W}} \quad \textit{Ans.}$$

For water at 20°C, Y ≈ 0.0728 N/m, ρ_g ≈ 9790 N/m^3, and θ ≈ 0°. Thus, for W = 0.5 mm,

$$h = \frac{2(0.0728\ \text{N/m})\cos 0°}{(9790\ \text{N/m}^3)(0.0005\ \text{m})} \approx 0.030\ \text{m} \approx \mathbf{30\ mm} \quad \textit{Ans.}$$

P1.73 A small submersible moves at velocity V in 20° C water at 2-m depth, where ambient pressure is 131 kPa. Its critical cavitation number is Ca ≈ 0.25. At what velocity will cavitation bubbles form? Will the body cavitate if V = 30 m/s and the water is cold (5° C)?

Solution: From Table A-5 at 20° C read pv = 2.337 kPa. By definition,

$$Ca_{crit} = 0.25 = \frac{2(p_a - p_v)}{\rho V^2} = \frac{2(131000 - 2337)}{(998\ \text{kg/m}^3)V^2}, \quad \text{solve } V_{crit} \approx \mathbf{32.1\ m/s} \quad \textit{Ans.}\ \text{(a)}$$

If we decrease water temperature to 5° C, the vapor pressure reduces to 863 Pa, and the density changes slightly, to 1000 kg/m^3. For this condition, if V = 30 m/s, we compute:

$$Ca = \frac{2(131000 - 863)}{(1000)(30)^2} \approx 0.289$$

This is *greater* than 0.25, therefore the body **will not cavitate for these conditions**. *Ans*. (b)

P2.44 Water flows downward in a pipe at 45°, as shown in Fig. P2.44. The mercury manometer reads a 6-in height. The pressure drop $p_2 - p_1$ is partly due to friction and partly due to gravity. Determine the total pressure drop and also the part due to friction only. Which part does the manometer read? Why?

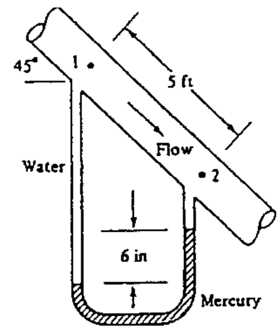


Fig. P2.44

Solution: Let "h" be the distance down from point 2 to the mercury-water interface in the right leg. Write the hydrostatic formula from 1 to 2:

$$p_1 + 62.4\left(5\sin 45° + h + \frac{6}{12}\right) - 846\left(\frac{6}{12}\right) - 62.4h = p_2,$$

$$p_1 - p_2 = \underbrace{(846 - 62.4)(6/12)}_{\text{....friction loss...}} - \underbrace{62.4(5\sin 45°)}_{\text{..gravity head..}} = 392 - 221$$

$$= \mathbf{171\,\frac{lbf}{ft^2}} \quad \textit{Ans.}$$

The manometer reads only the *friction loss* of 392 lbf/ft^2, not the gravity head of 221 psf.

P2.76 Panel BC in Fig. P2.76 is circular. Compute (a) the hydrostatic force of the water on the panel; (b) its center of pressure; and (c) the moment of this force about point B.

Solution: (a) The hydrostatic force on the gate is:

$$F = \gamma h_{CG} A$$
$$= (9790\ \text{N/m}^3)(4.5\ \text{m})\sin 50°(\pi)(1.5\ \text{m})^2$$
$$= \mathbf{239\ kN} \quad \textit{Ans.}\ \text{(a)}$$

(b) The center of pressure of the force is:

$$y_{CP} = \frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{\frac{\pi}{4}r^4\sin\theta}{h_{CG}A}$$

$$= \frac{\frac{\pi}{4}(1.5)^4\sin 50°}{(4.5\sin 50°)(\pi)(1.5^2)} = \mathbf{0.125\ m} \quad \textit{Ans.}\ \text{(b)}$$

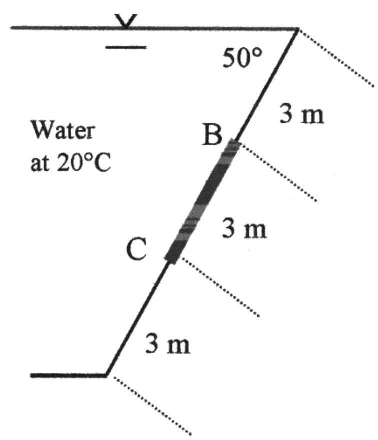


Fig. P2.76

Thus y is **1.625 m** down along the panel from B (or 0.125 m down from the center of the circle).

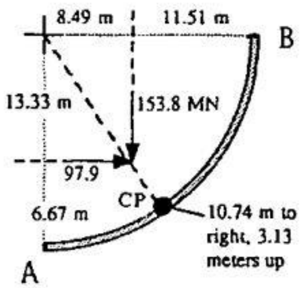
(c) The moment about B due to the hydrostatic force is,

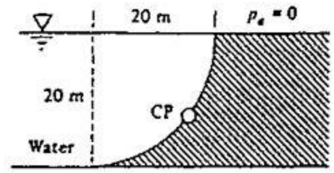
$$M_B = (238550\ \text{N})(1.625\ \text{m}) = 387{,}600\ \text{N}\cdot\text{m} = \mathbf{388\ kN\cdot m} \quad \textit{Ans.}\ \text{(c)}$$

P2.82 The dam in Fig. P2.82 is a quarter-circle 50 m wide into the paper. Determine the horizontal and vertical components of hydrostatic force against the dam and the point CP where the resultant strikes the dam.

Solution: The horizontal force acts as if the dam were vertical and 20 m high:

$$F_H = \gamma h_{CG} A_{vert} = \left(9790\frac{N}{m^3}\right)(10\ m)(20 \times 50\ m^2) = \mathbf{97.9\ MN}\ Ans.$$





C1.12 A solid aluminium disk (SG = 2.7) is 2 inches in diameter and 3/16 inch thick. It slides steadily down a 14° incline that is coated with a castor oil (SG = 0.96) film one hundredth of an inch thick. The steady slide velocity is 2 cm/s. Using Figure A.1 and a linear oil velocity profile assumption, estimate the *temperature* of the castor oil.

Solution: This problem reviews complicated units, volume and weight, shear stress, and viscosity. It fits the sketch in Fig. P1.45. The writer converts to SI units.

$$W = \rho_{alum}\, g\, A h = (2700\, kg/m^3)(9.81\, m/s^2)\{\pi[(1/12)(0.3048)]^2\}(3/16/12)(0.3048) = 0.256\, N$$

The weight component along the incline balances the shear stress, in the castor oil, times the bottom flat area of the disk.

$$W \sin\theta = \mu \frac{V}{h} A \,, \ \text{or}: \ (0.256N)\sin(14^\circ) = \mu\left[\frac{0.02\, m/s}{(1/100/12)(0.3048)\, m}\right]\pi[(1/12)(0.3048)m]^2$$

$$\text{Solve for} \quad \mu_{oil} \approx 0.39\, kg/(m-s)$$

Looking on Fig. A.1 for castor oil, this viscosity corresponds approximately to **30°C** *Ans.*
The specific gravity of the castor oil was a red herring and is not needed. Note also that, since *W* is proportional to disk bottom area *A*, that area cancels out and is not needed.