ME:5160 (58:160) Intermediate Mechanics of Fluids

Fall 2020 – HW10 Solution

6.71 It is desired to solve Prob. 6.62 for the most economical pump and cast-iron pipe system. If the pump costs \$125 per horsepower delivered to the fluid and the pipe costs \$7000 per inch of diameter, what are the minimum cost and the pipe and pump size to maintain the 3 ft^3/s flow rate? Make some simplifying assumptions.

Solution: For water at 20°C, take ρ = 1.94 $slug/ft^3$ and μ = 2.09E−5 slug/ft·s. For cast iron, take $\varepsilon \approx 0.00085$ ft. Write the energy equation (from Prob. 6.62) in terms of Q and d:

$$P_{in\ hp} = \frac{\rho g Q}{550}(\Delta z + h_f) = \frac{62.4(3.0)}{550}\left\{120 + f\left(\frac{2000}{d}\right)\frac{[4(3.0)/\pi d^2]^2}{2(32.2)}\right\} = 40.84 + \frac{154.2f}{d^5}$$

$$\text{Cost} = \$125P_{hp} + \$7000d_{inches} = 125(40.84 + 154.2f/d^5) + 7000(12d), \quad \text{with d in ft.}$$

$$\text{Clean up:} \quad \text{Cost} \approx \$5105 + 19278f/d^5 + 84000d$$

Regardless of the (unknown) value of f, this Cost relation does show a minimum. If we assume for simplicity that *f* is constant, we may use the differential calculus:

$$\frac{d(\text{Cost})}{d(d)}\bigg|_{f\approx const} = \frac{-5(19278)f}{d^6} + 84000, \quad \text{or} \quad d_{best} \approx (1.148\ f)^{1/6}$$

$$\text{Guess } f \approx 0.02, \quad d \approx [1.148(0.02)]^{1/6} \approx 0.533 \text{ ft}, \quad Re = \frac{4\rho Q}{\pi \mu d} \approx 665000, \quad \frac{\varepsilon}{d} \approx 0.00159$$

$$\text{Then} \quad f_{better} \approx 0.0224, \quad d_{better} \approx 0.543 \text{ ft (converged)}$$

Result: $d_{best} \approx 0.543$ ft ≈ **6.5 in,** $Cost_{min} \approx \$14300_{pump} + \$45600_{pipe} \approx$ **\$60,000**. *Ans.*

6.76 The small turbine in Fig. P6.76 extracts 400 W of power from the water flow. Both pipes are wrought iron. Compute the flow rate Q m^3/h. Why are there two solutions? Which is better?

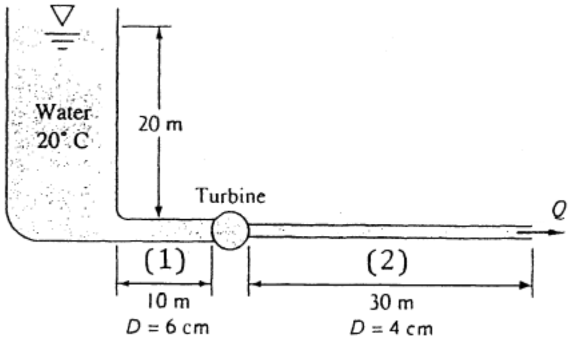

Fig. P6.76

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For wrought iron, take $\varepsilon \approx 0.046$ mm, hence $\varepsilon/d_1 = 0.046/60 \approx 0.000767$ and $\varepsilon/d_2 = 0.046/40 \approx 0.00115$. The energy equation, with $V_1 \approx 0$ and $p_1 = p_2$, gives

$$z_1 - z_2 = 20\text{ m} = \frac{V_2^2}{2g} + h_{f2} + h_{f1} + h_{turbine}, \quad h_{f1} = f_1\frac{L_1}{d_1}\frac{V_1^2}{2g} \quad \text{and} \quad h_{f2} = f_2\frac{L_2}{d_2}\frac{V_2^2}{2g}$$

$$\text{Also,}\quad h_{turbine} = \frac{P}{\rho g Q} = \frac{400\text{ W}}{998(9.81)Q} \quad \text{and} \quad Q = \frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2$$

