THE UNIVERSITY OF IOWA
Department of Mechanical Engineering

Fracture Mechanics	**Homework #1**	**Assigned: January 29, 2020**
ME:5159	**Total Points: 20**	**Due: February 10, 2020**

Problem 1:

Consider a plate containing a circular hole of radius a, as shown in the figure below. The plate, which has geometric dimensions large enough to be idealized as an infinite plate, is subjected to a uniform far-field tensile stress σ^{∞}. In the class, we showed that the linear-elastic stress field in this body is

$$\sigma_r = \frac{\sigma^{\infty}}{2}\left[\left(1-\frac{a^2}{r^2}\right)-\left(1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2}\right)\cos 2\theta\right]$$

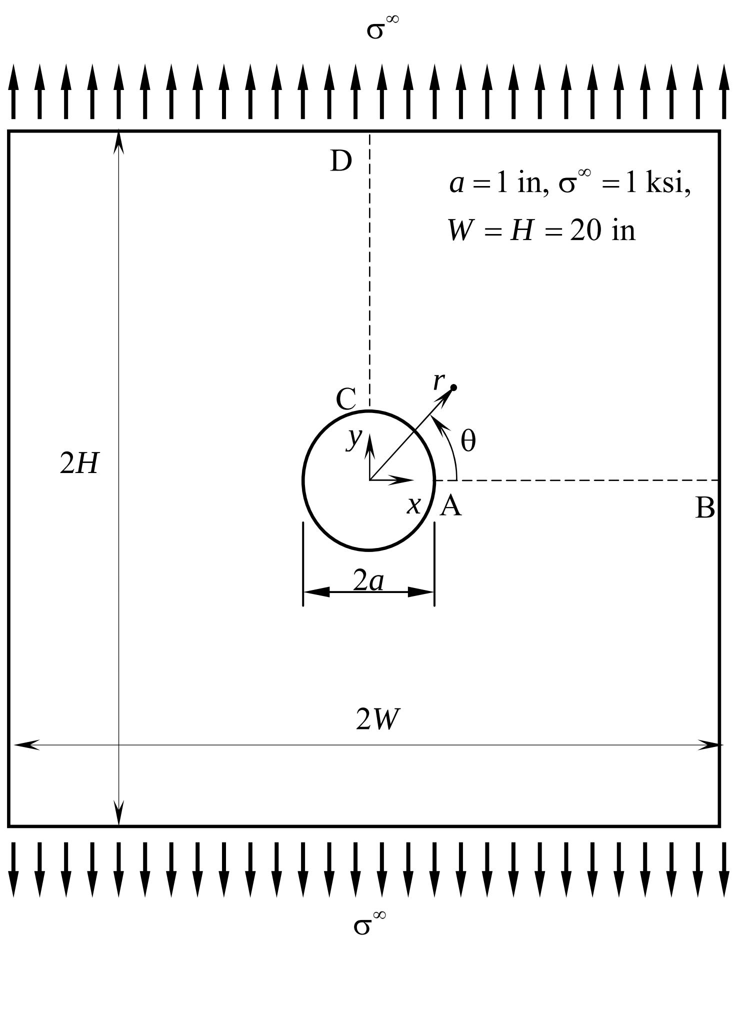
$$\sigma_\theta = \frac{\sigma^{\infty}}{2}\left[\left(1+\frac{a^2}{r^2}\right)+\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\tau_{r\theta} = \frac{\sigma^{\infty}}{2}\left(1-\frac{3a^4}{r^4}+\frac{2a^2}{r^2}\right)\sin 2\theta$$

where r and θ are polar coordinates with the center of the hole as the origin.

(1) Plot σ_r and σ_θ as a function of r at the horizontal (line AB) and vertical (line CD) planes through the center of the hole.

(2) Plot σ_θ as a function of θ ($0 \le \theta \le 2\pi$) at the free surface of the hole ($r = a$).



Problem 2:

Consider an infinite plate, shown in the figure below, containing an elliptical hole with $2a$ and $2b$ as the lengths of its major and minor axes. If the plate is subjected to a uniform far-field tensile stress σ^{∞}, we showed in the class that the normal stress at the tip of the ellipse (i.e., at point A) under linear-elastic condition is

$$\sigma_y\Big|_{x=a,y=0} = \sigma^{\infty}\left[1+2\frac{a}{b}\right].$$

Now, let ρ denote the radius of curvature at the tip of the ellipse. Show that $\sigma_y\Big|_{x=a,y=0}$ can also be expressed by

$$\sigma_y\Big|_{x=a,y=0} = \sigma^{\infty}\left[1+2\sqrt{\frac{a}{\rho}}\right].$$

Hint: using equation of ellipse in cartesian coordinates (x-y), calculate ρ from

$$\rho = \left|\frac{\left[1+\left(dy/dx\right)^2\right]^{3/2}}{d^2y/dx^2}\right|_{x=a,y=0}$$