ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2023 – HW12 Solution

P7.42 A light aircraft flies at 30 m/s (67 mi/h) in air at 20°C and 1 atm. Its wing is an NACA 0009 airfoil, with a chord length of 150 cm and a very wide span (neglect aspect ratio effects). Estimate the drag of this wing, per unit span length, (*a*) by flat plate theory; and (*b*) using the data from Fig. 7.25 for $\alpha = 0°$.

Solution: For air at 20°C and 1 atm, ρ = 1.2 kg/m^3 and μ = 1.8E-5 kg/m-s. First find the Reynolds number, based on chord length, to see where we are:

$$\mathrm{Re}_c = \frac{\rho U c}{\mu} = \frac{(1.2\,kg/m^3)(30\,m/s)(1.5\,m)}{1.8\mathrm{E}-5\,kg/m-s} = 3\times 10^6 \qquad \text{turbulent}$$

(*a*) For flat-plate theory, use Eq. (7.49*a*), which assumes transition at Re_x = 500,000:

$$C_d = \frac{0.031}{\mathrm{Re}_c^{1/7}} - \frac{1440}{\mathrm{Re}_c} = \frac{0.031}{(3\mathrm{E}6)^{1/7}} - \frac{1440}{3\mathrm{E}6} = 0.00368 - 0.00048 = 0.0032$$

$$Drag = C_d \frac{\rho}{2} U^2 (2bc) = (0.0032)(\frac{1.2}{2})(30)^2[2(1.0)(1.5)] = \mathbf{5.2}\ \frac{N}{m} \qquad Ans.(a)$$

(*b*) For the actual NACA 0009 airfoil, at Re_c = 3E6, in Fig. 7.25, read $C_d \approx 0.0065$. Then

$$Drag = C_d \frac{\rho}{2} U^2 (bc) = (0.0065)(\frac{1.2}{2})(30)^2[(1.0)(1.5)] = \mathbf{5.3}\ \frac{N}{m} \qquad Ans.(b)$$

The two are quite close. A thin airfoil at low angles is similar to a flat plate.

P7.50 Consider the flat-walled diffuser in Fig. P7.50, which is similar to that of Fig. 6.26*a* with constant width *b*. If *x* is measured from the inlet and the wall boundary layers are thin, show that the core velocity $U(x)$ in the diffuser is given approximately by

$$U = \frac{U_o}{1 + (2x\tan\theta)/W}$$

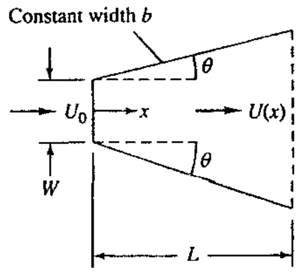


Fig. P7.50

where W is the inlet height. Use this velocity distribution with Thwaites' method to compute the wall angle θ for which laminar separation will occur in the exit plane when diffuser length $L = 2W$. Note that the result is independent of the Reynolds number.

Solution: We can approximate U(x) by the one-dimensional continuity relation:

$U_oWb = U(W + 2x\tan\theta)b$, or: $U(x) \approx U_o/[1 + 2x\tan\theta/W]$ (same as Görtler, Prob. 7.48)

We return to the solution from Görtler's (n = 1) distribution in Prob. 7.48:

$$\lambda = -0.09 \quad \text{if} \quad \frac{2x\tan\theta}{W} = 0.159 \ \text{(separation)}, \quad \text{or} \quad x = L = 2W,$$

$$\tan\theta_{sep} = \frac{0.159}{4} = 0.03975, \quad \boldsymbol{\theta_{sep} \approx 2.3°} \quad \textit{Ans.}$$

[This laminar result is much less than the <u>turbulent</u> value $\theta_{sep} \approx 8°–10°$ in Fig. 6.26c.]

P7.75 The helium-filled balloon in Fig. P7.75 is tethered at 20°C and 1 atm with a string of negligible weight and drag. The diameter is 50 cm, and the balloon material weighs 0.2 N, not including the helium. The helium pressure is 120 kPa. Estimate the tilt angle θ if the airstream velocity U is (a) 5 m/s or (b) 20 m/s.

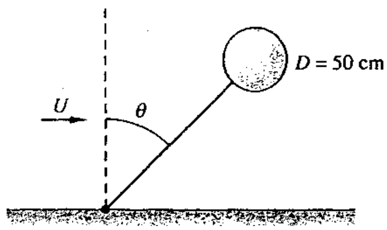


Fig. P7.75

Solution: For air at 20°C and 1 atm, take ρ = 1.2 kg/m^3 and μ = 1.8E−5 kg/m·s. For helium, R = 2077 J/kg·°K. The helium density = (120000)/[2077(293)] ≈ 0.197 kg/m^3.