If we rewrite the energy equation in terms of Q and multiply by Q, it is essentially a cubic polynomial, because for these rough walls the friction factors are almost constant:

$$Q\,h_{turbine} = \frac{400}{998(9.81)} = 20Q - \frac{8f_1L_1Q^3}{\pi^2 g d_1^5} - \frac{8f_2L_2Q^3}{\pi^2 g d_2^5} - \frac{8Q^3}{\pi^2 g d_2^4}$$

Solve by Excel or by iteration. There are three solutions, two of which are positive and the third is a meaningless negative value. The two valid (positive) solutions are:

$(a)\quad Q = 0.00437\,m^3/s = \mathbf{15.7}\,m^3/hr\ ;\ \text{Re}_1 = 92{,}500,\ f_1 = 0.0215\ ;\ \text{Re}_1 = 138{,}800,\ f_1 = 0.0221$

$(b)\quad Q = 0.00250\,m^3/s = \mathbf{9.0}\,m^3/hr\ ;\ \text{Re}_1 = 52{,}900,\ f_1 = 0.0232\ ;\ \text{Re}_1 = 79{,}400,\ f_1 = 0.0232$ **Ans.**

[The negative (meaningless) solution is Q = - 0.0069 m^3/hr.] Both solutions (a) and (b) are valid mathematically. **Solution (*b*) is preferred** – the same power for 43% less water flow, and the turbine captures 16.3 m of the available 20 m head. Solution (*a*) is also unrealistic, because a real turbine's power increases with water flow rate. Turbine (*a*) would generate more than 400 W.

P6.96 A fuel cell [Ref. 59] consists of air (or oxygen) and hydrogen micro ducts, separated by a membrane that promotes proton exchange for an electric current, as in Fig. P6.96. Suppose that the air side, at 20°C and approximately 1 atm, has five 1 mm by 1 mm ducts, each 1 m long. The total flow rate is 1.5E-4 kg/s. (*a*) Determine if the flow is laminar or turbulent. (*b*) Estimate the pressure drop.

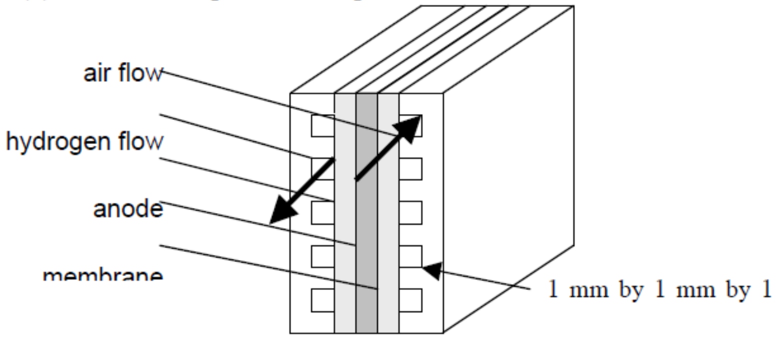

Fig. P6.96. Simplified diagram of an air-hydrogen fuel cell.
[*Problem courtesy of Dr. Pezhman Shirvanian*]

Solution: For air at 20°C and 1 atm, take $\rho = 1.20\ kg/m^3$ and $\mu = 1.8E\text{-}5\ kg/m\text{-}s$. The hydraulic diameter of a square duct is easy, the side length $a = 1$ mm. The mass flow through one duct is

$$\dot{m}_{1\,duct} = \frac{1.5\text{E}-4\,kg/s}{5} = 0.3\text{E}-4\frac{kg}{s} = \rho AV = (1.20\frac{kg}{m^3})[(0.001m)^2]V$$

$$\text{Solve for } V = 25.0\frac{m}{s}\text{, hence } \text{Re}_{Dh} = \frac{\rho VD_h}{\mu} = \frac{1.20(25.0)(0.001)}{0.000018} = 1667 \text{ (laminar)} \quad Ans.(a)$$

