ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2022 – HW13 Solution

P8.15 Hurricane Sandy, which hit the New Jersey coast on Oct. 29, 2012, was extremely broad, with wind velocities of 40 mi/h at 400 miles from its center. Its maximum velocity was 90 mi/h. Using the model of Fig. P8.14, at 20ºC with a pressure of 100 kPa far from the center, estimate (a) the radius *R* of maximum velocity, in mi; and (b) the pressure at *r* = *R*.

Solution: The air density is p/RT = (100,000)/[287(293)] = 1.19 kg/m^3. Convert 90 mi/h to 40.23 m/s and 40 mi/h to 17.9 m/s. The outer flow is irrotational, hence Bernoulli holds:

$$p_{r=R} + \frac{1}{2}\rho V_{r=R}^2 = p_{r=R} + \frac{1}{2}(1.19)(40.2)^2 = p_\infty + \frac{1}{2}\rho V_\infty^2 = 100,000 + 0$$

$$solve\ \ for \quad p_{r=R} = 100,000 - 960 = 99,040\,Pa \approx \mathbf{99\,kPa} \quad Ans.(b)$$

(a) The two velocities, plus the radii, fit irrotational vortex theory. Let the outer ring be r_2:

$$V_2 = 40\,mi/h = \frac{C}{r_2} = \frac{C}{400\,mi}\ , \ \text{hence} \ \ C = (40)(400) = 16,000\,mi^2/h$$

$$\text{Then} \quad V_R = 90\,mi/h = \frac{16,000\,m^2/h}{R}\ , \ \text{solve} \ \ R \approx \mathbf{178\,mi} \qquad Ans.(a)$$

P8.27 Water at 20°C flows past a half-body as shown in Fig. P8.27. Measured pressures at points A and B are 160 kPa and 90 kPa, respectively, with uncertainties of 3 kPa each. Estimate the stream velocity and its uncertainty.

Solution: Since Eq. (8.18) is for the upper surface, use it by noting that $V_C = V_B$ in the figure:

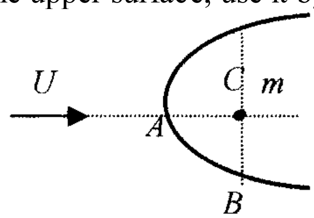


Fig. P8.27

$$\frac{r_C}{a} = \frac{\pi - \pi/2}{\sin(\pi/2)} = \frac{\pi}{2}, \quad V_C^2 = V_B^2 = U_\infty^2\left[1+\left(\frac{2}{\pi}\right)^2 + \frac{2}{(\pi/2)}\cos(\pi/2)\right] = 1.405U_\infty^2$$

$$\text{Bernoulli:} \quad p_A + \frac{\rho}{2}V_A^2 = 160000 + 0 = p_B + \frac{\rho}{2}V_B^2 = 90000 + \frac{998}{2}\left(1.405U_\infty^2\right)$$

Solve for $\mathbf{U_\infty \approx 10.0\ m/s}$ *Ans.*

The uncertainty in $(p_A - p_B)$ is as high as 6000 Pa, hence the uncertainty in U_∞ is **±0.4 m/s**. *Ans.*

P8.29 A uniform water stream, $U_\infty = 20$ m/s and $\rho = 998$ kg/m^3, combines with a source at the origin to form a half-body. At $(x, y) = (0, 1.2$ m$)$, the pressure is 12.5 kPa less than p_∞. (a) Is this point outside the body? Estimate (b) the appropriate source strength m and (c) the pressure at the nose of the body.

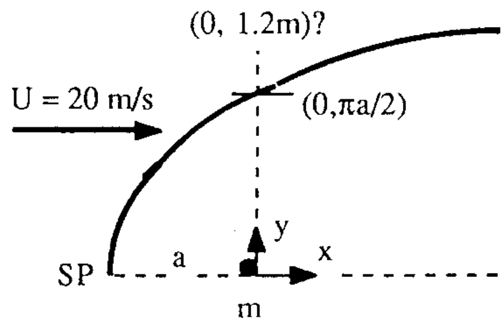


Fig P8.29

Solution: We know, from Fig. 8.5 and Eq. 8.18, the point on the half-body surface just above "m" is at y = πa/2, as shown, where a = m/U. The Bernoulli equation allows us to compute the necessary source strength m from the pressure at (x, y) = (0, 1.2 m):

$$p_\infty + \frac{\rho}{2}U_\infty^2 = p_\infty + \frac{998}{2}(20)^2 = p_\infty - 12500 + \frac{998}{2}\left[(20)^2 + \left(\frac{m}{1.2}\right)^2\right]$$