The balloon net buoyancy is independent of the flow velocity:

$$B_{net} = (\rho_{air} - \rho_{He})g\frac{\pi}{6}D^3 = (1.2 - 0.197)(9.81)\frac{\pi}{6}(0.5)^3 \approx 0.644 \text{ N}$$

The net upward force is thus Fz = (Bnet − W) = 0.644 − 0.2 = *0.444* N. The balloon drag *does* depend upon velocity. At 5 m/s, we expect laminar flow:

$$\text{(a) } U = 5\ \frac{m}{s}: \ \text{Re}_D = \frac{1.2(5)(0.5)}{1.8E\text{–}5} = 167000; \quad \textit{Table } 7.3: \ C_D \approx 0.47$$

$$Drag = C_D\frac{\rho}{2}U^2\frac{\pi}{4}D^2 = 0.47\left(\frac{1.2}{2}\right)(5)^2\frac{\pi}{4}(0.5)^2 \approx 1.384 \text{ N}$$

$$\text{Then} \quad \theta_a = \tan^{-1}\left(\frac{Drag}{F_z}\right) = \tan^{-1}\left(\frac{1.384}{0.444}\right) = \mathbf{72°} \quad \textit{Ans. } \text{(a)}$$

(b) At 20 m/s, Re = 667000 (*turbulent*), Table 7.3: CD ≈ 0.2:

$$Drag = 0.2\left(\frac{1.2}{2}\right)(20)^2\frac{\pi}{4}(0.5)^2 = 9.43 \text{ N}, \quad \theta_b = \tan^{-1}\left(\frac{9.43}{0.444}\right) = \mathbf{87°} \quad \textit{Ans. } \text{(b)}$$

These angles are too steep—the balloon needs more buoyancy and/or less drag.

P7.84 A Ping-Pong ball weighs 2.6 g and has a diameter of 3.8 cm. It can be supported by an air jet from a vacuum cleaner outlet, as in Fig. P7.84. For sea-level standard air, what jet velocity is required?

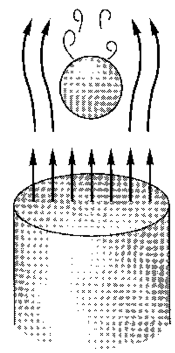


Fig P7.84

Solution: For sea-level air, take $\rho = 1.225$ kg/m^3 and $\mu = 1.78$E−5 kg/m·s. The ball weight must balance its drag:

$$W = 0.0026(9.81) = 0.0255\text{ N} = C_D \frac{\rho}{2} V^2 \frac{\pi}{4} D^2 = C_D \frac{1.225}{2} V^2 \frac{\pi}{4}(0.038)^2, \quad C_D = \text{fcn(Re)}$$

$$C_D V^2 = 36.7,\ \text{Use Fig. 7.16b, converges to } C_D \approx 0.47,\ \text{Re} \approx 23000,\ V \approx \mathbf{9\ m/s} \quad \textit{Ans.}$$

P7.108 The data in Fig. P7.108 are for lift and drag of a *spinning* sphere from Ref. 45, pp. 7–20. Suppose a tennis ball (W ≈ 0.56 N, D ≈ 6.35 cm) is struck at sea level with initial velocity $V_o = 30$ m/s, with "topspin" (front of the ball rotating downward) of 120 rev/sec. If the initial height of the ball is 1.5 m, estimate the horizontal distance travelled before it strikes the ground.

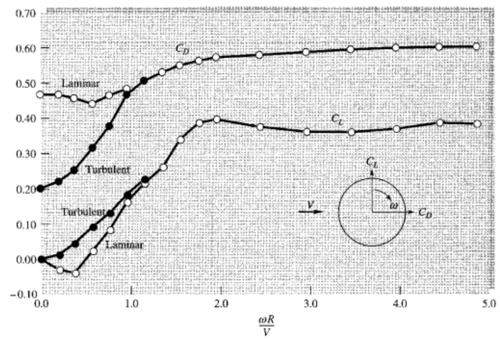
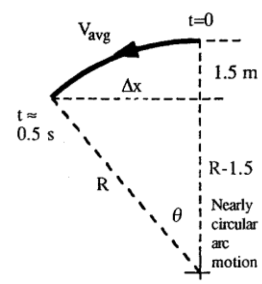


Fig P7.108

Solution: For sea-level air, take $\rho = 1.225$ kg/m³ and $\mu = 1.78E{-5}$ kg/m·s. For this short distance, the ball travels in nearly a circular arc, as shown at right. From Figure P7.108 we read drag and lift:



$$\omega = 120(2\pi) = 754\ \frac{\text{rad}}{\text{s}},$$

$$\frac{\omega R}{V} = \frac{754(0.03175)}{30} \approx 0.80,$$

$$\text{Read}\quad C_D \approx \mathbf{0.47},\quad C_L \approx \mathbf{0.12}$$

Initially, the accelerations in the horizontal and vertical directions are (*z* up, *x* to left)

$$a_{x,0} = -\frac{\text{drag}}{m} = -\frac{0.47(1.225/2)(30)^2(\pi/4)(0.0635)^2}{0.56/9.81} \approx \mathbf{-14.4\ m/s^2}$$

$$a_{z,0} = -g - \frac{\text{lift}}{m} = -9.81 - \frac{0.12(1.225/2)(30)^2(\pi/4)(0.0635)^2}{0.56/9.81} \approx \mathbf{-13.5\ m/s^2}$$

