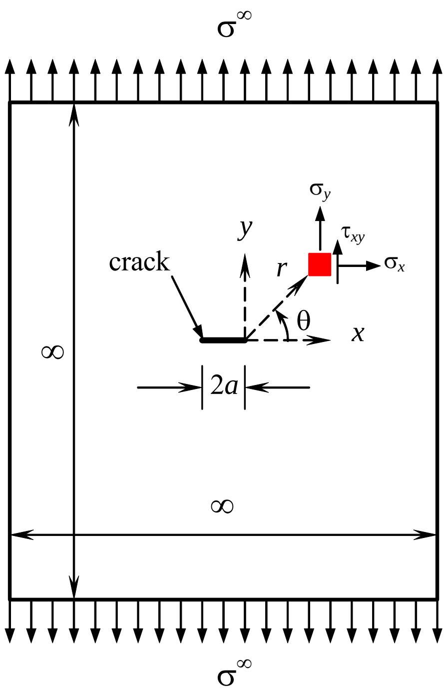
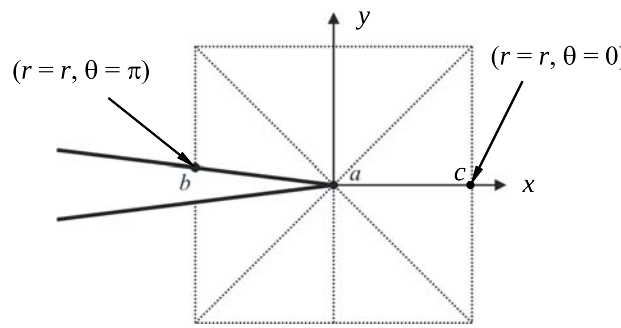
THE UNIVERSITY OF IOWA
Department of Mechanical Engineering

Fracture Mechanics	**Homework #2**	**Assigned: February 17, 2020**
ME:5159	**Total Points: 20**	**Due: February 28, 2020**

Problem 1:

A linear-elastic finite element analysis is performed for a wide (infinite) plate with a through crack (see figure below) subjected to mode-I tensile loading. The far-field tensile stress is σ^{∞} = 200 MPa and the crack length is $2a$ = 50 mm. Assuming *plane strain* condition, the stress normal to the crack plane (σ_y) at θ = 0 and relative displacement (u_y) at $\theta = \pi$ are calculated at node points near the crack tip as a function of r/a and are tabulated below. The elastic constants are as follows: E = 192 GPa and ν = 0.2.





$\theta = 0$		$\theta = \pi$	
r/a	σ_y/σ^{∞}	r/a	$u_y/2a$
0.005	11	0.005	9.99×10^{-5}
0.01	8.07	0.01	1.41×10^{-4}
0.02	6	0.02	1.99×10^{-4}
0.04	4.54	0.04	2.80×10^{-4}
0.06	3.89	0.06	3.41×10^{-4}
0.08	3.5	0.08	3.92×10^{-4}
0.1	3.24	0.1	4.36×10^{-4}
0.15	2.83	0.15	5.27×10^{-4}
0.2	2.58	0.2	6.00×10^{-4}
0.25	2.41	0.25	6.61×10^{-4}

Using the correlation methods, one can estimate the mode-I stress-intensity factor (SIF) for this problem from the following equations:

Stress-Correlation Method (Equation 4 in Lecture No. 9)

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \qquad (\text{for } \theta = 0) \tag{1}$$

Displacement-Correlation Method (Equation 3 in Lecture No. 12)

$$u_y = \frac{K_I}{\mu}\sqrt{\frac{r}{2\pi}}\left(2-2\nu\right) \qquad (\text{for } \theta = \pi) \tag{2}$$

1. Using Equations 1 and 2 and the tabulated finite element results of σ_y and u_y, estimate the SIFs as a function of r/a, and denote them as $K_{I,SC}$ and $K_{I,DC}$, respectively.

2. Recall that the exact solution ($K_{I,E}$) of SIF for this problem is (Equation 5 in Lecture No. 9)

$$K_{I,E} = \sigma^{\infty}\sqrt{\pi a} \tag{3}$$

   To evaluate the accuracy of stress- and displacement-correlation methods, define two error functions, such as,

$$e_{SC} = \frac{K_{I,E} - K_{I,SC}}{K_{I,E}} \times 100 \tag{4}$$

   and

$$e_{DC} = \frac{K_{I,E} - K_{I,DC}}{K_{I,E}} \times 100, \tag{5}$$

   respectively. Develop plots of e_{SC} and e_{DC} as a function of r/a. Which correlation method gives better accuracy? Why? Comments.

Problem 2:

Consider a material which has K_{Ic} = 40 MPa√m (36.4 ksi√in). Each of the five specimens in Table on page 81 of Lecture No. 10 has been fabricated from this material. Assume: B = 25.4 mm (1 inch), W = 50.8 mm (2 inches), S/W = 4, and a/W = 0.5. If failure occurs when $K_I > K_{Ic}$, estimate the failure load for each specimen. Which specimen has the highest failure load? Which has the lowest? Comments.