ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2022 – HW10 Solution

P6.80 The head-versus-flowrate characteristics of a centrifugal pump are shown in Fig. P6.80. If this pump drives water at 20°C through 120 m of 30-cm-diameter cast-iron pipe, what will be the resulting flow rate, in m^3/s?

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For cast iron, take $\varepsilon \approx 0.26$ mm, hence $\varepsilon/d = 0.26/300 \approx 0.000867$. The head loss must match the pump head:

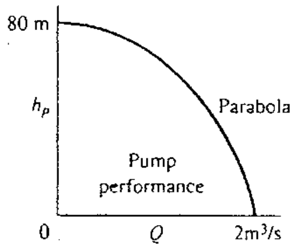


Fig. P6.80

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = \frac{8fLQ^2}{\pi^2 g d^5} = h_{pump} \approx 80 - 20Q^2, \quad \text{with Q in m}^3/\text{s}$$

$$\text{Evaluate} \quad h_f = \frac{8f(120)Q^2}{\pi^2(9.81)(0.3)^5} = 80 - 20Q^2, \quad \text{or:} \quad Q^2 \approx \frac{80}{20 + 4080f}$$

$$\text{Guess} \quad f \approx 0.02, \quad Q = \left[\frac{80}{20 + 4080(0.02)}\right]^{1/2} \approx 0.887\ \frac{m^3}{s}, \quad Re = \frac{4\rho Q}{\pi\mu d} \approx 3.76E6$$

$$\frac{\varepsilon}{d} = 0.000867, \quad f_{better} \approx 0.0191, \quad Re_{better} \approx 3.83E6, \quad \text{converges to } \mathbf{Q \approx 0.905}\ \mathbf{\frac{m^3}{s}} \quad \textit{Ans.}$$

P6.97 A heat exchanger consists of multiple parallel-plate passages, as shown in Fig. P6.97. The available pressure drop is 2 kPa, and the fluid is water at 20°C. If the desired total flow rate is 900 m^3/h, estimate the appropriate number of passages. The plate walls are hydraulically smooth.

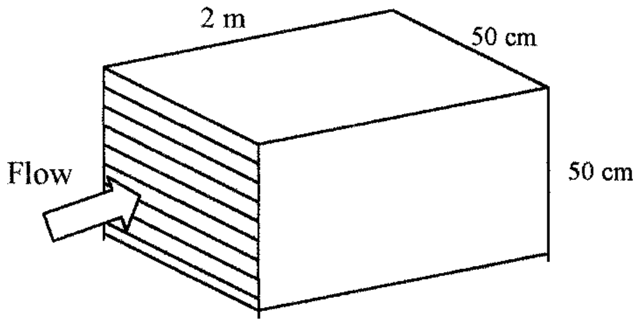


Fig. P6.97

Solution: For water, $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. Unlike Prob. 6.88, here we expect turbulent flow. If there are N passages, then b = 50 cm for all N and the passage thickness is $H = 0.5$ m/N. The hydraulic diameter is $D_h = 2H$. The velocity in each passage is related to the pressure drop by Eq. (6.58):

$$\Delta p = f\frac{L}{D_h}\frac{\rho}{2}V^2 \quad \text{where } f = f_{smooth} = fcn\left(\frac{\rho V D_h}{\mu}\right)$$

For the given data, $$2000\ Pa = f\frac{2.0\ m}{2(0.5\ m/N)}\frac{998\ kg/m^3}{2}V^2$$

Select N, find H and V and $Q_{total} = AV = b^2V$ and compare to the desired flow of 900 m^3/h. For example, guess $N = 20$, calculate $f = 0.0173$ and $Q_{total} = 2165$ m^3/h. The converged result is

$$Q_{total} = 908\ \text{m}^3/\text{h}, \quad f = 0.028,$$

$$\text{Re}_{D_h} = 14400, \quad H = 7.14\ \text{mm}, \quad \mathbf{N = 70\ passages} \quad \textit{Ans.}$$

***P6.102** A 70 percent efficient pump delivers water at 20□C from one reservoir to another 20 ft higher, as in Fig. P6.102. The piping system consists of 60 ft of galvanized-iron 2-in pipe, a reentrant entrance, two screwed 90□ long-radius elbows, a screwed-open gate valve, and a sharp exit. What is the input power required in horsepower with and without a 6□ well-designed conical expansion added to the exit? The flow rate is 0.4 ft^3/s.

