ACCELERATING NON-CARTESIAN SENSE FOR LARGE COIL ARRAYS: APPLICATION TO MOTION COMPENSATION IN MULTISHOT DWI

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ABSTRACT

Multi-shot sequences are often combined with techniques such as parallel imaging to achieve high fidelity images. For non-Cartesian sampling schemes, the reconstruction of such data becomes extremely computationally intensive. The problem of motion compensated SENSE reconstruction of non-Cartesian diffusion weighted images falls into a similar setting, when phase corrections are involved. Here we propose a new pipeline for the fast reconstruction of such data. The large array of composite sensitivity functions are replaced by a low-dimensional set of virtual sensitivity functions using a principal component analysis, thus enabling the evaluation of very few Fourier transforms. The time consuming gridding steps are replaced by a more efficient multiplication in the k-space, enabling further simplifications. Significant acceleration of reconstruction time is shown to be achieved with the new scheme. The algorithm in the general setting can accelerate SENSE reconstruction for large coil arrays.

***Index Terms*—** SENSE, non-linear phase correction, composite sensitivity

1. INTRODUCTION

Iterative sensitivity encoded (SENSE) reconstruction of multi channel non-Cartesian MRI data often provide excellent image quality [1]. However, a major drawback that restricts its utility in the recovery of multidimensional data is its computational complexity. Even though several acceleration techniques were introduced [2][3], the complexity of the scheme is still prohibitive when the number of coil elements are high. This is especially relevant as the number of channels in phased array coils is steadily increasing in the recent years.

Our interest in this problem is motivated by the similarity of this algorithm to motion compensated reconstruction of non-Cartesian multi-shot, multi-coil diffusion weighted MRI data. Single shot techniques have limited capability in increasing the spatial resolution in diffusion weighted imaging because of their sensitivity to geometric distortions from field inhomogeneities and T2* decay. Multi-shot sequences perform better in this context [4]. However, since multi-shot sequences acquire different regions of k-space using different excitations, even small motion of the subject between excitations can result in phase errors between different shots. These phase inconsistencies result in considerable distortions in the image. Hence, motion correction becomes necessary. In this context, non-Cartesian multi-shot sequences such as SNAILS becomes preferable[4]. Since these sequences sample the center k-space densely during each shot, they are self-navigated, thus eliminating the additional collection of navigator scans. The phases resulting from inter-shot motion are estimated from these central k-space regions. Such self-navigated multi-shot sequences are often combined with multi-channel acquisitions to achieve high SNR, improved resolution or faster acquisition.

Several motion compensation schemes have been proposed to recover the image from the self-navigated non-Cartesian data for diffusion weighted images (DWI). These methods fall into two broad categories: linear and non-linear phase correction methods. Linear phase errors resulting from rigid body motion results in shifts in k-space; they can be relatively easily corrected in k-space and can be de-coupled from the image reconstruction[5]. However non-linear phase errors resulting from non-rigid motions cannot be corrected this way. Liu et al, have shown that correction of non-linear phase errors can be reformulated as an iterative SENSE reconstruction scheme [6]. Specifically, they model the product of the motion-induced phase errors and coil sensitivities as a composite sensitivity encoding. Since the composite sensitivity functions are different for each coil and each shot, the number of sensitivity functions are often large. The computational complexity and reconstruction time scales linearly with the number of shots and coils. This is especially challenging in applications such as multi-direction diffusion weighted imaging (MDDWI), where a large number of images need to be reconstructed.

