53:244 (58:254) Energy Principles in Structural Mechanics

- *Introduction*, course perspective, course organization.
- *Material*: Derivation of basic principles of mechanics; principle of virtual work, principles of stationary and minimum potential energy; Hamilton's principle, and others that are derived from these basic principles. In addition, applications of these principles to rigid body and deformable body systems will be discussed.
- *Background Needed*: Vector and matrix theory and algebra, mechanics of deformable bodies, optimization concepts.
- *Newtonian* mechanics vs Variational mechanics (principle of virtual work).

1-1 MECHANICAL SYSTEM

- In this section, we shall discuss some general concepts related to a mechanical system, such definition of a m.s., configuration of a m.s., general problems of statics and dynamics, displacement of a m.s.; geometric terminology, configuration space, points in c.s., distance in c.s., neighborhood, deleted neighborhood, paths in c.s., continuity of paths, holonomic and nonholonomic systems.
- Any collection of *material points* is defined as a mechanical system (m.s.). For purpose of analysis, we will treat a m.s. at the macro level rather than the micro level. Two types of forces act on a m.s.:
  - ✓ *External forces* are due the presence of object external to the

m.s.

- ✓ *Internal forces* are due to action of elements of the m.s. on other elements of the system.

- *Configuration of a m.s.:* The simultaneous position of all the material points of a mechanical system relative to a reference frame I, is called a configuration of the m.s. A coordinate system is needed.
- *Displacement of a m.s.:* A m.s. is said to experience displacement if any of its material points is displaced.
- *Constraints on m.s.:* Geometrical constraints on the displacement of a m.s. are called constraints; e.g., a rigid body, a cantilever beam.

Geometric Terminology

- *Set*: Any collection of things is called a set.
- *Configuration space*: A collection of all the possible configurations of a mechanical system is a set, called the configuration space.
- A variable point in the configuration space is denoted as $\mathbf{X}$. Subscripts are used to indicate specific points in the configuration space, e.g., $\mathbf{X}_0$, $\mathbf{X}_1$. $\mathbf{X}_1 - \mathbf{X}_0$ designates displacement from $\mathbf{X}_0$ to $\mathbf{X}_1$.

Distance in Configuration Space (c.s.)

- *Displacement* of a m.s. relative to I occurs if any element of the m.s. moves relative to I. Displacement is a vector quantity, but here we define it as magnitude of the displacement vector.

- *The magnitude of displacement of a m.s.* is the least upper bound of the displacements of all the material points of the system.
- *Distance* s between any two points $\mathbf{X_1}$ and $\mathbf{X_2}$ in the c.s. is understood to be the magnitude of the displacement from $\mathbf{X_1}$ to $\mathbf{X_2}$; $s = |\mathbf{X_2} - \mathbf{X_1}|$.
- *Distance in the c.s.* has the following properties: Let $\mathbf{X_1}$, $\mathbf{X_2}$ and $\mathbf{X_3}$ be three distinct points in the c.s.

  1. $| X_1 - X_1| = 0$
  2. $s = |X_2 - X_1| = | X_1 - X_2| > 0$
  3. $|X_3 - X_2| + | X_2 - X_1| \geq |X_3 - X_1|$

- *Neighborhood* of a point $\mathbf{X_1}$ in the c.s. is defined as an open set of all the variable points $\mathbf{X}$ such that for any positive *r*: $|X - X_1| < r$.
- *Deleted neighborhood* is defined as a neighborhood that does not contain the point $\mathbf{X_1}$ itself: $0 < |X - X_1| < r$.

Paths in Configuration Space

- *Function notation*: The usual definition of function is used in the c.s. A configuration $\mathbf{X}$ of a m.s. is said to be a function of a real variable t in a closed interval [a,b] if to each value of t in this interval, there corresponds a single configuration $\mathbf{X}$. This is denoted as $\mathbf{X}(t)$.
- *Continuity requirements*. The usual definition of continuity of a function is used. A function $\mathbf{X}(t)$ is continuous at the point $\mathbf{X}_0 = \mathbf{X}(t_0)$

if to each $\varepsilon > 0$, there corresponds a $\delta > 0$ such that $|\mathbf{X}(t) - \mathbf{X}_0| < \varepsilon$ for all t in the interval $|t - t_0| < \delta$.

- *Path*. If $\mathbf{X}(t)$ is continuous on [a,b], then it represents a continuous curve in the c.s. This is called a *Jordan curve* or a *path*. $\mathbf{X}(a) = \mathbf{X}(b)$ implies a closed path in the c.s.

Nonholonomic Systems

- *Holonomic constraints* can be expressed as algebraic equations.
- *Nonholonomic constraints* are nonintegrable. They are expressed in the differential form.
- *Scleronomic constraints* are not explicitly dependent on time.
- *Rheonomic constraints* are explicitly dependent time.

1-2 GENERALIZED COORDINATES

- *Generalized coordinates* are real variables or parameters, the specification of which determines the configuration of the m.s. in the c.s. The number of g.c. can be finite or infinite. Let the g.c. for a finite dimensional system be represented as $(x_1, \ldots, x_n)$.
- *Regularity requirements*: g.c. must be independent of each other and satisfy the regularity requirements:

  (1) There must be one to one correspondence between the g.c. and points in the configuration space, i.e., there cannot be any constraint between them (we must be able to displace each

coordinate independent of the others). That is, there cannot be any constraint of the form $f(x_1, ..., x_n) = 0$.

(2) The limit $\Delta s/\Delta x_i$ must exist as Δx_i goes to zero, and must be different from zero. That is, if any g.c. is given an increment Δx_i, the corresponding displacement of the m.s. Δs must be an infinitesimal of the same order of magnitude.

- *Degrees of freedom*

  = (# of coordinates - # of nonintegrable constraints)

  For a holonomic system, # of d.o.f. = # of g.c.

- ***HW #1**: 1.2*