Summary of Reynolds Stresses and TKE Levels for Different Flows

Geometry	$\sqrt{k}/U_{max}$	$\sqrt{\overline{uu}}/U_{max}$	$\overline{vv}/\overline{uu}$	$\overline{ww}/\overline{uu}$	$\overline{uv}/\overline{uu}$	$\overline{uw}/\overline{uu}$	$\overline{vw}/\overline{uu}$
Wall y+<50	0.089	0.1 (y+=12)	0.24	0.35	0.15	/	/
BL (0.1<y/δ<0.7)	/	$\sqrt{k}/U_{max}$	$\overline{vv}+\overline{ww}=\overline{uu}$		/	/	/
BL (y/δ>0.7)							
BL (flat plate, y/δ<0.8, $Re_x=10^7$)	0.14	0.12	0.12	0.29	0.11	/	/
Wake	0.98	0.90	0.89	0.89	/	/	/
Jet	0.21	0.29	0.56	0.63	0.25	/	/
Plane mixing layer	0.19	0.17	0.60	0.77	0.33	0	0
Separated turbulent boundary layer	0.11	0.13	0.23	0.41	0.00108	/	/
Backward-facing step	0.18	0.18	0.44	0.63	0.0031	/	/
NACA0024 ($Re=2.26\times10^6$)	0.55	0.71	0.40	0.60	0.40	0.33	0.07
Landing Gear ($Re=6\times10^5$)	0.50	/	/	/	/	/	/
Flat plate at high incidence ($Re_c=2\times10^4$)	0.55	0.50	/	/	/	/	/
Sphere ($Re=1\times10^4$)	0.32	0.24	/	/	/	/	/

* non-dimensionalized by U_{max}^2

TKE Budget and Reynolds Stress for Canonical Flows

C-convection; P-production; T-Transport; VD- viscous diffusion; VP-velocity pressure gradient; ε-dissipation

1. DNS of a **plane mixing layer** (Rogers & Moser, Physics of fluids, 1994)

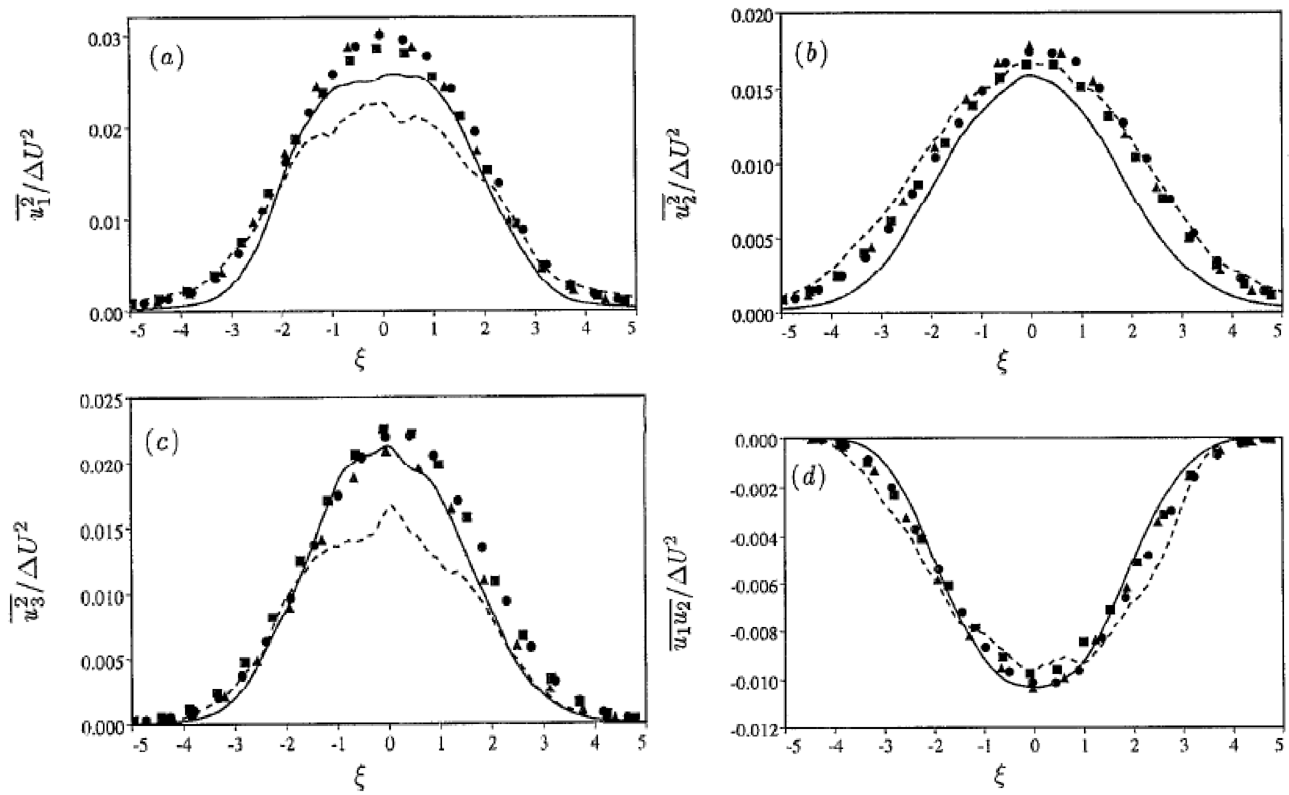


Figure 1.Comparison of the time-averaged ($\xi = x_2/\delta_m$) simulation results for the components of the Reynolds stress tensor (-) with the results of Bell and Mehta (1990) and the simulation profiles at τ= 187.5 (----).

