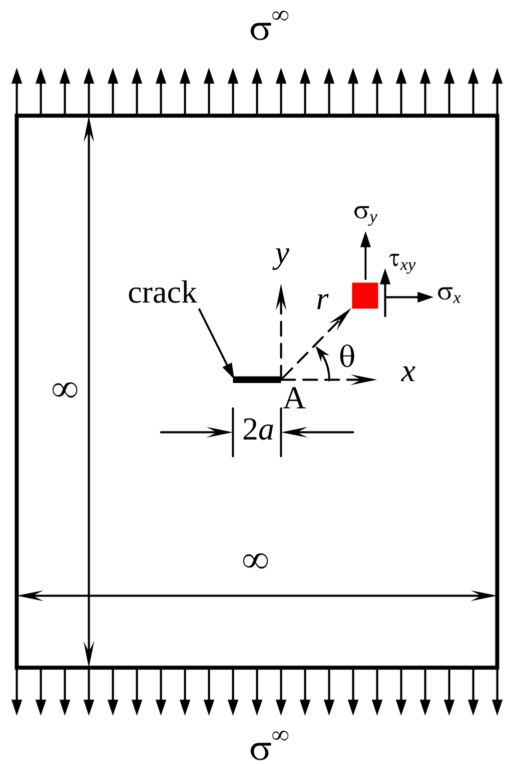
THE UNIVERSITY OF IOWA
Department of Mechanical Engineering

Fracture Mechanics	**Homework #4**	**Assigned: March 25, 2020**
ME:5159	**Total Points: 30**	**Due: April 03, 2020**

Problem 1:

A weightless, homogeneous, isotropic, infinite plate with a crack of length 2*a* is subjected to a far-field applied stress σ^∞, as shown in the figure below. With the crack-tip A as the origin of the coordinate system, the linear-elastic crack-tip stress field (*i.e.*, when $r \to 0$) can be obtained as

$$\begin{aligned}
\sigma_x &= \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \\
\sigma_y &= \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right], \quad 0 \le |\theta| \le \pi \\
\tau_{xy} &= \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}
\end{aligned}$$



where $K_I = \sigma^\infty\sqrt{\pi a}$ is the mode-I stress intensity factor. For the plate material with uniaxial yield strength S_y, let $r_y(\theta)$ denote the plastic zone size as a function of θ. Using the linear-elastic stress field above and assuming no redistribution of stresses due to crack-tip plasticity,

(a) Determine the plastic zone size $r_y(\theta)$ in terms of K_I and S_y for (1) plane stress and (2) plane strain conditions.

(b) From the results of (a) and ν = 1/3, sketch the plastic zone boundary in terms of $r_y(\theta)\cos\theta \big/ \left[K_I^{\,2}/(\pi S_y^{\,2})\right]$ vs. $r_y(\theta)\sin\theta \big/ \left[K_I^{\,2}/(\pi S_y^{\,2})\right]$ plot at crack tip A for $0 \le |\theta| \le \pi$. Which stress state gives conservative prediction of the plastic zone size? Comments.

Assume *von Mises yield criterion* for your analysis.

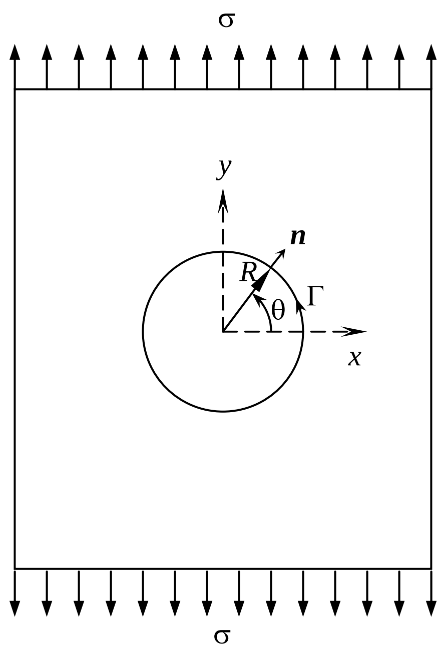
Problem 2:

Solve Problem 1 assuming *Tresca yield criterion*. Compare results of Problems 1 and 2. Comments.

Problem 3:

Consider an uncracked, linear-elastic (E, ν) specimen in plane stress, which is subject to uniaxial tensile stress of magnitude σ, as shown in the figure below. Let Γ denote a closed circular contour of radius R indicated in the figure. Confirm that the J-integral for this uncracked body is

$$J = \int_{\Gamma} \left[W dy - \boldsymbol{T} \cdot \frac{\partial \boldsymbol{u}}{\partial x} d\Gamma \right] = 0 .$$



$$\boldsymbol{T} = \boldsymbol{\sigma}\boldsymbol{n}$$

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z \end{bmatrix}; \quad \boldsymbol{n} = \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix}$$

$$\boldsymbol{u} = \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix}$$