Here, we introduce a new pipeline to considerably accelerate the reconstruction of non-Cartesian multi-coil, multi-shot data. The central idea of the proposed scheme is to approximate the composite sensitivity functions as a linear combination of few basis functions. Since the composite sensitivity functions are smooth and have high redundancy between them, very few basis functions are sufficient to accurately represent them. Previously researchers have modeled the sensitivity functions using polynomials in conventional SENSE reconstructions [7]. We use the principal components of the sensitivity functions to obtain a more compact representation. This approximation enables us to express the Fourier transform of the sensitivity weighted images as the linear combination of Fourier transforms of the image, weighted by the basis functions. Since the number of basis functions are far fewer, a significant reduction in the number of FFT operations can be achieved. We then replace the gridding operations as in [2] and perform more algebraic reductions to further simplify the reconstruction scheme. The comparison of the proposed scheme with the motion compensated SENSE reconstructions demonstrate a 10-12 fold speed-up with minor loss in image quality. In addition to enabling fast recovery of multi-direction DWIs, the proposed scheme will be very useful in the recovery of MRI data from large coil arrays.

2. BACKGROUND

2.1. Iterative SENSE reconstruction

As described in [2], the SENSE reconstruction for arbitrary k-space trajectories can be posed as an optimization problem

$$\hat{f} = \arg\min_{f} \|\mathbf{Ef} - \mathbf{m}\|^2, \quad (1)$$

where $\mathbf{f}$ is the vectorized image, $\mathbf{m}$ is the measured multichannel data, and $\mathbf{E}$ is the encoding matrix. The normal equations corresponding to (1) are given by $\mathbf{E}^H\mathbf{E}\ \mathbf{f} = \mathbf{E}^H\mathbf{m}$, where the encoding matrix $\mathbf{E}$ is the combination of non-uniform Fourier transforms matrix $\mathbf{Q}$ and sensitivity weightings $\mathbf{S}_i; i = 1, ..N_c$ corresponding to N_c coils.

$$\mathbf{E} = \begin{bmatrix} \mathbf{QS}_1 \\ \vdots \\ \mathbf{QS}_{N_c} \end{bmatrix} \quad (2)$$

$\mathbf{S}_i; i = 1, ..N_c$ are the diagonal matrices, whose diagonal entries are the sensitivity weights $s_i(\mathbf{x})$. The problem in (1) is often solved using conjugate gradients (CG) algorithm, the straight-forward implementation of which requires N_c non-uniform Fourier transforms and inverse Fourier transforms.

To speed up the CG steps in SENSE reconstructions, several researchers have proposed to exploit the Toeplitz structure of $\mathbf{E}^H\mathbf{E}$ [2]. This property simplifies $\mathbf{g} = \mathbf{E}^H\mathbf{E}\mathbf{f}$ to $\mathbf{g} = \mathcal{F}^{-1}\mathcal{W}\mathcal{F}f$, where $\mathcal{F}$ is the Cartesian Fourier transform and $\mathcal{W}$ is a point by point multiplication in the Fourier domain. Using this simplification, $\mathbf{E}^H\mathbf{E}\ \mathbf{f}$ can be efficiently implemented as ,

$$g = \sum_{i=1}^{M} \left(\mathcal{S}_i^* \circ \mathcal{F}^{-1} \circ \mathcal{W} \circ \mathcal{F} \circ \mathcal{S}_i\right)\ f, \quad (3)$$

where $\mathcal{S}_i f$ corresponds to point by point multiplication of f by the i^{th} coil sensitivity function $s_i(\mathbf{x})$. Since (3) can be efficiently implemented using FFTs, significant speedups have been reported.

2.2. Motion compensated multishot reconstruction

Since multi-shot schemes acquire different interleaves sequentially using different excitations, the motion of the specimen between excitations result in artifacts. Anderson and Gore have shown that, for DWI, these distortions can be modeled as weighting the images from each shot by different phase images. Thus, the single coil, multi-shot acquisition scheme can be modeled similar to multichannel reconstruction [4]:

$$\mathbf{E} = \begin{bmatrix} \mathbf{Q}_1\mathbf{\Phi}_1 \\ \vdots \\ \mathbf{Q}_{N_i}\mathbf{\Phi}_{N_i} \end{bmatrix} \quad (4)$$

Here $\mathbf{Q}_i, i = 1, .., N_{\text{i}}$ are the non-Cartesian Fourier matrices corresponding to the N_i interleaves. The entries of the diagonal matrices $\mathbf{\Phi}_i; i = 1, .., N_{\text{i}}$ are the phase weights corresponding to different shots $e^{j\phi_i(\mathbf{x})}$. The phase weights for each shot are estimated by subtracting the navigator phase of each shot from the phase of image reconstructed for each shot.