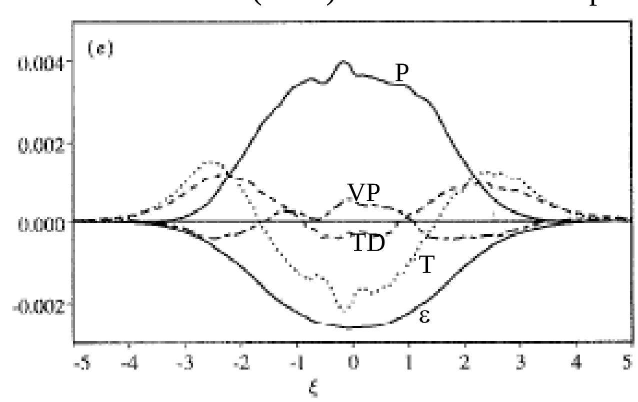


Figure 2. TKE budget for a plane mixing layer: δ_m is the momentum thickness.

Main conclusions:

(1) **Reynolds normal stresses (+)**: peak in the center of the mixing layer;
$\overline{uu}$ (0.030)> $\overline{ww}$ (0.023)> $\overline{vv}$ (0.018)

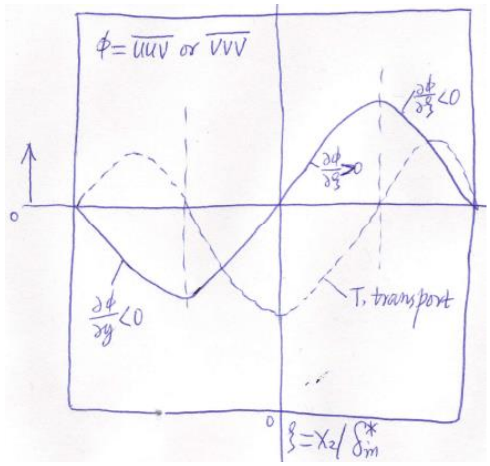
(2) **Reynolds shear stress (-)**: peaks in the center of the mixing layer; -$\overline{uv}$ (0.010) < $\overline{vv}$ (0.018)

(3) **TKE (+)**: peaks in the center of the mixing layer (0.036)

A. **Production** (+): $P_{ij} = -\overline{u_i u_j}\frac{\partial U_i}{\partial x_j} = -\overline{u_i v}\frac{\partial U_i}{\partial y} = -\overline{uv}\frac{\partial U}{\partial y} - \overline{vv}\frac{\partial V}{\partial y} - \overline{wv}\frac{\partial W}{\partial y}$ **dominant and main producing term**, peaks in the center of the layer

B. **Dissipation** (-): $\varepsilon_{ij} = -\frac{1}{\mathrm{Re}}\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_k}} = -\frac{1}{\mathrm{Re}}\overline{\left(\frac{\partial u_i}{\partial y}\right)^2} = -\frac{1}{\mathrm{Re}}\overline{\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial y}\right)^2}$ main consuming term; peaks in the center of the layer, $\varepsilon/P = 0.66$

C. **Transport** (+ or -): $T_{ij} = -\frac{1}{2}\frac{\partial \overline{u_i u_i u_j}}{\partial x_j} = -\frac{1}{2}\left(\frac{\partial \overline{uuv}}{\partial y}+\frac{\partial \overline{vvv}}{\partial y}\right)$ peaks in the center of the layer with $T/P = 0.57$. Zero at two locations that correspond to the half-thickness of the layer. , i.e., move TKE from the middle of the shear layer (bounded by half-thickness of the layer $\xi = \pm 1.5$) to the edge of the shear layer.



D. **Convection**: $C_{ij} = -U_j\frac{\partial k}{\partial x_j} = -V\frac{\partial k}{\partial y}$ not available.

E. **Velocity-pressure gradient** (+ or -): $VP_{ij} = -\frac{\partial \overline{p' u_j}}{\partial x_j} = -\frac{\partial \overline{p' V}}{\partial y}$, peaks in the center of the layer with $VP/P = 0.15$. Zero at the same location of the zero for transport and shows opposite sign of transport.

F. **Viscous diffusion**: $D_{ij} = \frac{1}{\mathrm{Re}}\frac{\partial^2 k}{\partial x_j^2} = \frac{1}{\mathrm{Re}}\frac{\partial^2 k}{\partial y^2}$ an order of magnitude smaller than any other term across the entire layer and thus neglected.