Solve for $\boldsymbol{m}$ **- 6.0** $\frac{\mathbf{m^2}}{\mathbf{s}}$ *Ans.* (b) while $a = \frac{m}{U} = \frac{6.0}{20} = 0.3$ m

The body surface is thus at y = πa/2 = **0.47 m** above m. Thus the point in question, y = 1.2 m above m, is **<u>outside the body</u>**. *Ans.* (a)

At the nose SP of the body, (x, y) = (–a, 0), the velocity is zero, hence we predict

$$p_\infty + \frac{\rho}{2}U_\infty^2 = p_\infty + \frac{998}{2}(20)^2 = p_{nose} + \frac{\rho}{2}(0)^2, \quad \text{or} \quad \mathbf{p_{nose} \approx p_\infty + 200\ kPa} \quad \textit{Ans. (c)}$$

P8.44 Suppose that circulation is added to the cylinder flow of Prob. P8.43 sufficient to place the stagnation points at $\theta = 35°$ and $145°$. What is the required vortex strength K in m^2/s? Compute the resulting pressure and surface velocity at (a) the stagnation points, and (b) the upper and lower shoulders. What will be the lift per meter of cylinder width?

Solution: Recall that Prob. P8.43 was for water at 20°C flowing at 6 m/s past a 1-m-diameter cylinder, with $p_\infty = 200$ kPa. From Eq. (8.35),

$$\sin\theta_{stag} = \sin(35°) = \frac{K}{2U_\infty a} = \frac{K}{2(6\text{ m/s})(0.5\text{ m})}, \quad \text{or:} \quad K = \mathbf{3.44}\ m^2/s \quad \textit{Ans.}$$

(a) At the stagnation points, velocity is zero and pressure equals stagnation pressure:

$$p_{stag} = p_\infty + \frac{\rho}{2}U_\infty^2 = 200{,}000\ Pa + \frac{998\text{ kg/m}^3}{2}(6\text{ m/s})^2 = \mathbf{218{,}000\ Pa} \quad \textit{Ans.}\text{ (a)}$$

(b) At any point on the surface, from Eq. (8.37),

$$p_{stag} = 218000 = p_{surf} + \frac{\rho}{2}\left(-2U_\infty\sin\theta + \frac{K}{a}\right)^2 = p_{surf} + \frac{998}{2}\left[-2(6)\sin\theta + \frac{3.44}{0.5}\right]^2$$

At the upper shoulder, $\theta = 90°$,

$$p = 218000 - \frac{998}{2}(-5.12)^2 \approx \mathbf{204{,}900\ Pa} \quad \textit{Ans.}\text{ (b—upper)}$$

At the lower shoulder, $\theta = 270°$,

$$p = 218000 - \frac{998}{2}(-18.88)^2 \approx \mathbf{40{,}100\ Pa} \quad \textit{Ans.}\text{ (b—lower)}$$

***P8.48** Wind at U_∞ and p_∞ flows past a Quonset hut which is a half-cylinder of radius a and length L (Fig. P8.48). The internal pressure is p_i. Using inviscid theory, derive an expression for the upward force on the hut due to the difference between p_i and p_s.

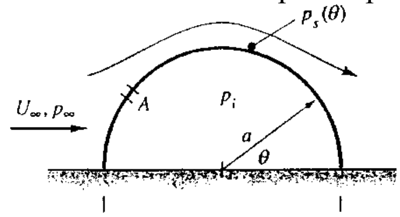


Fig P8.48

Solution: The analysis is similar to Prob. P8.46 on the previous page. If p_o is the stagnation pressure at the nose (θ = 180°), the surface pressure distribution is

$$p_s = p_o - \frac{\rho}{2}U_s^2 = p_o - \frac{\rho}{2}(2U_\infty \sin\theta)^2 = p_o - 2\rho U_\infty^2 \sin^2\theta$$

Then the net upward force on the half-cylinder is found by integration:

$$F_{up} = \int_0^\pi (p_i - p_s)\sin\theta\, ab\, d\theta = \int_0^\pi (p_i - p_o + 2\rho U_\infty^2 \sin^2\theta)\sin\theta\, ab\, d\theta,$$

or: $\mathbf{F_{up} = (p_i - p_o)2ab + \frac{8}{3}\rho U_\infty^2\, ab}$ *Ans.* $\left(\text{where } p_o = p_\infty + \frac{\rho}{2}U_\infty^2\right)$

P8.75 Using the four-source image pattern needed to construct the flow near a corner shown in Fig. P8.72, find the value of the source strength m which will induce a wall velocity of 4.0 m/s at the point $(x, y) = (a, 0)$ just below the source shown, if $a = 50$ cm.

