Fuzzy Logic

Andrew Kusiak
Intelligent Systems Laboratory
2139 Seamans Center
The University of Iowa
Iowa City, IA 52242 – 1527
andrew-kusiak@uiowa.edu
http://www.icaen.uiowa.edu/~ankusiak

(Based on the material provided by Professor V. Kecman)

What is Fuzzy Logic?

Fuzzy logic is a tool for embedding

human knowledge

(experience, expertise, heuristics)

Why Fuzzy Logic ?

Human knowledge is fuzzy: expressed in 'fuzzy' linguistic terms, e.g., young, old, large, cheap.

Example

Temperature is expressed as cold, warm or hot.
No quantitative meaning.

Fuzzy Logic

"Fuzzy logic may be viewed as a bridge between the excessively wide gap between the *precision* of classical crisp logic and the *imprecision* of both the real world and its human interpretation"

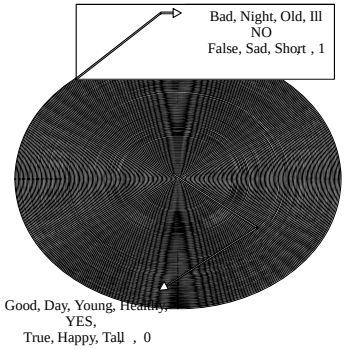
Paraphrasing L. Zadeh

Fuzzy Logic

- Fuzzy logic attempts to model the way of reasoning of the human brain.
- Almost all human experience can be expressed in the form of the IF - THEN rules.
- Human reasoning is pervasively approximate, non-quantitative, linguistic, and dispositional (meaning, usually qualified).

The World is Not Binary!

Gradual transitions and ambiguities at the boundaries



When and Why to Apply FL?

Criteria

- Human knowledge is available
- Mathematical model is unknown or impossible to obtain
- Process substantially nonlinear
- Lack of precise sensor information

When and Why to Apply FL?

Criteria

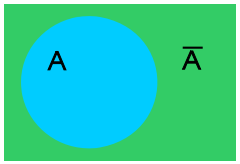
- At higher levels of hierarchical control systems
- In decision making processes

How to Transfer Human Knowledge Into the Model ?

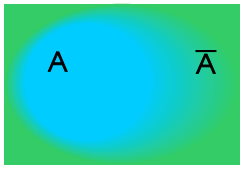
- Knowledge should be structured.
- Possible shortcomings:
  - Knowledge is subjective
  - 'Experts' may bounce between extreme points of view:
    - Have problems with structuring the knowledge, or
    - Too aware in his/her expertise, or
    - Tend to hide 'knowledge', or ...
- Solution: Find a 'good' expert.

Fuzzy Sets vs Crisp Sets

Crisp Sets



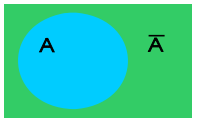
Fuzzy Sets

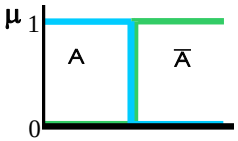


Venn Diagrams

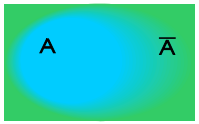
Fuzzy Sets vs Crisp Sets

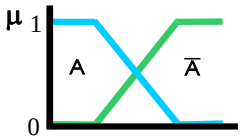
Crisp Sets





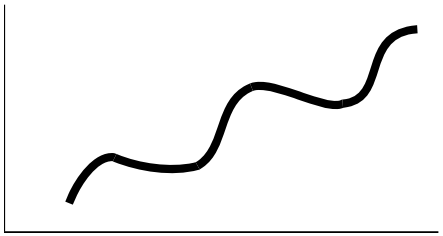
Fuzzy Sets





μ - membership degree, possibility distribution, grade of belonging

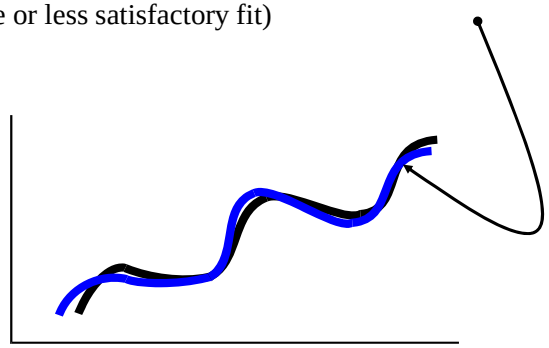
Modeling or Approximating a Function: Curve or Surface Fitting



