**The exam is closed book and closed notes.**

The velocity potential for a cylinder with radius *a* rotating in a uniform stream of ideal fluid with velocity *U* is:

$$ϕ=Ur\left(1+\frac{a^{2}}{r^{2}}\right)\cos(θ)+\frac{Γ}{2π}θ$$

where Γ is the circulation. (a) Find the tangential velocity $v\_{θ}$ at the surface of the cylinder (r=a). (b) For what value of the circulation (as a function of *a* and *U*) will the stagnation point be located at point B on the cylinder?

Note: $v\_{θ}=-\frac{1}{r}\frac{∂ϕ}{∂θ}$



**Solution:**

 (a)

**(+3)**

$$v\_{θ}=-\frac{1}{r}\frac{∂ϕ}{∂θ}$$

$$=-\frac{1}{r}\left[Ur\left(1+\frac{a^{2}}{r^{2}}\right)\left(-\sin(θ)\right)+\frac{Γ}{2π}\right]$$

$$=U\left(1+\frac{a^{2}}{r^{2}}\right)\sin(θ)-\frac{Γ}{2πr}$$

At the cylinder surface (r=a):

$v\_{θ}=2U\sin(θ)-\frac{Γ}{2πa}$

**(+2)**

(b)

For stagnation point:

**(+2)**

$$v\_{θ}=0 ⇒ \sin(θ\_{stag})=\frac{Γ}{4πUa}$$

At point B:

$$θ\_{stag}=\frac{3π}{2}$$

Therefore:

**(+3)**

$$\sin(\left(\frac{3π}{2}\right))=\frac{Γ}{4πUa}$$

$$-1=\frac{Γ}{4πUa}$$

$$Γ=-4πUa$$