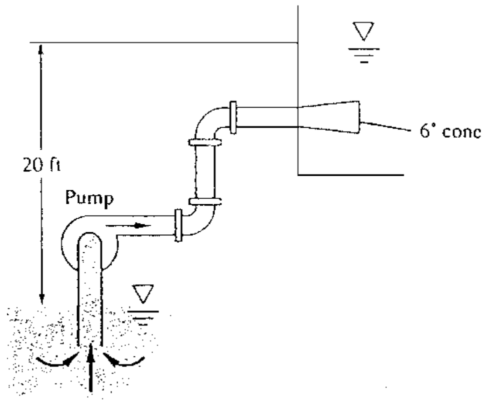


Fig. P6.102

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E–}5$ slug/ft·s. For galvanized iron, $\varepsilon \approx 0.0005$ ft, whence $\varepsilon/d = 0.0005/(2/12 \text{ ft}) \approx 0.003$. Without the 6° cone, the minor losses are:

$$K_{\text{reentrant}} \approx 1.0; \quad K_{\text{elbows}} \approx 2(0.41); \quad K_{\text{gate valve}} \approx 0.16; \quad K_{\text{sharp exit}} \approx 1.0$$

$$\text{Evaluate} \quad V = \frac{Q}{A} = \frac{0.4}{\pi(2/12)^2/4} = 18.3\ \frac{\text{ft}}{\text{s}}; \quad \text{Re} = \frac{\rho V d}{\mu} = \frac{1.94(18.3)(2/12)}{2.09\text{E–}5} \approx 284000$$

At this Re and roughness ratio, we find from the Moody chart that $f \approx 0.0266$. Then

$$\text{(a)} \quad h_{\text{pump}} = \Delta z + \frac{V^2}{2g}\left(f\frac{L}{d} + \Sigma K\right) = 20 + \frac{(18.3)^2}{2(32.2)}\left[0.0266\left(\frac{60}{2/12}\right) + 1.0 + 0.82 + 0.16 + 1.0\right]$$

$$\text{or} \quad h_{\text{pump}} \approx 85.6 \text{ ft}, \quad \text{Power} = \frac{\rho g Q h_p}{\eta} = \frac{(62.4)(0.4)(85.6)}{0.70}$$

$$= 3052 \div 550 \approx \mathbf{5.55\ hp} \quad \textit{Ans.}\text{ (a)}$$

(b) If we replace the sharp exit by a 6° conical diffuser, from Fig. 6.23, $K_{exit} \approx 0.3$. Then

$$h_p = 20 + \frac{(18.3)^2}{2(32.2)}\left[0.0266\left(\frac{60}{2/12}\right) + 1.0 + .82 + .16 + 0.3\right] = 81.95 \text{ ft}$$

then $\quad \text{Power} = (62.4)(0.4)(81.95)/0.7 \div 550 \approx$ **5.31 hp** (4% less) *Ans.* (b)

P6.113 The parallel galvanized-iron pipe system of Fig. P6.113 delivers water at 20°C with a total flow rate of 0.036 m^3/s. If the pump is wide open and not running, with a loss coefficient $K = 1.5$, determine (a) the flow rate in each pipe and (b) the overall pressure drop.

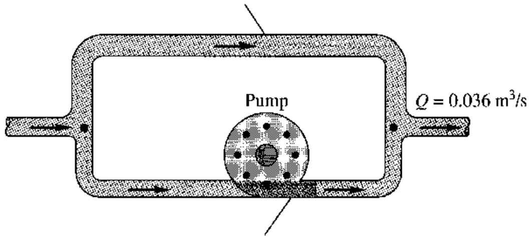


Fig. P6.11

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For galvanized iron, $\varepsilon = 0.15$ mm. Assume turbulent flow, with Δp the same for each leg:

$$h_{f1} = f_1 \frac{L_1}{d_1}\frac{V_1^2}{2g} = h_{f2} + h_{m2} = \frac{V_2^2}{2g}\left(f_2 \frac{L_2}{d_2} + 1.5\right),$$

$$\text{and} \quad Q_1 + Q_2 = (\pi/4)d_1^2 V_1 + (\pi/4)d_2^2 V_2 = Q_{total} = 0.036 \text{ m}^3/\text{s}$$