The more general case of multi-channel, multi-shot acquisitions can be derived from the above settings by considering the composite sensitivity functions as defined in [6] $s_{i,j}(\mathbf{x}) = e^{j\phi_i(\mathbf{x})}s_j(\mathbf{x}), i = 1, .., N_i, j = 1, .., N_c$. Thus, the new encoding matrix becomes:

$$\mathbf{E} = \begin{bmatrix} \mathbf{Q}_1\mathbf{S}_{1,1} \\ \vdots \\ \mathbf{Q}_1\mathbf{S}_{1,N_c} \\ \vdots \\ \mathbf{Q}_{N_i}\mathbf{S}_{N_i,N_c} \end{bmatrix} \quad (5)$$

The main challenge with this scheme is the large number (N_iN_c) of composite sensitivity functions; each CG step involves N_iN_c FFTs and IFFTs. The computational complexity of this scheme is often prohibitive, when applying to multi-dimensional data like MDDWIs.

3. THEORY

The main focus of this work is to accelerate the iterative reconstruction of non-Cartesian multi-shot, multi-channel acquisitions. The central idea of our work is to approximate the composite sensitivity functions $s_i(\mathbf{x}); i = 1, .., N_iN_c$ as a linear combination of N_b basis functions:

$$s_i(\mathbf{x}) \approx \sum_{j=1}^{N_b} a_{i,j}\ v_j(\mathbf{x}); i = 1, .., N_iN_c \quad (6)$$

Since the composite sensitivity functions are smooth and have significant redundancy between them, the above approximation is often very good for even small values of N_b. Since the composite sensitivities are obtained by the multiplication of coil sensitivity functions by smooth phase functions, the redundancy between the N_iN_c composite sensitivity functions is expected to be very high.

3.1. Simplification of $\mathbf{E}^H\mathbf{E}$ f

Using (6), $\mathcal{F} \circ \mathcal{S}_i\,(f)$ simplifies to the linear combination of the Fourier transforms of the images weighted by the basis functions:

$$\mathcal{F} \circ \mathcal{S}_i\,(f) = \sum_{j=1}^{N_b} a_{i,j}\ \mathcal{F} \circ \mathcal{V}_j\ f. \tag{7}$$

Here, $\mathcal{V}_j\ f$ corresponds to the point by point multiplication of f by the j^{th} basis function.

We will now consider a general setting where we have M sensitivity functions $S_i; i = 1, .., M$. The corresponding sensitivity weighted images $s_i(\mathbf{x})f(\mathbf{x})$ are acquired using the Fourier sampling operator $\mathbf{Q}_i$. This is a generalization of the actual setup, but will enable us to obtain simpler expressions. Our main goal is to simplify the computation of the operation $\mathbf{g} = \mathbf{E}^H\mathbf{E}\ \mathbf{f}$, specified by

$$g = \sum_{i=1}^{M} \left(\mathcal{S}_i^* \circ \mathcal{F}^{-1} \circ \mathcal{W}_i \circ \mathcal{F} \circ \mathcal{S}_i\right)\ f. \tag{8}$$

This expression is very similar to (3), except that each of the sensitivity weighted images are assumed to be acquired with a different trajectory. Hence the Fourier domain weighting operators $\mathcal{W}_i$ depend on i. Note that the computation of (8) requires M FFT and M IFFT operations, which is computationally expensive for large number of coils or composite sensitivity functions. The approximation in (6) enables us to simplify (8) as

$$g = \sum_{i=1}^{M} \underbrace{\sum_{k=0}^{N_b} a_{i,k}^* \left(\mathcal{V}_k^* \circ \mathcal{F}^{-1}\right)}_{\mathcal{S}_i^*\circ\mathcal{F}^{-1}} \circ \mathcal{W}_i \circ \underbrace{\sum_{j=0}^{N_b} a_{i,j}\mathcal{F} \circ \mathcal{V}_j}_{\mathcal{F}\circ\mathcal{S}_i}\ f \tag{9}$$