(*b*) We could go with the simply circular-duct approximation, $f = 64/\text{Re}$, but we have a more exact laminar-flow result in Table 6.4 for a square duct:

$$f_{square} = \frac{56.91}{\text{Re}_{Dh}} = \frac{56.91}{1667} = 0.0341$$

$$Then\ \Delta p = f\frac{L}{D}\frac{\rho}{2}V^2 = (0.0341)(\frac{1m}{0.001m})(\frac{1.2kg/m^3}{2})(25\frac{m}{s})^2 = \mathbf{12{,}800\ Pa} \quad Ans.(b)$$

6.105 The system in Fig. P6.105 consists of 1200 m of 5 cm cast-iron pipe, two 45° and four 90° flanged long-radius elbows, a fully open flanged globe valve, and a sharp exit into a reservoir. If the elevation at point 1 is 400 m, what gage pressure is required at point 1 to deliver 0.005 m^3/s of water at 20°C into the reservoir?

Solution: For water at 20°C, take $\rho = 998\ kg/m^3$ and $\mu = 0.001$ kg/m·s. For cast iron, take $\varepsilon \approx 0.26$ mm, hence $\varepsilon/d = 0.0052$. With the flow rate known, we can compute V, Re:

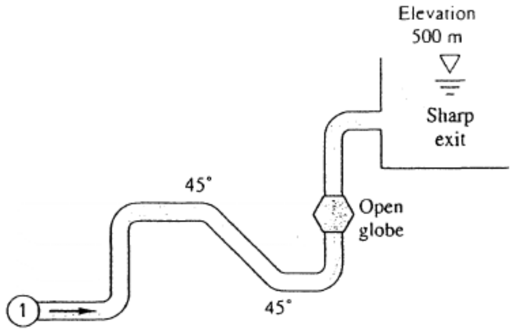

Fig. P6.105

$$V = \frac{Q}{A} = \frac{0.005}{(\pi/4)(0.05)^2} = 2.55\ \frac{m}{s};\quad Re = \frac{998(2.55)(0.05)}{0.001} \approx 127000,\quad f_{Moody} \approx 0.0315$$

The minor losses may be listed as follows:

45° long-radius elbow: $K \approx 0.2$; 90° long-radius elbow: $K \approx 0.3$

Open flanged globe valve: $K \approx 8.5$; submerged exit: $K \approx 1.0$

Then the energy equation between (1) and (2—the reservoir surface) yields

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = 0 + 0 + z_2 + h_f + \sum h_m,$$

$$\text{or:}\quad p_1/(\rho g) = 500 - 400 + \frac{(2.55)^2}{2(9.81)}\left[0.0315\left(\frac{1200}{0.05}\right) + 0.5 + 2(0.2) + 4(0.3) + 8.5 + 1 - 1\right]$$

$$= 100 + 253 = 353\ \text{m},\quad \text{or:}\quad \mathbf{p_1} = (998)(9.81)(353) \approx \mathbf{3.46\ MPa}\quad \textit{Ans.}$$

6.116 For the series-parallel system of Fig. P6.116, all pipes are 8-cm-diameter asphalted cast iron. If the total pressure drop $p_1 - p_2$ = 750 kPa, find the resulting flow rate Q m^3/h for water at 20°C. Neglect minor losses.

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For asphalted cast iron, $\varepsilon \approx$ 0.12 mm, hence ε/d = 0.12/80 ≈ 0.0015 for all three pipes. The head loss is the same through AC and BC:

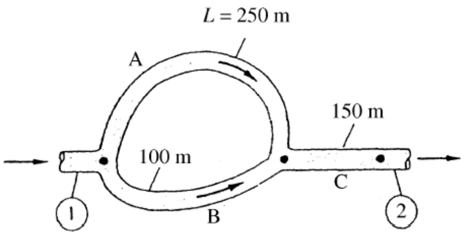

Fig. P6.116

$$\frac{\Delta p}{\rho g} = h_{fA} + h_{fC} = h_{fB} + h_{fC} = \left(f\frac{L}{d}\frac{V^2}{2g}\right)_A + \left(f\frac{L}{d}\frac{V^2}{2g}\right)_C = \left(f\frac{L}{d}\frac{V^2}{2g}\right)_B + \left(f\frac{L}{d}\frac{V^2}{2g}\right)_C$$