2. DNS of a **separated turbulent boundary layer** (DNS, Na and Moin, JFM 1998)

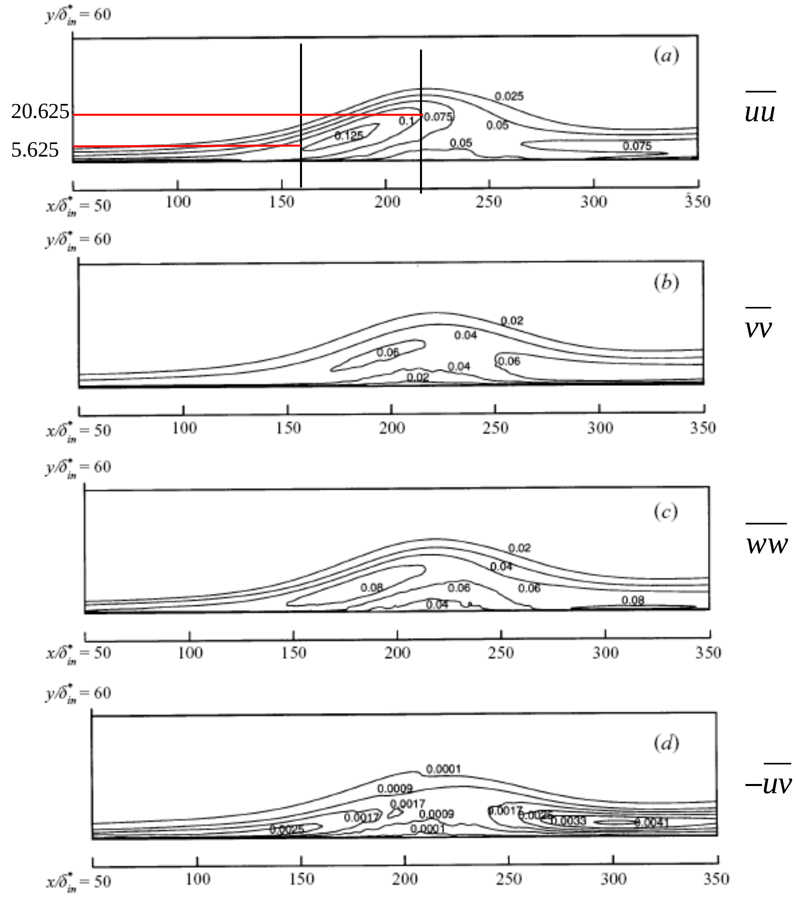


Figure 3. Contour of Reynolds stresses

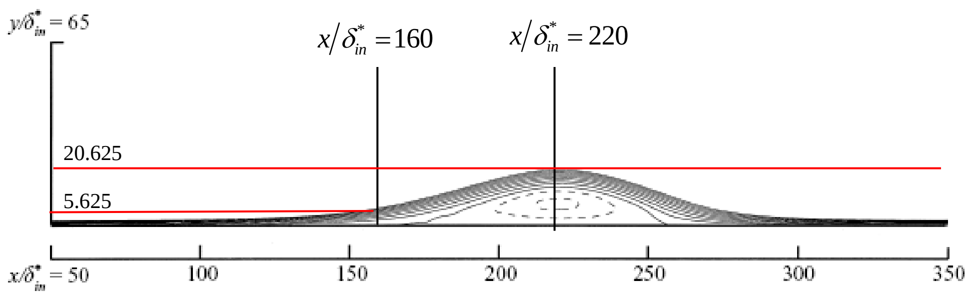


Figure 4. Streamlines of the mean flow

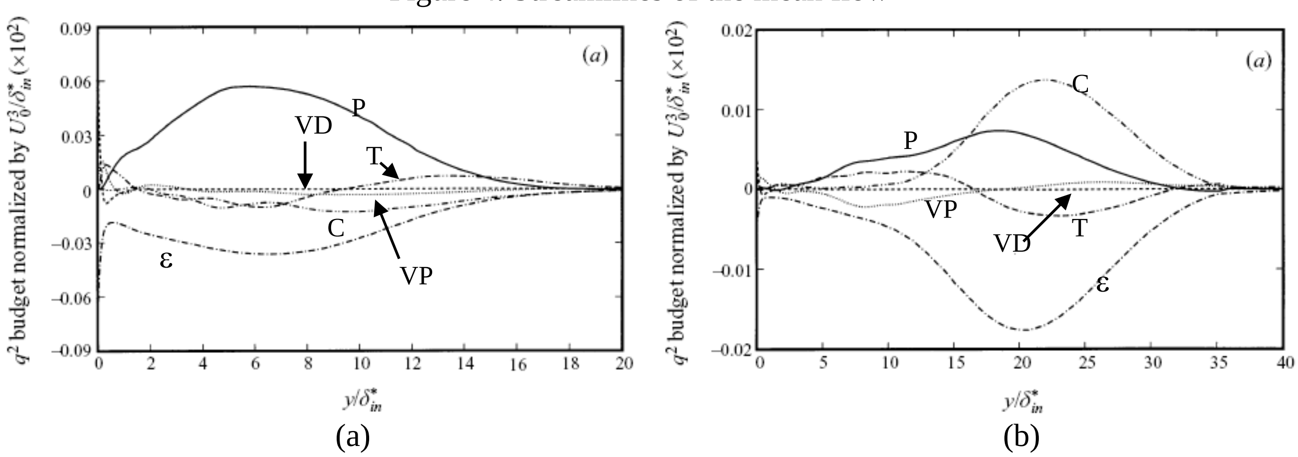
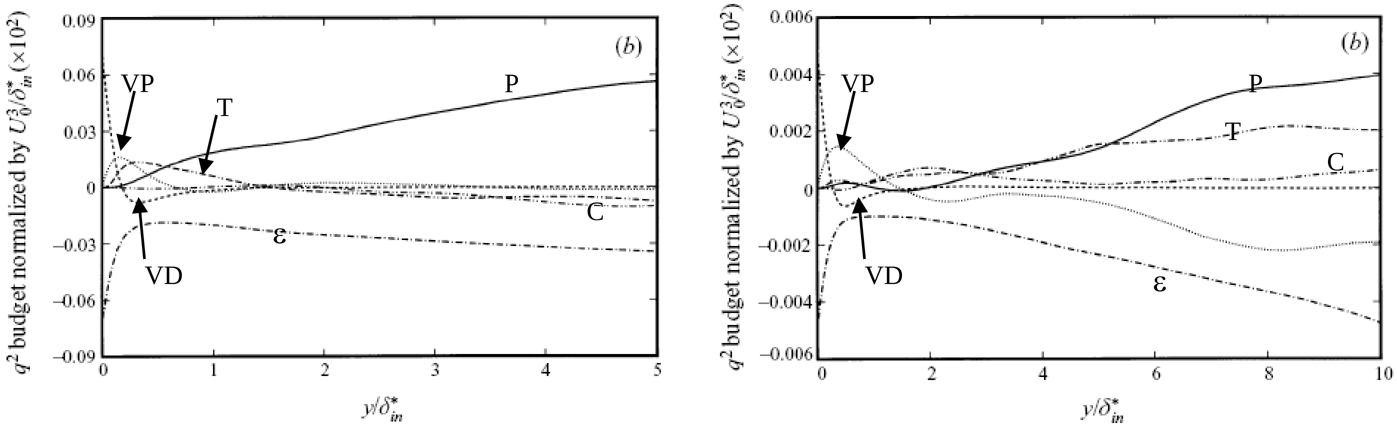


Figure 5. TKE budget for a **separated turbulent boundary layer under APG (entire region)**: (a) in the detachment region ($x/\delta_{in}^* = 160$); (b) in the middle of the separation bubble ($x/\delta_{in}^* = 220$)



(a) (b)

