1-6 THE PRINCIPLE OF VIRTUAL WORK

- Fourier's inequality is a *sufficient* condition but <u>not</u> necessary for equilibrium. *Necessary condition*: must be satisfied; however, if it is satisfied, the point may not be an equilibrium point. *Sufficient condition*: its satisfaction indicates the point to be in equilibrium.

  ⇒ Some systems can be in equilibrium, but Fourier's inequality $W' \leq 0$ is violated. For example, consider the two-cylinder system. The gravity performs positive work when the smaller cylinder at the top is given a small k.a.v.d. However, the smaller system at the top is an equilibrium configuration.

- The above example indicates that equilibrium exists if W' is stationary in the sense that it is an infinitesimal of higher order than the displacement in the c.s.

- *The principle of virtual work/the principle of virtual displacements*: Let X be any point in the neighborhood of the point X_0 in the c.s. of a m.s. Let s be the distance in the c.s. The point X_0 in the configuration space is an equilibrium point if $\lim s \to 0\ (W(X,X_0)/s) \leq 0$ for every admissible path of approach of X to X_0 (k.a.v.d). This is still a *sufficient* condition but not necessary. (e.g., a particle subjected to $F = F_0 sgn(x)$).

Variational Form of the Principle of Virtual Work

- In many cases, it is possible to express the work function W(X,Xo) as

(1-5) $W = \delta W + \frac{1}{2}\delta^2 W + O(s^3)$

- δW and $\delta^2 W$ are linear and quadratic homogeneous functionals in the geometric variables [f(**x**) is a homogeneous function of degree n if and only if $s^n f(\mathbf{x}) = f(n\mathbf{x})$]. For finite dimensional systems, (1-5) is obtained by Taylor's expansion.
- $\delta W \leq 0$ can be used instead of lim $s \to 0$ $(W/s) \leq 0$ as the principle of v.w.; i.e., the equilibrium criterion. Thus to check $\delta W \leq 0$, we assume the virtual displacement to be very small such that all the higher order terms can be neglected in the displacement.
- *Unchecked system* (*reversible system*): a system with a continuously varying force when a virtual displacement is executed.
- For an unchecked system, the virtual work is an infinitesimal of higher order than the displacement of the particle in the c.s. since the force acquires only an infinitesimal magnitude during an infinitesimal k.a.v.d.
- For an unchecked system, lim $s \to 0$ $(W/s) = 0$ is *necessary and sufficient* for equilibrium. It is more convenient to express it as $\delta W = 0$.
- It is important to note that W is the virtual work for any k.a.v.d. It may contain first order, second order, and higher order terms. Many times, we directly write δW assuming small k.a.v.d. and set it to zero

to determine the equilibrium states. It is not appropriate to call δW as the virtual work (although it is usually called that); it is the first variation of the V.W.

1-7 GENERALIZED FORCE

- For a finite dimensional system, the first variation of the virtual work for the small virtual displacement δx_i of the *g.c.* x_i is a linear function

  (1-6) $$\delta W = Q_i \delta x_i$$

  where Q_i is called the *generalized force* (summation is implied for repeated indices).
- The p.v.w. $\delta W = 0$ leads to the following condition: the configuration for which all components of *generalized force* vanish ($Q_i = 0$) is an equilibrium configuration, which is <u>necessary and sufficient for unchecked holonomic system</u>; it is only <u>sufficient for both checked holonomic systems and nonholonomic systems</u>.

Generalized External Force

- The virtual work corresponding to an arbitrary k.a.v.d. may be separated as

  $$W = W_e + W_i$$

  where W_e and W_i are the least upper bounds of the v.w. of external and internal forces, respectively. W_e may also be expressed in the form (1-6). For a finite dimensional system, δW_e is a linear form in

δx_i

(1-7) $\delta W_e = P_i \delta x_i$

where P_i are some functions of x_j that are called components of generalized external force.

- Even though loading on a system is constant, g.e.f. (P_i) need not be constant (e.g., the link and spring system).
- The generalized force depends not only on the nature of the loads but also on the choice of g.c., e.g., spring-rigid-bar mechanism.
- **HW#3**: For the two cylinder system, show that the V.W. of the gravity force of the smaller cylinder is an infinitesimal of higher order than the virtual displacement in the c.s.