Since d is the same, $V_A + V_B = V_C$ and f_A, f_B, f_C are found from the Moody chart. Cancel g and introduce the given data:

$$\frac{750000}{998} = f_A\frac{250}{0.08}\frac{V_A^2}{2} + f_C\frac{150}{0.08}\frac{V_C^2}{2} = f_B\frac{100}{0.08}\frac{V_B^2}{2} + f_C\frac{150}{0.08}\frac{V_C^2}{2}, \quad V_A + V_B = V_C$$

$$\text{Guess } f_{rough} \approx 0.022 \text{ and solve laboriously: } V_A \approx 2.09\ \frac{m}{s},\ V_B \approx 3.31\ \frac{m}{s},\ V_C \approx 5.40\ \frac{m}{s}$$

Now compute $Re_A \approx 167000$, $f_A \approx 0.0230$, $Re_B \approx 264000$, $f_B \approx 0.0226$, $Re_C \approx 431000$, and $f_C \approx 0.0222$. Repeat the head loss iteration and we converge: $V_A \approx 2.06$ m/s, $V_B \approx 3.29$ m/s, $V_C \approx 5.35$ m/s, $Q = (\pi/4)(0.08)^2(5.35) \approx$ **0.0269 m³/s**. *Ans.*

C6.4 Suppose you build a house out in the 'boonies,' where you need to run a pipe to the nearest water supply, which fortunately is about 1 km above the elevation of your house. The gage pressure at the water supply is 1 MPa. You require a minimum of 3 gal/min when your end of the pipe is open to the atmosphere. To minimize cost, you want to buy the smallest possible diameter pipe with an extremely smooth surface.
(a) Find the total head loss from pipe inlet to exit, neglecting minor losses.
(b) Which is more important to this problem, the head loss due to elevation difference, or the head loss due to pressure drop in the pipe?
(c) Find the minimum required pipe diameter.

Solution: Convert 3.0 gal/min to 1.89E–4 m^3/s. Let 1 be the inlet and 2 be the outlet and write the steady-flow energy equation:

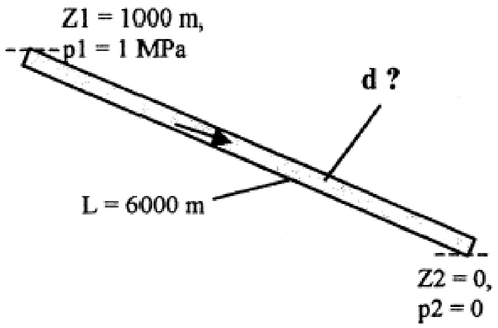

Fig. C6.4

$$\frac{p_{1gage}}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_{2gage}}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_f$$

or: $h_f = z_1 - z_2 + \frac{p_{1gage}}{\rho g} = 1000 \text{ m} + \frac{1E6 \text{ kPa}}{998(9.81)} = 1000 + 102 = 1102 \text{ m}$ *Ans.* (a)

(b) Thus, *elevation drop* of 1000 m is more important to head loss than $\Delta p/\rho g$ = 102 m.

(c) To find the minimum diameter, iterate between flow rate and the Moody chart:

$$h_f = f\frac{L}{d}\frac{V^2}{2g}, \quad L = 6000 \text{ m}, \quad \frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right), \quad V = \frac{Q}{\pi d^2/4},$$

$$Q = 1.89\text{E–4 } \frac{\text{m}^3}{\text{s}}, \quad \text{Re} = \frac{Vd}{\nu}$$

We are given h_f = 1102 m and ν_{water} = 1.005E–6 m^2/s. We can iterate, if necessary with Excel, which can swiftly arrive at the final result:

$f_{smooth} = 0.0266$; Re = 17924; V = 1.346 m/s; $\mathbf{d_{min}}$ **= 0.0134 m** *Ans.*(c)