Figure 6. TKE budget for a **separated turbulent boundary layer under APG (CLOSE TO THE WALL)**: (a) in the detachment region ($x/\delta_{in}^* = 160$); (b) in the middle of the separation bubble ($x/\delta_{in}^* = 220$)

Main conclusions:

(1) **Reynolds normal stresses (+)**: peak near the toe and reach local maxima on the high-speed side of the free shear line (the separation streamline that divides the separation bubble from outer region).
$\overline{uu}$ (0.125)> $\overline{ww}$ (0.08)> $\overline{vv}$ (0.06)

(2) **Reynolds shear stress (+)**: maxima are significantly reduced up to the middle of the separation bubble. It increases thereafter and reaches its maximum value downstream of the reattachment region.
$-\overline{uv}$ (0.0041) < $\overline{vv}$ (0.06)

(3) **TKE (+):** peaks near the toe (0.133)

(4) **TKE budget near the toe** $x/\delta_{in}^* = 160$:

A. **Production** (+): **dominant term**, peaks on the high-speed side of the free shear line; decrease to zero on the wall.

B. **Dissipation** (-): peaks on the high-speed side of the free shear line ($\varepsilon/P = 0.64$); significant consuming term near the wall and balanced mainly by viscous diffusion.

C. **Transport** (+ or -): peaks on the high-speed side of the free shear line ($T/P = 0.15$). Zero at two locations. One is very close to the wall ($y/\delta_{in}^* = 1.5$) and the other is on the high-speed side ($y/\delta_{in}^* = 9.0$ even farther away from the wall than P and ε), i.e., move TKE from the shear layer towards the wall and outer region.

D. **Convection** (-): peaks at $y/\delta_{in}^* = 9.0$ even farther away from the wall than P and ε, $C/P = 0.22$; decrease to zero on the wall; The main contribution to the convection term is from the longitudinal component C_{11}:

$$C_{11} = -U\frac{\partial \overline{uu}}{\partial x} - V\frac{\partial \overline{uu}}{\partial y}$$, where the first term $-U\frac{\partial \overline{uu}}{\partial x}$ is dominant.

At $x/\delta_{in}^* = 160$, U>0, $\frac{\partial \overline{uu}}{\partial x} > 0$, so C<0.

E. **Velocity-pressure gradient** (+ or -): almost zero across the shear layer and only significant near the Wall (+).

F. **Viscous diffusion (+)**: an order of magnitude smaller than any other term across the entire layer except near the wall where it becomes a significant producing term and balanced by ε.

(5) **TKE budget in the middle of the separation bubble** $x/\delta_{in}^{*} = 220$:

A. **Production** (+): peaks on the high-speed side of the free shear line, $P/\varepsilon = 0.43$; decrease to zero on the wall.

B. **Dissipation** (-): **dominant term** peaks on the high-speed side of the free shear line; significant consuming term near the wall and balanced mainly by viscous diffusion.

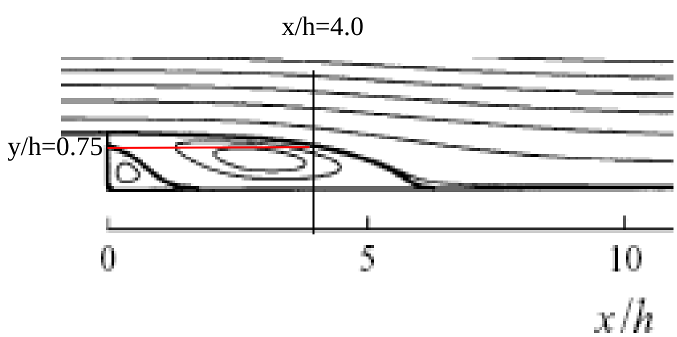
C. **Transport** (+ or -): peaks on the high-speed side of the free shear line with $T/\varepsilon = 0.22$. Zero at two locations. One is very close to the free shear line ($y/\delta_{in}^{*} = 16$) and the other is on the high-speed side ($y/\delta_{in}^{*} = 31$ even farther away from the solid surface than P, ε, and C), i.e., move TKE from the region bounded by the free shear line and the high-speed side of the free shear line into the separation bubble and outer region.

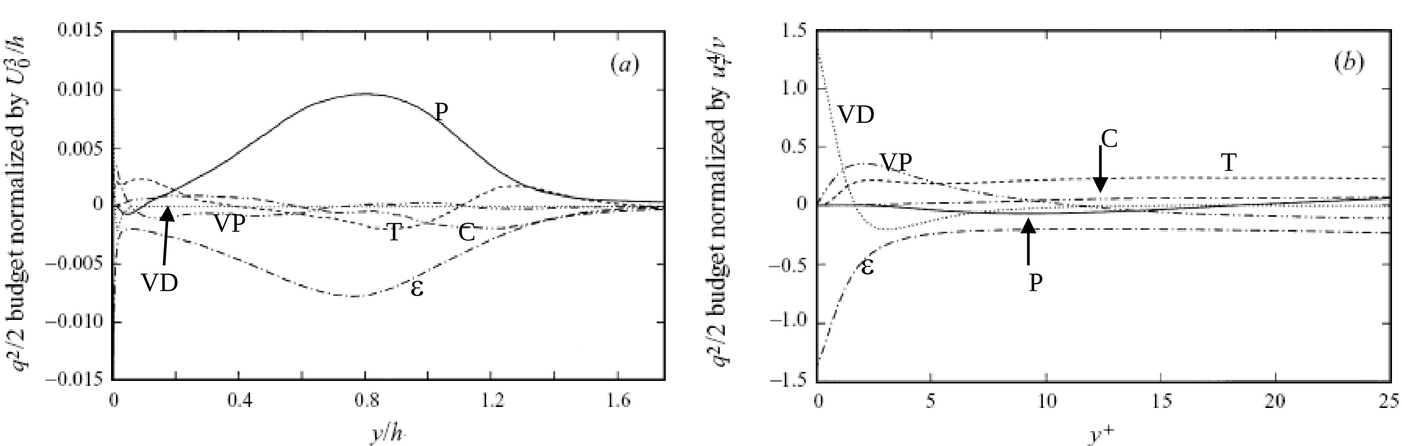
D. **Convection** (+): peaks at almost the same location as dissipation, $C/\varepsilon = 0.78$; The main contribution to the convection term is from the longitudinal component C11: $C_{11} = -U\dfrac{\partial \overline{uu}}{\partial x} - V\dfrac{\partial \overline{uu}}{\partial y}$, where the first term $-U\dfrac{\partial \overline{uu}}{\partial x}$ is dominant. At $x/\delta_{in}^{*} = 220$, U>0, $\dfrac{\partial \overline{uu}}{\partial x} < 0$, so C>0.