Terms used in other disciplines:
regression (L or NL), estimation, identification, filtering

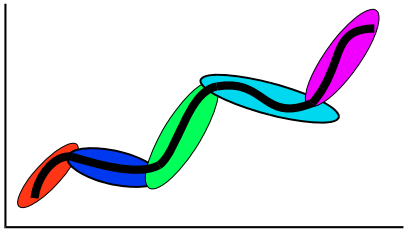
Modeling a Function

Standard mathematical approach of curve fitting
(more or less satisfactory fit)



Modeling a Function

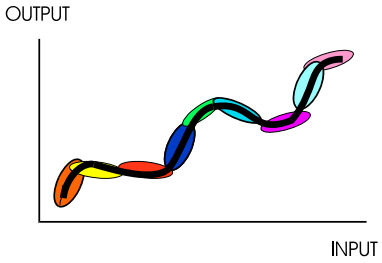
Curve fitting by using fuzzy rules (patches)
Surface approximation for 2 inputs or
a hyper-surface (3 or more inputs)



Small number of rules - Large patches or rough approximation

Modeling a Function

Fuzzy patches

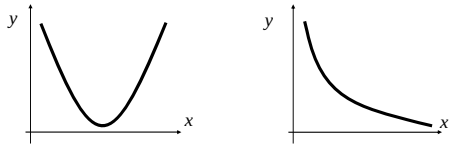


More rules - more smaller patches and better approximation

What is the origin of the patches and how do they work?

Example 1

Consider modeling two different functions by fuzzy rules

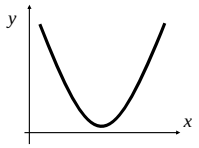


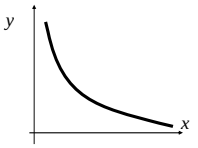
Example 1

- Lesser number of rules decreases the approximation accuracy. An increase in a number of rules, increases the precision at the cost of a computation time needed to process these rules.
- This is the most classical soft computing dilemma - A trade-off between the imprecision and uncertainty on one hand and low solution cost, tractability and robustness on the other.
- The appropriate rules for the two functions are:

Example 1

Appropriate rules





IF *x* is *low* THEN *y* is *high.*

IF *x* is *medium* THEN *y* is *low.*

IF *x* is *large* THEN *y* is *high.*

IF *x* is *low* THEN *y* is *high.*

IF *x* is *medium* THEN *y* is *medium.*

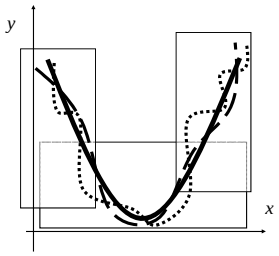
IF *x* is *large* THEN *y* is *low.*

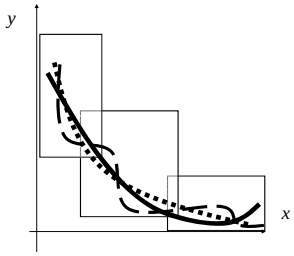
Example 1

These rules define three large rectangular patches that cover the functions. They are shown in the next slide together with two possible approximators for each function.

Example 2

Modeling two different functions by fuzzy rules





The two original functions (solid lines in both graphs) covered by three patches produced by IF-THEN rules and modeled by two possible *approximators* (*dashed* and *dotted curves*).

Example 2

COMMENTS

- Humans do not (or only rarely) think in terms of nonlinear functions.
- Humans do not 'draw these functions in their mind'.
- We neither try 'to see' them as geometrical artifacts.
- In general, we do not process geometrical figures, curves, surfaces or hypersurfaces while performing tasks or expressing our knowledge.

