1-10 POTENTIAL ENERGY OF A SYSTEM OF PARTICLES

- *Potential energy* of two and n particles systems

For two particles system (F is force ofattraction).

dW = **F**•d**r** = -Fdr;   dV = -dW

(1-16) $$V = \int F dr$$

For an n-particle system, there are n(n-1)/2 *bonds*. This is also a conservative system. The total potential energy V is the summation of p.e. of each of the bonds.

$$V = \sum_{i=1}^{M} V_i; \qquad V_i = \int F_i dr_i; \qquad M=n(n-1)/2$$

Force Field

- Let us consider a system composed of one movable particle and several fixed particles. The force **F** which acts on the movable particle due to its bond between the fixed particle, is a vector point function or a vector field. The potential energy V of this system is a function of the position of the movable particle x_i:

$$dV = \frac{\partial V}{\partial x_i} dx_i$$

$$dV = -dW = -F_i dx_i$$

(1-18) **F =** -grad V

- A *force field* that possesses a *potential energy function* is said to be conservative.
- If x_i are rectangular coordinates, and F_i are the components of the vector **F**, an infinitesimal vector d**x** is the direction of **F** conforms to the differential relation:

(1-17) $$\frac{dx_1}{F_1} = \frac{dx_2}{F_2} = \frac{dx_3}{F_3}; \qquad \text{or} \qquad \frac{dx}{F_x} = \frac{dy}{F_y} = \frac{dz}{F_z}$$

- The general integral of these equations represents a system of curves called "*lines of force*", or *flow lines*.

Newtonian Potential

- Newton's law of gravitation: the force of attraction between any two point masses is

(1-19) $$F = k\frac{mm'}{r^2}$$

- The potential energy of this system using (1-16), is

(1-20) $$V = -k\frac{mm'}{r}$$

- Let k = 1, m' = 1. Then potential energy of a unit mass in a force field that is generated by n fixed masses m_i is

  (1-21) $$V = -\sum \frac{m_i}{r_i}$$

  r_i = distance between the ith fixed mass and the movable mass.

- The potential energy of a unit mass in a force field that is generated by a *continuous fixed distributed mass* with variable density ρ:

  (1-22) $$V = -\int \frac{?\, ds}{r}$$

  where r is the distance from a particle of the distributed mass to the movable particle.

- Newtonian potential functions (Eqs. 1-21 and 1-22) are solutions of Laplace's differential equation $\nabla^2 V=0$. Any function that satisfies Laplace's equation is called a *Harmonic* function.

- Example of distributed electric charge.

1-11 STABILITY

- *Infinitesimal Theory of Stability*: Infinitesimal disturbances of a stable system cause only infinitesimal displacement in the c.s.
- Small amount of energy supplied from external sources may sometimes cause the system to experience large displacement; however, this does not necessarily mean that the system is unstable. Example: moving the table on the floor.
- If external forces must perform positive work to produce any small displacement of a given system, then equilibrium of system is stable. However, the above example shows that this condition is merely *sufficient* for stability; it is not a necessary condition.
- *Theory of Stability for Conservative Systems*:

  T + V = constant, implies that infinitesimal increment in T is accompanied by only an infinitesimal increment in V. Therefore, if a motionless conservative system is in a configuration of *minimum potential energy*, an infinitesimal initial velocity causes only an infinitesimal displacement in c.s. And so, stable equilibrium exists. Conversely, if the p.e. is not a minimum and if the system is holonomic, an impulse that directs the system along a path for which V decreases causes the kinetic energy to increase continually (since T+V=constant); therefore the equilibrium is unstable.

- Law of minimum potential energy for conservative holonomic systems: Stable equilibrium $\Rightarrow$ Potential energy is at a *relative minimum*.
- Relative minimum implies that the system is stable in a weak sense, because a better relative minimum may be found nearby. Example: bead on a frictionless rigid wire.
- Examples: Weights connected by a frictionless mechanism with rigid members. Marble in a cup. Frictionless chain of rigid bars with hinged ends. Stationary cup of water.
- For conservative holonomic system with *finite degrees of freedom* the potential energy V is a single-valued function of the generalized coordinates $x_1, x_2, \cdots x_n$. If there are

increments h_1, h_2,$\cdots h_n$ of the x's, we can express the increment of V by using Taylor's theorem.

$$\Delta V = \delta V + 1/2\ \delta^2 V + o(s^3)$$

- From the principle of virtual work, the necessary and sufficient condition for equilibrium of conservative holonomic unchecked systems is that δV vanishes for all h_i. So we can use $\delta^2 V$ to determine whether the potential energy is at a relative minimum or not.
  - $\delta^2 V$ is positive definite $\Rightarrow$ V is at a relative minimum $\Rightarrow$ stable.
  - $\delta^2 V$ is negative definite, or negative semidefinite $\Rightarrow$ V is not at a relative minimum $\Rightarrow$ not stable.
  - $\delta^2 V$ is positive semidefinite $\Rightarrow$ cannot determine stability of equilibrium.
- Example: A thin hemispherical shell of constant thickness balanced on a hemispherical dome of same radius.