Solution: The flow pattern is formed by four equal sources m in the 4 quadrants, as in the figure at right. The sources above and below the point A(a, 0) cancel each other at A, so the velocity at A is caused only by the two left sources. The velocity at A is the sum of the two horizontal components from these 2 sources:

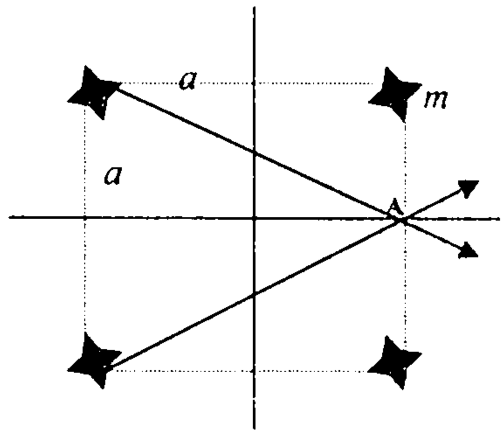


Fig P8.75

$$V_A = 2\frac{m}{\sqrt{a^2+(2a)^2}}\frac{2a}{\sqrt{a^2+(2a)^2}} = \frac{4ma}{5a^2} = \frac{4m}{5(0.5m)} = 4\ \frac{\text{m}}{\text{s}} \quad \text{if } \boldsymbol{m = 2.5\ \frac{m^2}{s}} \qquad \textit{Ans.}$$

C8.4 Find a formula for the stream function for flow of a doublet of strength λ at a distance a from a wall, as in Fig. C8.4. (a) Sketch the streamlines. (b) Are there any stagnation points? (c) Find the maximum velocity along the wall and its position.

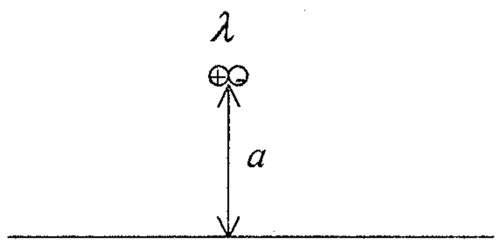
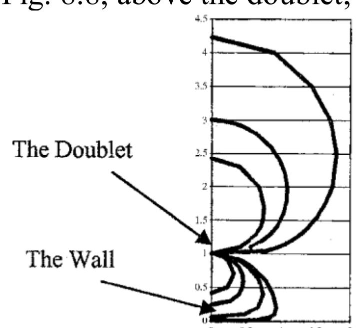


Fig C8.4

Solution: Use an image doublet of the same strength and orientation at the $(x, y) = (0, -a)$. The stream function for this combined flow will form a "wall" at $y = 0$ between the two doublets:

$$\psi = -\frac{\lambda(y+a)}{x^2+(y+a)^2} - \frac{\lambda(y-a)}{x^2+(y-a)^2}$$

(a) The streamlines are shown on the next page for one quadrant of the doubly-symmetric flow field. They are fairly circular, like Fig. 8.8, above the doublet, but they flatten near the wall.



Problem C8.4

(b) There are **no stagnation points** in this flow field. *Ans.* (b)

(c) The velocity along the wall ($y = 0$) is found by differentiating the stream function:

$$u_{wall} = \frac{\partial \psi}{\partial y}\Big|_{y=0} = -\frac{\lambda}{x^2+a^2} + \frac{2\lambda a^2}{(x^2+a^2)^2} - \frac{\lambda}{x^2+a^2} + \frac{2\lambda a^2}{(x^2+a^2)^2}$$

The maximum velocity occurs at $x = 0$, that is, right between the two doublets:

$$u_{w,\max} = \frac{2\lambda}{a^2} \quad \textit{Ans. (c)}$$