Example 2

COMMENTS

- Even more, our expertise or understanding of some functional dependencies is often not a structured piece of knowledge at all.
- We typically perform complex tasks without being able to express how they are executed.

Example 2

Classroom Exercise

Explain to your colleague in the form of IF-THEN rules how to ride a bike.

Car Example

Fuzzy Control of the Distance Between Two Cars

The steps in fuzzy modeling are always the same.

i) Define the variables of relevance, interest or importance:
- In engineering we call them input and output variables

ii) Define the subsets' intervals:
- Small - medium, or negative - positive, or
- Left - right (labels of dependent variables)

Car Example

Fuzzy Control of the Distance Between Two Cars

iii) Choose the shapes and the positions of fuzzy subsets, i.e.,

- Membership functions, i.e., attributes

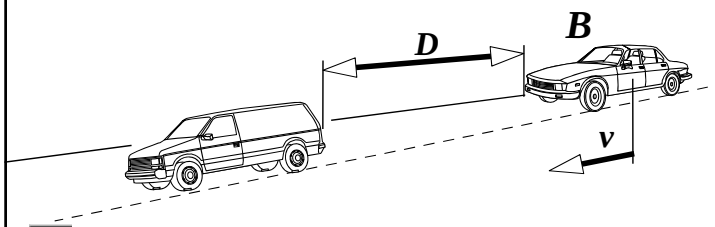
iv) Set the rule form, i.e., IF - THEN Rules

v) Perform computation and (if needed) tune (learn, adjust, adapt) the positions and the shapes of both the input and the output attributes of the model

Car Example

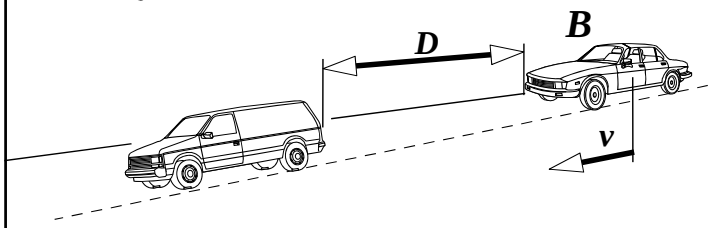
Fuzzy Control of the Distance Between Two Cars

INPUTS: D = DISTANCE, v = SPEED
OUTPUT: B = BRAKING FORCE



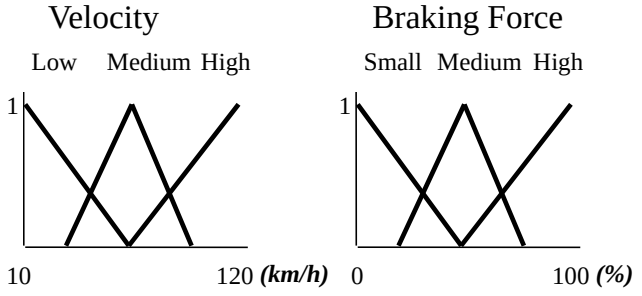
Car Example

Fuzzy Control of the Distance Between Two Cars



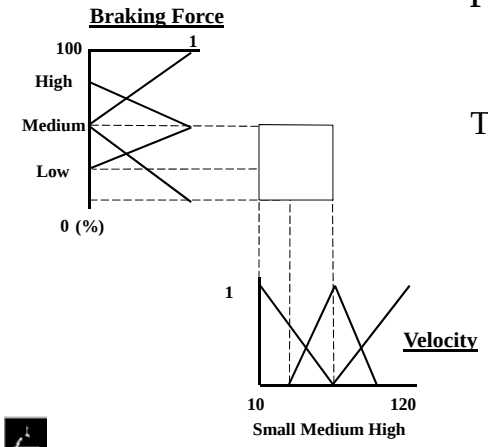
Analyze the rules for a given distance **D** and for different velocity ***v***, i.e., ***B*** = *f*(***v***)