$$\begin{aligned} &= \sum_{i=1}^{M} \sum_{j,k=0}^{N_b} a_{i,j}a_{i,k}^* \left(\mathcal{V}_k^* \circ \mathcal{F}^{-1} \circ \mathcal{W}_i \circ \mathcal{F} \circ \mathcal{V}_j\right)\ f \\ &= \sum_{k=1}^{N_b} \sum_{j=1}^{N_b} \mathcal{V}_k^* \circ \mathcal{F}^{-1} \circ \underbrace{\left(\sum_{i=1}^{M} a_{i,j}a_{i,k}^* \circ \mathcal{W}_i\right)}_{\mathcal{U}_{j,k}} \circ \mathcal{F} \circ \mathcal{V}_j\ f \end{aligned} \tag{10}$$

Note that the computation of the above expression requires only N_b FFT and IFFT operations compared to M FFT and IFFT computations in (8). Since the number of basis functions N_b is often far smaller than the number of composite sensitivity functions M, the above computation is considerably more efficient. Note that we now have N_b^2 weighting functions in the Fourier domain, which is often smaller than the number of composite sensitivity functions M.

The simplification of the operator $\mathbf{E}^H\mathbf{E}$ in (10) is graphically illustrated in Fig 1(b). The classical implementation is shown in Fig 1(a), which involves M FFT and IFFT operations. The simplification, indicated by (10) reduces the number of Fourier domain weighting operations to N_b^2. In the specific example we are considering in this paper $M = 176$, while N_b is varied between 5 and 10. When $N_b = 5$, the number of FFTs reduces from 176 to 5, while the number of Fourier domain weighting operations reduces from 176 to 25. This demonstrates the significant reduction in computational complexity offered by the proposed scheme.

3.2. Choice of basis functions

As discussed previously, polynomial basis functions are widely used for the approximation and smoothing of coil sensitivity function in classical SENSE reconstruction. To minimize the number of functions, and hence reduce the computational complexity, we propose to use principal component analysis to determine the optimal basis functions. Specifically, we stack the vectorized composite sensitivity functions into the columns of a matrix $\mathbf{B}$ and perform a singular value decomposition of the matrix $\mathbf{B} = \mathbf{U\Sigma V}^H$. The left singular vectors $\mathbf{U}$ correspond to the basis functions, while $\mathbf{A} = \mathbf{\Sigma V}^H$ are the coefficients. We choose the number of basis functions to obtain a desired approximation quality, which can be easily performed by plotting the singular vectors of $\mathbf{\Sigma}$.

4. RESULTS

To test the proposed algorithm, in-vivo DWI data of a health human volunteer was collected using a multi-shot variable density spiral sequence with 8 channel head coil on a 3T GE scanner with the following specifications: FOV 19.2cm, matrix 192x192, 1x1mm^2 resolution, b=1200 s/mm^2, 22 interleaves, and TE / TR = 40 / 2000ms.

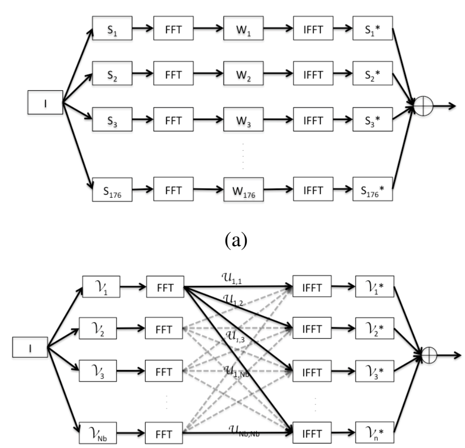

Fig. 1. (a) Original, (b) proposed implementation of E^HEI

Figure 2 shows the results from various schemes of reconstruction. The data were first reconstructed using CG-NUFFT with no motion correction. Due to motion between shots,