The term ax serves to slow down the ball from 30 m/s, when hit, to about 24 m/s when it strikes the floor about 0.5 s later. The average velocity is (30 + 24)/2 = **27** m/s. The term az causes the ball to curve in its path, so one can estimate the radius of curvature and the angle of turn for which Δz = 1.5 m. Then, finally, one estimates Δx as desired:

$$\frac{V^2_{avg}}{R} = a_z,\quad \text{or:}\quad R \approx \frac{(27)^2}{13.5} \approx 54\ \text{m};\quad \theta = \cos^{-1}\left(\frac{54-1.5}{54}\right) \approx 13.54°$$

$$\text{Finally,}\quad \Delta x_{ball} = R\sin\theta = (54)\sin(13.54°) \approx \mathbf{12.6\ m}\quad \textit{Ans.}$$

A more exact numerical integration of the equations of motion (not shown here) yields the result Δx ≈ 13.0 m at t ≈ 0.49 s.

C7.2 Air at 20°C and 1 atm flows at Vavg = 5 m/s between long, smooth parallel heat-exchanger plates 10 cm apart, as shown below. It is proposed to add a number of widely spaced 1-cm-long thin 'interrupter' plates to increase the heat transfer, as shown. Although the channel flow is turbulent, the boundary layer over the interrupter plates is laminar. Assume all plates are 1 m wide into the paper. Find (a) the pressure drop in Pa/m without the small plates present. Then find (b) the number of small plates, per meter of channel length, which will cause the overall pressure drop to be 10 Pa/m.

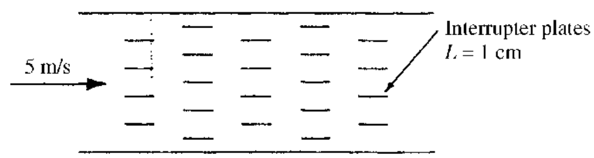


Fig. C7.2

Solution: For air, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. (a) For wide plates, the hydraulic diameter is Dh = $2h$ = 20 cm. The Reynolds number, friction factor, and pressure drop for the bare channel (no small plates) is:

$$Re_{Dh} = \frac{\rho V_{avg} D_h}{\mu} = \frac{(1.2)(5.0)(0.2)}{1.8E\text{–}5} = 66{,}700 \;\; (\textit{turbulent})$$

$$f_{Moody,smooth} \approx 0.0196$$

$$\Delta p_{bare} = f \frac{L}{D_h} \frac{\rho}{2} V_{avg}^2 = (0.0196)\left(\frac{1.0 \text{ m}}{0.2 \text{ m}}\right)\left(\frac{1.2}{2}\right)(5.0)^2 = \mathbf{1.47} \;\; \frac{\mathbf{Pa}}{\mathbf{m}} \quad \textit{Ans.} \text{ (a)}$$

Each small plate (neglecting the wake effect if the plates are in line with each other) has a *laminar* Reynolds number:

$$Re_L = \frac{\rho V_{avg} L_{plate}}{\mu} = \frac{(1.2)(5.0)(0.01)}{1.8E\text{–}5} = \mathbf{3333} < 5E5, \quad \therefore \; \textit{laminar}$$

$$C_{D,laminar} = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{3333}} \approx 0.0230$$

$$F_{1plate} = C_D \frac{\rho}{2} V_{avg}^2 A_{2\,sides} = (0.0230)\left(\frac{1.2}{2}\right)(5.0)^2 (2 \times 0.01 \times 1) = 0.0069 \;\; \frac{\text{N}}{\text{plate}}$$

Each plate force must be supported by the channel walls. The effective pressure drop will be the bare wall pressure drop (assumed unchanged) plus the sum of the interrupter-plate forces divided by the channel cross-section area, which is given by (h × 1 m) = 0.1 m^2. The extra pressure drop provided by the plates, for this problem, is (10.0 − 1.47) = **8.53** Pa/m. Therefore we need

$$\textit{No. of plates} = \frac{\Delta p_{needed}}{(F/A)_{1plate}} = \frac{8.53 \text{ Pa/m}}{(0.0069 \text{ N/plate})/(0.1 \text{ m}^2)} \approx \mathbf{124 \ plates} \quad \textit{Ans.} \text{ (b)}$$

This is the number of small interrupter plates *needed for each meter of channel length* to build up the pressure drop to 10.0 Pa/m.