ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2023 – HW1 Solution

P1.22 The *Ekman number*, Ek, arises in geophysical fluid dynamics. It is a dimensionless parameter combining seawater density ρ, a characteristic length L, seawater viscosity μ, and the Coriolis frequency $\Omega \sin\phi$, where Ω is the rotation rate of the earth and ϕ is the latitude angle. Determine the correct form of Ek if the viscosity is in the numerator.

Solution : First list the dimensions of the various quantities:

$$\{\rho\} = \{ML^{-3}\} \ ; \quad \{L\} = \{L\} \ ; \quad \{\mu\} = \{ML^{-1}T^{-1}\} \ ; \quad \{\Omega \sin\phi\} = \{T^{-1}\}$$

Note that $\sin\phi$ is itself dimensionless, so the Coriolis frequency has the dimensions of Ω. Only ρ and μ contain mass {M}, so if μ is in the numerator, ρ must be in the denominator That combination μ/ρ we know to be the kinematic viscosity, with units $\{L^2T^{-1}\}$. Of the two remaining variables, only $\Omega \sin\phi$ contains time $\{T^{-1}\}$, so it must be in the denominat So far, we have the grouping $\mu/(\rho\,\Omega \sin\phi)$, which has the dimensions $\{L^2\}$. So we put the length-squared into the denominator and we are finished:

$$\text{Dimensionless Ekman number:} \quad \text{Ek} = \frac{\mu}{\rho L^2 \Omega \sin\phi} \qquad \textit{Ans.}$$

P1.41 An aluminum cylinder weighing 30N, 6cm in diameter and 40cm long, is falling concentrically through a long vertical sleeve of diameter 6.04cm. The clearance is filled with SAE 50 oil at 20°C. Estimate the *terminal* (zero acceleration) fall velocity. Neglect air drag and assume a linear velocity distribution in the oil. [HINT: You are given diameters, not radii.]

Solution: From Table A.3 for SAE 50 oil, μ = 0.86 kg/m-s. The clearance is the difference in *radii*: 3.02 – 3.0cm = 0.02cm = 0.0002m. At terminal velocity, the cylinder weight must balance the viscous drag on the cylinder surface:

$$W = \tau_{wall} A_{wall} = (\mu \frac{V}{C})(\pi DL) , \quad \text{where } C = \text{clearance} = r_{sleeve} - r_{cylinder}$$

$$\text{or:} \qquad 30\,N = [0.86 \frac{kg}{m-s})(\frac{V}{0.0002\,m})]\ \pi(0.06\,m)(0.40\,m)$$

$$\text{Solve for } V = \mathbf{0.0925}\,m/s \qquad \textit{Ans.}$$