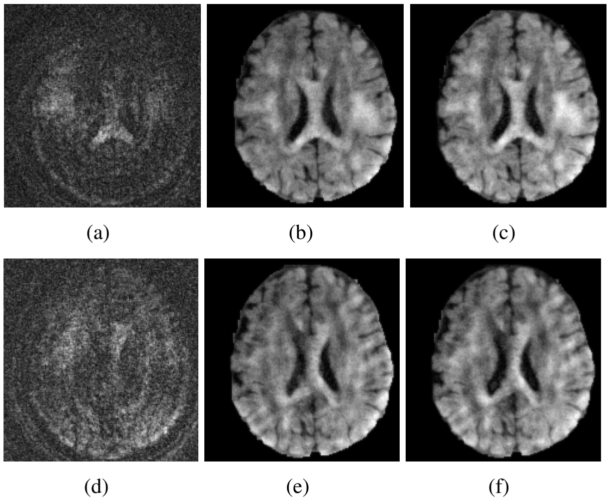

Fig. 2. (a),(d) SENSE reconstruction without motion correction of two diffusion-weighted images (b&e), (c&f) are combined motion correction and image reconstruction from schemes shown in Fig 1 (a) and (b) respectively

the reconstructed image was highly corrupted as can be seen from Fig 2(a). Combined motion correction and reconstruction using the implementation of $E^H EI$ as shown in Fig 1(a) was performed on the data. As one can seen from Fig 2(b), high quality image free of motion artifacts was reconstructed. However, there are 22*8=176 composite sensitivity matrices, hence that many FFTs, gridding and IFFTs. CG takes about 10 iterations to converge, resulting in a total computation time of 178 sec. Next, the simplified implementation with few basis functions as shown in Fig 1(b) was tested, where the 176 composite sensitivity maps were represented using 10 basis functions. This reduced the number of FFTs and IFFTs to 10, and number of multiplications to 100 as opposed to 176 gridding steps. Reconstruction time for 10 iterations was 14 sec. One can see that comparable quality of motion correction and reconstruction was achieved using new schemes with drastic reduction in computation time of about 12 times. In Fig 3, the reconstruction time and error is plotted as a function of number of basis functions used. The plot shows that with 10 basis functions, the error is not significantly increased, however the computation time is considerably lowered. For applications such as reconstruction of high angular resolution diffusion imaging, the savings becomes highly beneficial.

5. CONCLUSION

Combined motion correction and image reconstruction are desirable for multi-shot multi-coil non-cartesian trajectories. However, SENSE reconstruction becomes computationally challenging due to large number of virtual sensitivity encodings involved. Here we proposed a new pipeline for the joint motion correction and SENSE reconstruction of non-Catresian data by approximating the virtual sensitivites using a reduced number of basis functions. The common set of basis functions enabled to perform the FFTs and IFFTs more efficiently only on those basis functions rather than on individual sensitivity encodings. By replacing the gridding steps with multiplications in the Fourier domain, the intermediate steps between the FFTs and IFFTs could be combined into multiplication by a weighting matrix that can be efficiently computed. Significant acceleration in reconstruction time was obtained.

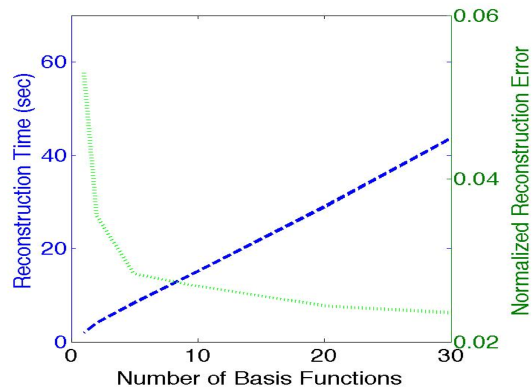

Fig. 3. Plot of reconstruction time and error as a function of number of basis functions used. For comparison, the classical implementation takes 178 sec for reconstruction.

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