Car Example



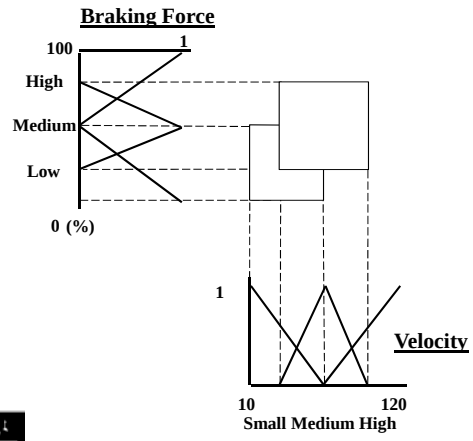
IF the Velocity is **Low, THEN** the Braking Force is **Small**
IF the Velocity is **Medium, THEN** the Braking Force is **Medium**
IF the Velocity is **High, THEN** the Braking Force is **High**

Car Example



The fuzzy patch

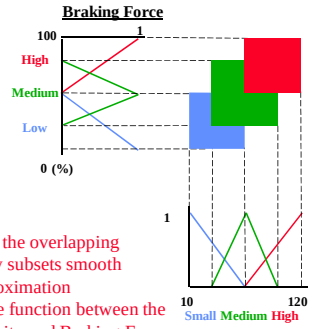
Car Example



The fuzzy patches

Car Example

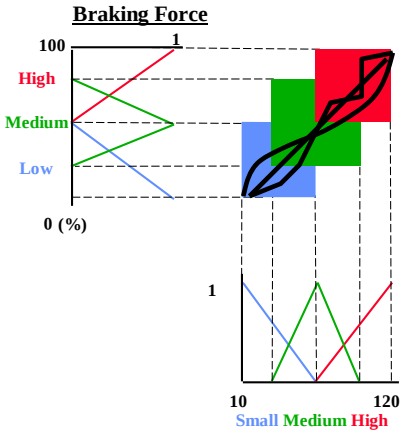
The fuzzy patches



Note the overlapping fuzzy subsets smooth approximation of the function between the Velocity and Braking Force

The Fuzzy Patches Define the Function

Car Example



Three possible dependencies between the Velocity and Breaking force.

Each of us drives differently

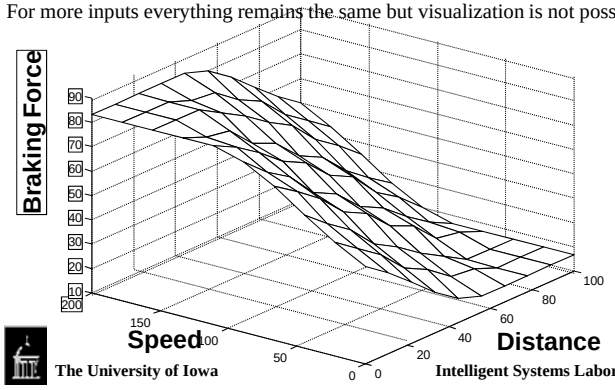
FUNCTIONAL DEPENDENCE OF THE VARIABLES

SURFACE OF KNOWLEDGE

Fuzzy Control of the Distance Between Two Cars

Visualization of 2 INPUTS: D and *v*, and 1 OUTPUT B is possible.

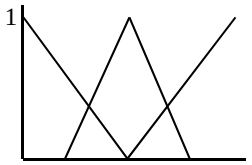
For more inputs everything remains the same but visualization is not possible.