E. **Velocity-pressure gradient** (+ or -): almost zero across the shear layer and only significant near the wall (+).

F. **Viscous diffusion (+)**: an order of magnitude smaller than any other term across the entire layer except near the wall where it becomes a significant producing term and balanced by ε.

3. DNS of a **backward-facing step** flow (Le, Moin, and Kim, JFM 1997)





(a) (b)

Figure 7. TKE budget for a **backward-facing step** flow at *x/h*=4.0: (a) overall; (b) near the wall.

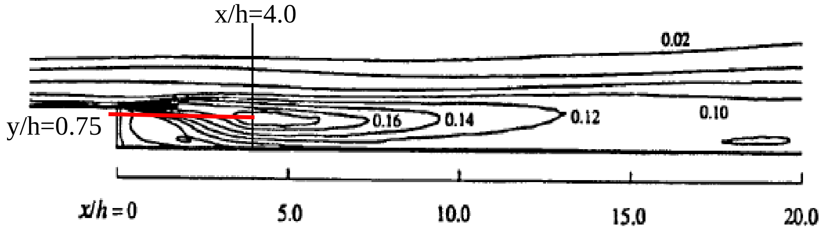


Figure 5.57. Longitudinal turbulence intensity contours ($\sqrt{\overline{u'^2}}/U_0$).

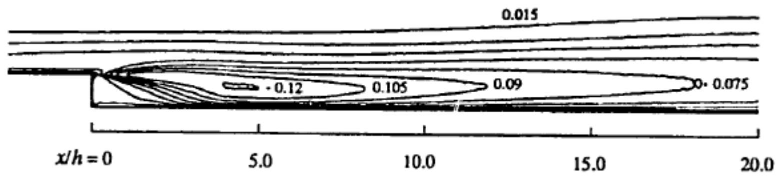


Figure 5.59. Vertical turbulence intensity contours ($\sqrt{\overline{v'^2}}/U_0$).

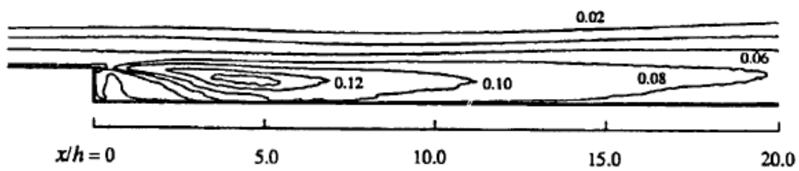


Figure 5.61. Spanwise turbulence intensity contours ($\sqrt{\overline{w'^2}}/U_0$).

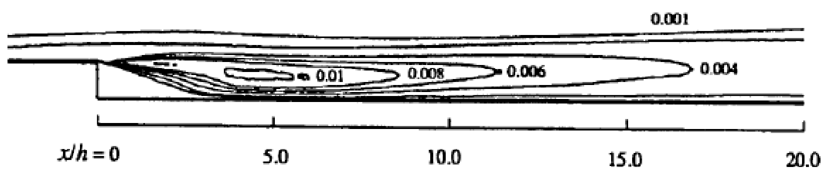


Figure 5.63. Reynolds shear stress contours ($-\overline{u'v'}/U_0^2$).

(1) **Reynolds normal stresses (+)**: peak on the high-speed side of the free shear line, NOT near the separation point. $\overline{uu}$ (0.18)> $\overline{ww}$ (0.14)> $\overline{vv}$ (0.12)

(2) **Reynolds shear stress (+)**: peak on the high-speed side of the free shear line, NOT near the separation Point. -$\overline{uv}$ (0.01) < $\overline{vv}$ (0.12).

(3) **TKE (+):** peaks on the high-speed side of the free shear line (0.22).

(4) **TKE budget in the middle of the separation bubble** $x/h = 4$:

A. **Production** (+): **dominant term,** peaks on the high-speed side of the free shear line and decrease to zero on the wall. Production is mostly due to the production of the longitudinal stress**,** $P_{11} = -\overline{uu}\dfrac{\partial U}{\partial x}$**,** which is different from what found by Piirto *et al.* **(**Measuring Turbulence Energy with PIV in a Backward-facing Step Flow, Experiments in Fluids Vol. 35, 2003.**)** who concluded that "Production is mostly due to the Production of the longitudinal stress, $P_{21} = -\overline{uv}\dfrac{\partial V}{\partial x}$ "

B. **Dissipation** (-): peaks on the high-speed side of the free shear line ($\varepsilon/P = 0.78$); significant consuming term near the wall and balanced mainly by viscous diffusion.

C. **Transport** (+ or -): peaks on the high-speed side of the free shear line ($T/P = 0.21$) and decreases to zero on the wall; Also zero at y/h=0.3 and on the high-speed side (y/h=1.1) even farther away from the wall than P and ε), i.e., Transport moves TKE from 0.3<y/h<1.1 towards the wall and the outer region.

D. **Convection** (+): peaks on the high-speed side of the free shear line ($C/P = 0.21$) and decreases to zero on the wall.

E. **Velocity-pressure gradient** (+ or -): at least one-order smaller than other terms and only significant near the wall (+).

F. **Viscous diffusion**: an order of magnitude smaller than any other term across the entire layer except near the wall where it becomes a significant producing term and balanced by ε.