When the friction factors are correctly found from the Moody chart, these two equations may be solved for the two velocities (or flow rates). Begin by guessing f ≈ 0.020:

$$(0.02)\left(\frac{60}{0.05}\right)\frac{V_1^2}{2(9.81)} = \frac{V_2^2}{2(9.81)}\left[(0.02)\left(\frac{55}{0.04}\right) + 1.5\right], \quad \text{solve for } V_1 \approx 1.10V_2$$

$$\text{then} \quad \frac{\pi}{4}(0.05)^2(1.10V_2) + \frac{\pi}{4}(0.04)^2 V_2 = 0.036. \quad \text{Solve } V_2 \approx 10.54 \frac{m}{s}, \; V_1 \approx 11.59 \frac{m}{s}$$

$$\text{Correct } Re_1 \approx 578000, \quad f_1 \approx 0.0264, \quad Re_2 \approx 421000, \quad f_2 \approx 0.0282, \text{ repeat.}$$

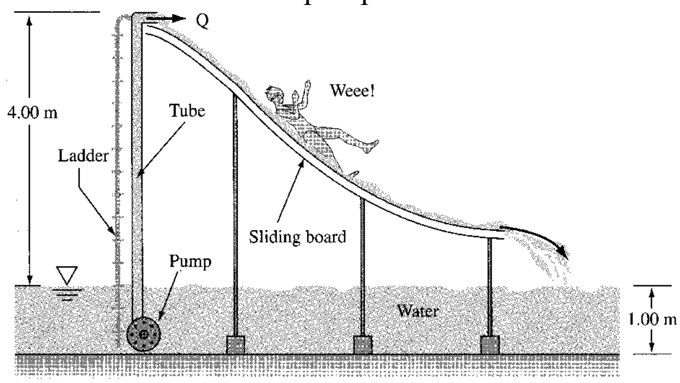
The 2nd iteration converges: $f_1 \approx 0.0264$, $V_1 = 11.69$ m/s, $f_2 \approx 0.0282$, $V_2 = 10.37$ m/s,

$Q_1 = A_1V_1 =$ **0.023 m³/s,** $Q_2 = A_2V_2 =$ **0.013 m³/s.** *Ans.* (a)

The pressure drop is the same in either leg:

$$\Delta p = f_1 \frac{L_1}{d_1}\frac{\rho V_1^2}{2} = \left(f_2 \frac{L_2}{d_2} + 1.5\right)\frac{\rho V_2^2}{2} \approx \mathbf{2.16E6\ Pa} \quad \textit{Ans. (b)}$$

C6.3 The water slide in the figure is to be installed in a swimming pool. The manufacturer recommends a continuous water flow of 1.39E–3 m^3/s (about 22 gal/min) down the slide to ensure that customers do not burn their bottoms. An 80%-efficient pump under the slide, submerged 1 m below the water surface, feeds a 5-m-long, 4-cm-diameter hose, of roughness 0.008 cm, to the slide. The hose discharges the water at the top of the slide, 4 m above the water surface, as a free jet. Ignore minor losses and assume $\alpha = 1.06$. Find the brake horsepower needed to drive the pump.



FigC6.3

Solution: For water take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. Write the steady-flow energy equation from the water surface (1) to the outlet (2) at the top of the slide:

$$\frac{p_a}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_a}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_f - h_{pump}, \quad \text{where } V_2 = \frac{1.39E\text{–}3}{\pi(0.02)^2} = 1.106 \ \frac{m}{s}$$

$$\textit{Solve for} \quad h_{pump} = (z_2 - z_1) + \frac{V_2^2}{2g}\left(\alpha_2 + f\frac{L}{d}\right)$$

Work out Red = ρVd/μ = (998)(1.106)(0.04)/0.001 = 44200, ε/d = 0.008/4 = 0.002,

whence fMoody = 0.0268. Use these numbers to evaluate the pump head above:

$$h_{pump} = (5.0 - 1.0) + \frac{(1.106)^2}{2(9.81)}\left[1.06 + 0.0268\left(\frac{5.0}{0.04}\right)\right] = 4.27 \text{ m},$$

$$\text{whence} \quad \mathbf{BHP_{required}} = \frac{\rho g Q h_{pump}}{\eta} = \frac{998(9.81)(1.39E\text{–}3)(4.27)}{0.8} = \mathbf{73\ watts} \quad \textit{Ans.}$$