Example: Room Temperature Control

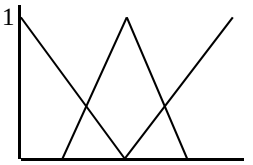
Room Temperature

Cold Warm Hot



Fan Speed

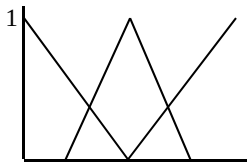
Slow Medium Fast



Example: Room Temperature Control

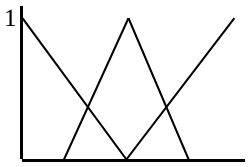
Room Temperature

Cold Warm Hot



Fan Speed

Slow Medium Fast

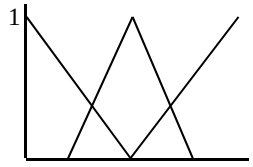


If Room Temperature is Cold, then Fan Speed is Slow

Example: Room Temperature Control

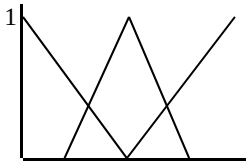
Room Temperature

Cold Warm Hot



Fan Speed

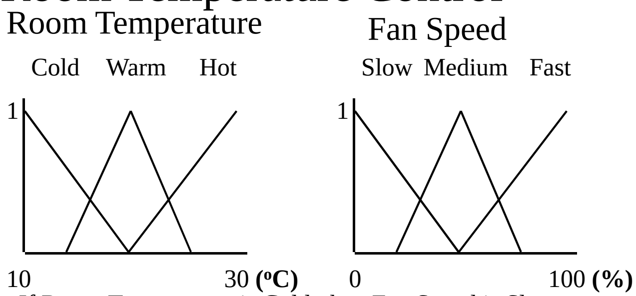
Slow Medium Fast



If Room Temperature is Cold, then Fan Speed is Slow

If Room Temperature is Warm, then Fan Speed is Medium

Example: Room Temperature Control

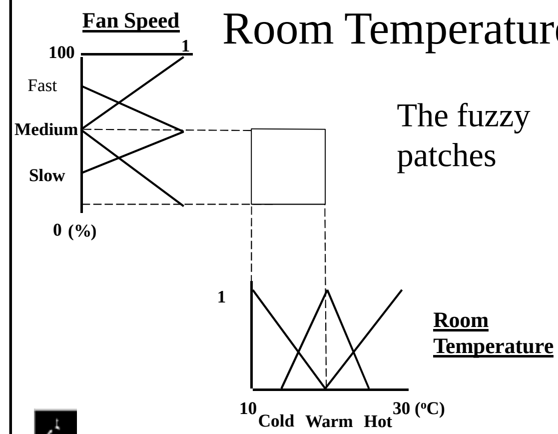


If Room Temperature is Cold, then Fan Speed is Slow

If Room Temperature is Warm, then Fan Speed is Medium

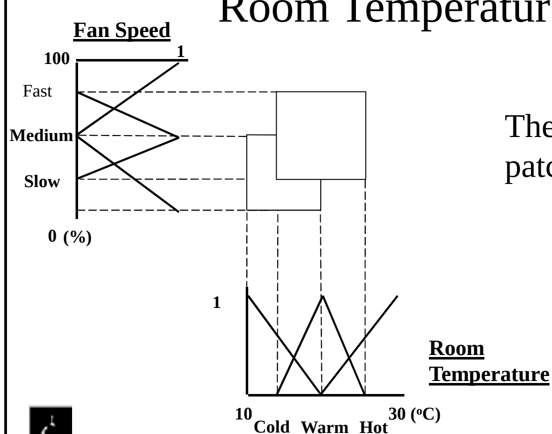
If Room Temperature is Hot, then Fan Speed is Fast