Turbulent Kinetic Energy Budget and Reynolds Stress Budget Analysis

1. Canonical Flow (c—convection; p—production; ε—dissipation; T—transport; VP—velocity-pressure gradient)

Geometry	Parameter/ Approach	TKE budget					Reynolds stress Budget					Comments	Reference
		C	P	ε	T	VP	C	P	ε	T	VP		
Axisymmetric wake of sphere	$Re_D = 8,600$ EFD	Dominant, Peaks near axis	P=0.2 ε P=0.15C	Peaks at axis; ε =0.5C	T=0.5C	N/A	N/A	N/A	N/A	N/A	N/A	Turbulence is strongly influenced by conditions upstream. Reynolds stress reach self-similarity for (50<x/d<150).	Uberoi and Freymuth (1970), Physics of fluids, 13, 2205-2210.
Plane mixing layer	Re=2x10^4 DNS	N/A	Dominant term, peaks at the center of the mixing layer (ML)	Peaks at the center of ML, ε =0.71P	Peaks at center of ML, T=0.65P	N/A	$\overline{u^2}$ N/A	Dominant, peaks at center of ML	Peaks at center of ML, ε =0.25P	Peaks at center of ML, ε =0.25P	Peaks at center of ML, VP=0.5P	All terms peaks at center of mixing layer; Production is always dominant terms for $\overline{u^2}$ and $\overline{uv}$, while VP is the dominant term for $\overline{v^2}$ and $\overline{w^2}$. Flow rate of TKE increases linearly with x, which in contrast to jet and wakes.	Rogers and Moser, 1994, Physics of fluids 6 (2)
							$\overline{v^2}$ N/A	N/A	Peaks at center of ML, ε =0.5VP	Peaks at center of ML, T =0.5VP	Dominant, peaks at center of ML		
							$\overline{w^2}$ N/A	N/A	Peaks at center of ML, ε =0.6VP	Peaks at center of ML, T =0.36VP	Dominant, peaks at center of ML		
							$\overline{uv}$ N/A	Dominant, peaks at center of ML	Peaks at center of ML, ε =0.63P	Peaks at center of ML, T=0.5P	N/A		
Turbulent boundary layer	R_θ=1410 DNS	C=0 (y+<50) Peaks at BL edge	Peaks at y+=12; P=ε (y+=40~ y/δ=0.4); P=0 (y>δ)	Peaks at wall; ε=P (y+=40~ y/δ=0.4)	Peaks at BL edge	Peaks at y/δ=0.8; VP=0.5C; VP=0 for most BL	C=0 (y+<50) Peaks at BL edge	P11 is the dominant term of $\overline{u^2}$	Peaks at wall except for shear stress	Peaks at BL edge	VP dominant terms compared to 0 in TKE budget	Effect of pressure fluctuation is to redistribute energy from $\overline{u^2}$ to $\overline{v^2}$ and $\overline{w^2}$	Spalart (1988), JFM, 187, pp. 61-98.
Fully developed channel flow	Re=13,750 DNS	C=0 (y+<50)	Peaks at y+=12, where $P = 1.8\varepsilon$	Peaks at wall, $\varepsilon = P$	Peaks at y+=5 (>0) and y+=12 (<0)	VP=0	N/A	N/A	N/A	N/A	N/A	Peak P occurs where viscous stress and the Reynolds stress equal. Transports energy toward the wall and the log-law region.	Kim et al., 1987, JFM, 177, 133-166.

1. Canonical Flow (c—convection; p—production; ε—dissipation; t—transport; vp—velocity-pressure gradient)

Geometry	Parameter/ Approach	TKE budget					Reynolds stress Budget						Comments	Reference
		C	P	ε	T	VP		C	P	ε	T	VP		
Axisymmetric jet	EFD	Peaks at centerline; C=0.74 ε	Peaks at $r/r_{1/2}=0.6$ P=0.82 ε	Dominant; peaks at centerline	Peaks at $r/r_{1/2}=0.5$ T=0.35 ε	N/A	$\overline{u^2}$	Peaks at centerline, C=0.65 ε	Dominant term, Peaks at $r/(x-x_0)=0.06$ P=1.48ε	Peaks at $r/(x-x_0)=0.06$	Peaks at $r/(x-x_0)=0.06$ T=0.2ε	Peaks at centerline, VP=0.65ε	At edge, turbulence production goes to zero and turbulent transport balances dissipation, Reynolds stress decay. Reynolds stress decay when approaching the edge and exhibit significant anisotropy.	Panchapakesan and Lumley (1993), JFM, 246, 197-223. Hussein et al. (1994), JFM, 258, 31-75.
							$\overline{v^2}$	Peaks at centerline, C=0.46 ε	Peaks at centerline, P=0.20ε	Dominant term, peaks at centerline	Peaks at $r/(x-x_0)=0.1$ T=0.5ε	Peaks at centerline, VP=0.72ε		
							$\overline{w^2}$	Peaks at centerline, C=0.46 ε	Peaks at centerline, P=0.10ε	Peaks at centerline	Peaks at centerline T=0.58ε	Dominant term, Peaks at centerline, VP=1.42ε		
							$\overline{uv}$	Peaks at $r/(x-x_0)=0.05$ C=0.1T	Peaks at $r/(x-x_0)=0.05$ C=0.17T	N/A	Dominant term, peaks at $r/(x-x_0)=0.05$	N/A		

