Problem 2:

The equation of an ellipse in the cartesian co-ordinate system (*x*-*y*) is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ . \tag{1}$$

Taking derivative w.r.t. *x* on both sides of Equation 1,

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0 \ . \tag{2}$$

Equation 2 and its further derivative w.r.t. *x* yield

$$\frac{dy}{dx} = -\frac{b^2}{a^2}\frac{x}{y} \tag{3}$$

$$\frac{d^2y}{dx^2} = -\frac{b^4}{a^2 y^3} \tag{4}$$

Hence,

$$\begin{aligned} \rho &= \left| \frac{\left[1+\left(dy/dx\right)^2\right]^{3/2}}{d^2y/dx^2} \right|_{x=a,\, y=0} \\ &= \left| \frac{\left[1+\left(-\frac{b^2 x}{a^2 y}\right)^2\right]^{3/2}}{-\frac{b^4}{a^2 y^3}} \right|_{x=a,\, y=0} \\ &= \left| \frac{\left[a^4 y^2 + b^4 x^2\right]^{3/2}}{a^4 b^4} \right|_{x=a,\, y=0} \\ &= \frac{b^2}{a} \end{aligned} \quad , \tag{5}$$

which gives

$$b = \sqrt{\rho a} \qquad (b \text{ is positive}) \tag{6}$$

Therefore,

$$\boxed{\sigma_y\Big|_{x=a,\, y=0} = \sigma^\infty\left[1+2\frac{a}{b}\right] = \sigma^\infty\left[1+2\sqrt{\frac{a}{\rho}}\right]} \tag{7}$$