Example: Room Temperature Control



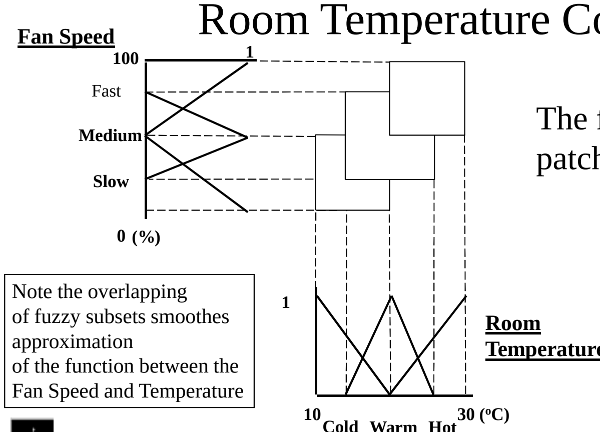
The fuzzy patches

Example: Room Temperature Control



The fuzzy patches

Example: Room Temperature Control

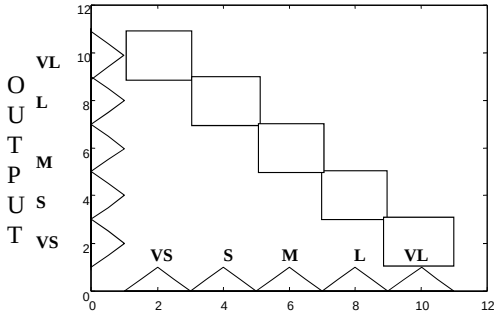


The fuzzy patches

Note the overlapping of fuzzy subsets smoothes approximation of the function between the Fan Speed and Temperature

Example:
Room Temperature Control

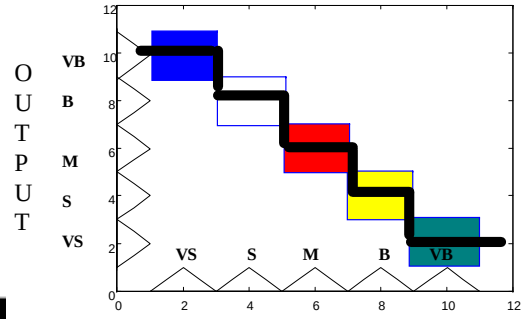
There must be some overlapping of the input fuzzy subsets (membership or characteristic functions) if we want to obtain a smooth model



If there was no overlapping. one would obtain the stepwise function as shown next

Example:
Room Temperature Control

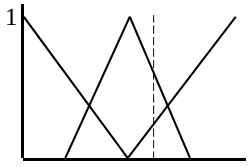
There must be some overlapping of the input fuzzy subsets (membership or characteristic functions) if we want to obtain a smooth model



Output Computation: Fuzzification, Inference and Defuzzifaction

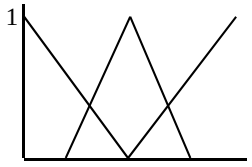
Room Temperature

Cold Warm Hot



Fan Speed

Slow Medium Fast



R1: If Room Temperature is Cold, Then Fan Speed is Slow

R2: If Room Temperature is Warm, Then Fan Speed is Medium

R3: If Room Temperature is Hot,Then Fan Speed is Fast

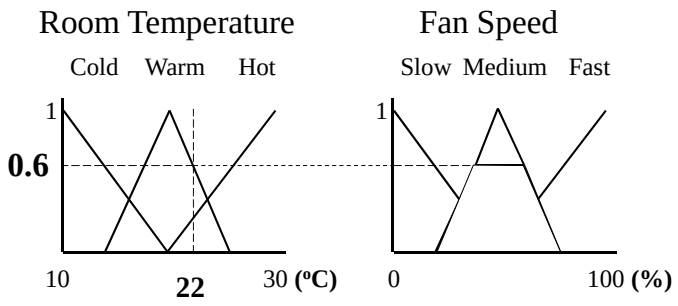
Example:
Room Temperature Control

- After the fuzzy modeling is done there is an operational phase:

  Compute the fan speed when the room temperature = 22 ºC

- NOTE: 22 ºC belongs to the subsets 'Warm' and 'Hot'