2. Separated Flow (separated turbulent boundary layer under Adverse Pressure Gradient)

Geometry	Parameter/ Approach	TKE budget						Reynolds stress Budget						Comments	Reference
			C	P	ε	T	VP		C	P	ε	T	VP		
Turbulent boundary layer (separated)	$\mathrm{Re}_\theta=300$ DNS	x*= 160	C= 0.2P	Dominant, peaks at y*=6	Two peaks at wall (ε=1.2P); at y*=6 (0.64P)	Peaks at y*=5, T=0.2P	Peaks at y*=0.2, VP=0.25P	$\overline{u^2}$	N/A	N/A	N/A	N/A	N/A	x*=160 is located at the detached region; x*=220 is located at the middle of separation bubble; x*=270 is located at the reattachment region; x*=320 is located at far downstream. $x^*=x/\delta^*_{in}$ $y^*=y/\delta^*_{in}$	Na and Moin, 1998, JFM.
		x*= 220	Peaks at y*=20; C=0.7 8 ε	Peaks at y*=18; P=0.44ε	Dominant, Peaks at y*=20	Peaks at y*=20; T=0.17 ε	Peaks at y*=8; VP=0.11 ε	$\overline{u^2}$	Dominant, peaks at y*=25	Peaks at y*=20, P=0.08C	Peaks at y*=21, ε =0.63C	Peaks at y*=25 T=0.4C	Peaks at y*=21, VP =0.92C		
		x*= 270	Peaks at y*=11 C=P	dominant term ; Peaks at y*=9;	Peaks at wall; ε =2.2P	Peaks at y*=5; T=0.3P	Peaks at y*=0.25; VP=0.65P	$\overline{u^2}$	N/A	N/A	N/A	N/A	N/A		
		x*= 320	Peaks at y*=12 ;0.19P	Peaks at y*=0.4; dominant term	Peaks at wall; ε =1.4P	Peaks at y*=0.5; T=0.38P	Peaks at y*=0.5; VP=0.19P	$\overline{u^2}$	Peaks at wall, C=0	Dominant, peaks at y*=0.45	Peaks at wall, ε =0.88P	Peaks at y*=0.2, T=0.25P	Peaks at wall, VP=0.88P		

3. Separated Flow (backward-facing step, DNS by Huang Le, 1995, Ph.D. thesis)
HSSL stands for "high-speed side of the free shear layer line"

<table>
<tr><th rowspan="2">Geometry</th><th rowspan="2">Parameter/ Approach</th><th colspan="5">TKE budget (x/h=-2, before separation)</th><th colspan="6">Reynolds stress Budget (x/h=-2, before separation)</th><th rowspan="2">Comments</th></tr>
<tr><th>C</th><th>P</th><th>ε</th><th>T</th><th>VP</th><th></th><th>C</th><th>P</th><th>ε</th><th>T</th><th>VP</th></tr>
<tr><td rowspan="4">Backward-facing step</td><td rowspan="4">Re_h=10^5
DNS</td><td rowspan="4">0</td><td rowspan="4">Dominant, y/h=1.03</td><td rowspan="4">Peaks at wall, ε=1.35P</td><td rowspan="4">Peaks at y/h =1.019
T=0.54P</td><td rowspan="4">Peaks at y/h=1.01
VP= 0.07P</td><td>$\overline{u^2}$</td><td>0</td><td>Dominant, peaks at (y-h)/h =0.04</td><td>Peaks at wall, ε=viscous diffusion</td><td>Peaks at (y-h)/h =0.02, T=0.5P</td><td>Peaks (y-h)/h >0.1 VP=0.13P</td><td rowspan="4">For normal stress $\overline{u^2}$ and shear stress $\overline{uv}$, turbulence production is dominant term. For $\overline{v^2}$ and $\overline{w^2}$, velocity-pressure gradient term is dominant.</td></tr>
<tr><td>$\overline{v^2}$</td><td>Peaks at (y-h)/h =0.7 C=0.19VP</td><td>0</td><td>Peaks at (y-h)/h =0.1 E=0.57 VP</td><td>0</td><td>Dominant, Peaks at (y-h)/h =0.2</td></tr>
<tr><td>$\overline{w^2}$</td><td>0</td><td>N/A</td><td>Peaks at wall, ε=VP</td><td>0</td><td>Dominant, peaks at y/h=0.025</td></tr>
<tr><td>$\overline{uv}$</td><td>0</td><td>Dominant, peaks at (y-h)/h =0.08</td><td>0</td><td>Peaks at (y-h)/h =0.04
T=0.54P</td><td>Peaks at (y-h)/h =0.05
VP=P</td></tr>
</table>

<table>
<tr><th rowspan="2">Geometry</th><th rowspan="2">Parameter/ Approach</th><th colspan="5">TKE budget (x/h=4, recirculation region)</th><th colspan="6">Reynolds stress Budget (x/h=4, recirculation region)</th><th rowspan="2">Comments</th></tr>
<tr><th>C</th><th>P</th><th>ε</th><th>T</th><th>VP</th><th></th><th>C</th><th>P</th><th>ε</th><th>T</th><th>VP</th></tr>
<tr><td rowspan="4">Backward-facing step</td><td rowspan="4">Re_h=10^5
DNS</td><td rowspan="4">0</td><td rowspan="4">Dominant term, peaks y/h=0.8</td><td rowspan="4">Peaks at y/h=0.8
ε=0.83P</td><td rowspan="4">Peaks y/h=0.9
T=0.28P</td><td rowspan="4">Peaks y/h=0.18
VP=0.11P</td><td>$\overline{u^2}$</td><td>Peaks y/h=1.3
C=0.11P</td><td>Dominant term, peaks at HSSL</td><td>Peaks at HSSL
ε=0.34P</td><td>Peaks at y/h=0.1
T=0.14P</td><td>Peaks at HSSL
VP=0.69P</td><td rowspan="4">For normal stress $\overline{u^2}$ and shear stress $\overline{uv}$, turbulence production is dominant term. For $\overline{v^2}$ and $\overline{w^2}$, velocity-pressure gradient term is dominant.</td></tr>
<tr><td>$\overline{v^2}$</td><td>Peaks y/h=1.2
C=0.25VP</td><td>Peaks at y/h=0.62
P=0.4VP</td><td>Peaks at y/h=0.7
ε=0.88VP</td><td>Peaks at y/h=0.02
T=0.83VP</td><td>Dominant term, peaks at y/h=0.8</td></tr>
<tr><td>$\overline{w^2}$</td><td>Peaks y/h=1.0
C=0.21VP</td><td>N/A</td><td>Peaks at y/h=0.8
ε=0.8P</td><td>Peaks at y/h=0.85
T=0.2VP</td><td>Dominant term, peaks at y/h=0.8</td></tr>
<tr><td>$\overline{uv}$</td><td>0</td><td>Dominant term, peaks at y/h=0.7</td><td>0</td><td>Peaks at y/h=0.70
T=0.22P</td><td>Peaks at y/h=0.60
T=0.72P</td></tr>
</table>

