1-3 ELEMENTARY PRINCIPLES OF DYNAMICS

- *Inertial reference frame* I (Newtonian, Galilean): does not ***rotate*** or ***accelerate*** w.r.t. remote stars. Earth is not an inertial reference frame but for all practical purposes, it is considered to be so.

WORK

- *Work* that a force **F** performs on the particle as it displaces d**r**:

  $$dW = \mathbf{F} \cdot d\mathbf{r} = \mathbf{F} \cdot \mathbf{v}\, dt$$

- Work of **F**(t) on the particle during the interval t_0 to t_1:

  (1-1) $\quad W = \int_{t_0}^{t_1} \boldsymbol{F} \bullet \boldsymbol{v} dt \qquad$ **F·v** = power

- *Work is a relative quantity* (depends on definition of I) since **v** depends on the choice of I.
- Work on a m.s.: Work W performed on a particle is said to be performed on the m.s. to which the particle belongs.
- If *several forces* act on particles of the system, the work that they perform on the m.s. is the sum of work of the individual particles.
- *Continuous* shifting of point of application of force: *The work that force performs on the m.s. is understood to be the sum of the works that it performs on each particle on which it acts*. Eq. (1-1) remains valid for a force that is applied in any continuous way to a m.s.
- Work Eq. (1-1) applied to a m.s.; **v** is *interpreted as the instantaneous velocity of the particle on which the force acts and not that of the point of application of the force.* ***Examples***: rigid ball

rolling on a rigid surface, brick sliding on the pavement.

- The above examples show that Eq. (1-1) is more general than

$$W = \int_{t_0}^{t_1} \boldsymbol{F} \bullet \boldsymbol{dr}$$

- The last equation applies only if **F** acts continuously on the same particle.
- *Newton's third law*: To every action, there is a reaction.
- *Internal force*: Force that a component of the m.s. exerts on another component of the m.s.
- *External force*: Force exerted on the m.s. by external objects. Force exerted on an external object by the m.s.
- Work of external and internal forces: Total $W = W_e + W_i$.

Law of Kinetic Energy

- *The work of all the forces (internal and external) that act on a m.s. equals the increase of kinetic energy of the system.* $W = \Delta T$ is restricted to I.
- For a particle: $F = m d\mathbf{v}/dt$; $dW = F \cdot d\mathbf{r} = m(d\mathbf{v}/dt) \cdot d\mathbf{r} = m\mathbf{v} \cdot d\mathbf{v}$

$$dW = d/dt(1/2\ mv^2)\ dt = dT; \text{ or } W = \Delta T$$

1-4 FIRST LAW OF THERMODYNAMICS

- *Principle of conservation of energy*: The work that is performed on a m.s. by external forces plus the heat that flows into the system from outside equals the increase of kinetic energy plus the increase of internal energy.

- T1: $W_e + Q = \Delta T + \Delta U$.
- ΔU = change in internal energy. Internal energy is *dependent on state of the system*. In general, the internal energy may depend on factors other than configuration and temperature. Only *changes in the internal energy can be measured* or determined. This is seen by Hysteresis loop example.
- *Internal energy U of a system is indeterminate;* only changes in U can be measured. *Example: hysteresis loop*: Work W_e supplied for one cycle = area enclosed by the loop; if Q = 0 (i.e., no heat escapes), then $W_e = \Delta U$ by T1. The temperature is changed by the cycle. Heat extracted to restore the initial temperature $\neq \Delta U$, since some energy is locked in the rod by residual stresses at the microscopic level. This shows that U depends on factors other than strain and temperature.
- Since $W_e + W_i = \Delta T$, T1 can be written as $W_i = Q - \Delta U$.
- Rigid body or frictionless incompressible fluid: $W_i = 0$, T1: $Q = \Delta U$.
- Adiabatic Process: Q = 0; T1: $W_i = - \Delta U$.

1-5 FOURIER'S INEQUALITY

- A motionless m.s. remains at rest if $W' \leq 0$ for all k.a.v.d. (a *sufficient condition but not necessary*). W' is called the *virtual work*.
- *Virtual Displacement*: arbitrary but small k.a.d. obtained by *freezing time (and performed adiabatically)*. The virtual displacement is

performed at an *infinitesimal speed* (too small to be measured). Note that *virtual displacement is required to be small but not necessarily infinitesimal*. We will see later on that the virtual work can be expanded to have first, second and higher order variations. The force **F** may vary when the virtual displacement is executed.

- *Equilibrium*: A m.s. is said to be in equilibrium if it remains at rest in an inertial reference frame. Example: brick resting on a table.
- Let X be a configuration neighboring X_0. v.w. W' depends on the path from X to X_0. The set of all the v.w. is bounded from above. Let $W(X,X_0)$ be the L.U.B. for all W' from X to X_0. Then Fourier inequality becomes $W(X,X_0) \leq 0$.
- *Fourier's Inequality*: The point X_0 in the c.s. represents an equilibrium state if $W(X,X_0) \leq 0$ for all points X in a neighborhood of X_0.
- For *adiabatic process*: T1: $W_i' = -\Delta U$; Fourier's inequality becomes: $W_e' \leq \Delta U$.
- **HW #2**: Show that the force of the track performs no work on the wheel that is rotating without slip.