Fuzzification and Inference

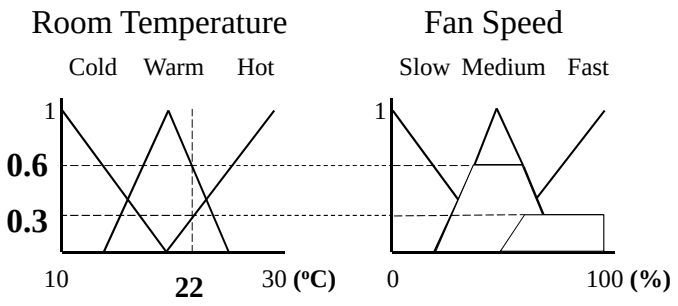


If Room Temperature is Cold, Then Fan Speed is Slow

If Room Temperature is Warm, Then Fan Speed is Medium

If Room Temperature is Hot, Then Fan Speed is Fast

Fuzzification and Inference

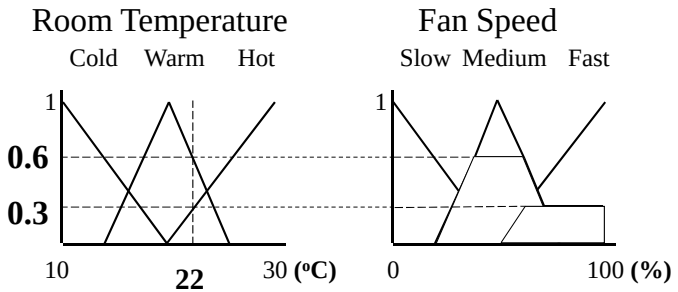


If Room Temperature is Cold, then Fan Speed is Slow

If Room Temperature is Warm, then Fan Speed is Medium

If Room Temperature is **Hot,** then Fan Speed is **Fast**

Fuzzification and Inference



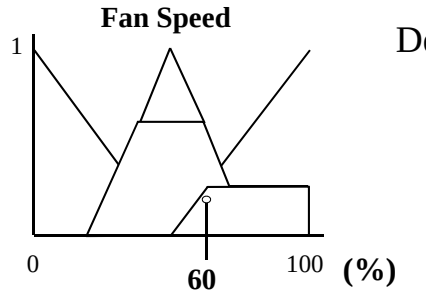
WHAT IS THE OUTPUT VALUE?

If Room Temperature is Cold then Fan Speed is Slow

If Room Temperature is Warm then Fan Speed is Medium

If Room Temperature is **Hot** then Fan Speed is **Fast**

Defuzzification

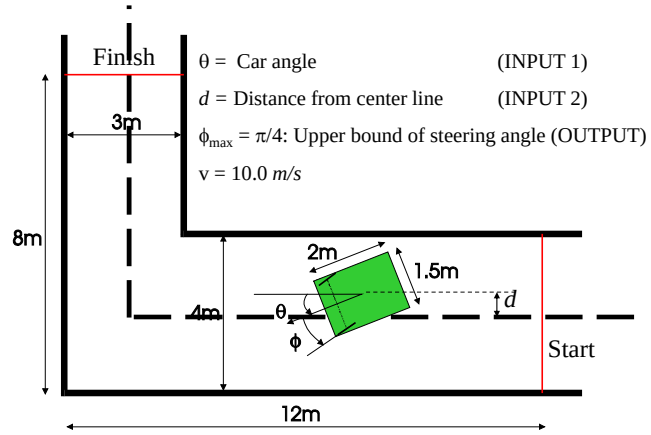


- The result of the fuzzy inference is a fuzzy subset composed of the slices of fan speed: Medium (blue) and Fast (red)
- How to find a crisp (useful in the real world application) value?
- One of several methods used to obtain a crisp output value is 'the center of area formula'

Example: Vehicle Turning Problem

- Generic fuzzy logic controller
  - Developed in Matlab
  - User friendly
  - Multiple inputs
- Many other commercial applications are possible

Configuration of the Vehicle Turning Problem



θ = Car angle (INPUT 1)

d = Distance from center line (INPUT 2)

$\phi_{max} = \pi/4$: Upper bound of steering angle (OUTPUT)

v = 10.0 *m/s*

Conclusions

- Fuzzy logic can be implemented wherever there is structured human knowledge, expertise, heuristics, experience.
- Fuzzy logic is not needed whenever there is an analytical closed-form model that, using a reasonable number of equations, can solve a problem in a reasonable time, at the reasonable costs and with higher accuracy.

Conclusions

Possible obstavles

- Finding good (dependable) expert
- Right choice of the variables
- Increasing the number of *inputs*, as well as the number of fuzzy subsets per input variable, the number of rules increases exponentially (curse of dimensionality)
- Good news is that there are plenty of real life problems and situations that can be solved with small number of rules only