Geometry	Parameter/ Approach	TKE budget (**x/h=7,** reattachment region)					Reynolds stress Budget (**x/h=7,** reattachment region)						Comments
		C	P	ε	T	VP		C	P	ε	T	VP	
Backward-facing step	Re_h=10^5 DNS	0	Dominant term, peaks at y/h=0.75	Peaks at y/h=0.7, ε=0.67P	Peaks y/h=0.7 T=0.55P	0	$\overline{u^2}$	Peaks at y/h=0.5, C=0.2P	Dominant term, peaks at HSSL (y/h=0.8)	Peaks on wall, ε_{SL}=0.33 P	Peaks at HSSL, T=0.25P	Peaks at HSSL, VP=0.6P	For normal stress $\overline{u^2}$ and shear stress $\overline{uv}$, turbulence production is dominant term. For $\overline{v^2}$ and $\overline{w^2}$, velocity-pressure gradient term is dominant.
							$\overline{v^2}$	Peaks at y/h=1.25, C=0.26VP	Peaks at y/h=0.5 P=0.43VP	Peaks at y/h=0.6, ε=0.63VP	Peaks at y/h=0.05 T=1.14VP	Dominant term, peaks at y/h=0.05	
							$\overline{w^2}$	0	N/A	Peaks at y/h=0.7 ε=VP (in SL)	0	Dominant term, peaks at y/h=0.7	
							$\overline{uv}$	Peaks at y/h=0.5 C=0.11P	Dominant term, peaks at y/h=0.0075	0	Peaks at y/h=0.05, T=0.5P	Peaks at y/h=0.0075, VP=0.67P	

Geometry	Parameter/ Approach	TKE budget (**x/h=10,** behind reattachment)					Reynolds stress Budget (**x/h=10,** behind reattachment)						Comments
		C	P	ε	T	VP		C	P	ε	T	VP	
Backward-facing step	Re_h=10^5 DNS	0	Dominant term, peaks at y/h=0.8	Peaks at wall. At y/h=0.8, ε=P	Peaks at y/h=0.8 T=0.5P	0	$\overline{u^2}$	Peaks at y/h=0.5, C=0.2P	Dominant term, peaks at HSSL	Peaks at wall, ε_{SL}=0.2P	Peaks at y/h=0.75, T=0.18P	Peaks at HSSL, VP=0.5P	For normal stress $\overline{u^2}$ and shear stress $\overline{uv}$, turbulence production is dominant term. For $\overline{v^2}$ and $\overline{w^2}$, velocity-pressure gradient term is dominant.
							$\overline{v^2}$	Peaks at y/h=1.5 C=0.29VP	Peaks at y/h=0.6, P=0.36VP	Peaks at y/h=0.7, ε=0.7VP	Peaks at y/h=0.05 T=1.29VP	Dominant term, peaks at y/h=0.05	
							$\overline{w^2}$	0	N/A	Peaks at y/h=0.7 ε=VP	0	Dominant term, peaks at y/h=0.7	
							$\overline{uv}$	Peaks at y/h=0.6 C=0.11P	Dominant term, peaks at y/h=0.75	0	Peaks at y/h=0.005 T=0.57P	Peaks at y/h=0.05 VP=0.9P	

<table>
<tr><th rowspan="2">Geometry</th><th rowspan="2">Parameter/ Approach</th><th colspan="5">TKE budget (x/h=18, recovery region)</th><th colspan="6">Reynolds stress Budget (x/h=18, recovery region)</th><th rowspan="2">Comments</th></tr>
<tr><th>C</th><th>P</th><th>ε</th><th>T</th><th>VP</th><th></th><th>C</th><th>P</th><th>ε</th><th>T</th><th>VP</th></tr>
<tr><td rowspan="4">Backward-facing step</td><td rowspan="4">Re_h=10^5
DNS</td><td rowspan="4">0</td><td rowspan="4">Dominant term, peaks at y/h=0.05</td><td rowspan="4">Peaks at wall,
ε=P</td><td rowspan="4">Peaks at y/h=1.0
T=0.5P</td><td rowspan="4">0</td><td>$\overline{u^2}$</td><td>0</td><td>Dominant term, peaks at y/h=0.05</td><td>Peaks at wall,
ε=0.42P</td><td>Peaks at y/h=1.2
T=0.01P</td><td>Peaks at y/h=1.5
VP=0.2P</td><td rowspan="4">For normal stress $\overline{u^2}$ and shear stress $\overline{uv}$, turbulence production is dominant term. For $\overline{v^2}$ and $\overline{w^2}$, velocity-pressure gradient term is dominant.</td></tr>
<tr><td>$\overline{v^2}$</td><td>Peaks at y/h=0.6
C=0.19VP</td><td>Peaks at y/h=0.5
P=0.08VP</td><td>Peaks at y/h=0.75
ε=0.61VP</td><td>Peaks at y/h=0.05
T=0.76VP</td><td>Dominant term, peaks at y/h=1.0</td></tr>
<tr><td>$\overline{w^2}$</td><td>C=T=0.2 VP</td><td>N/A</td><td>ε=VP</td><td>T=C=0.2 VP</td><td>Dominant term, peaks near wall, y/h=0.01</td></tr>
<tr><td>$\overline{uv}$</td><td>Peaks at y/h=0.8,
C=0.14P</td><td>Dominant term, peaks at y/h=1,0.1</td><td>0</td><td>Peaks at y/h=1,
T=0.29P</td><td>Peaks at y/h=0.05,
VP=1.35P</td></tr>
</table>