Energy Output

for Wind Power Management

Spring 2015

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Outline

- Variability in wind
- Distribution plotting
- Mean power of the wind
- Betz' law
- Power density
- Power curves
- The power coefficient
- Calculator guide
- The power calculator
- Annual energy output

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Characterizing Wind Variability

- Why is wind variability important?
- Suppose we know that the average annual wind speed is 6.3 m/s
- We could choose a turbine with the largest power production at 6.3 m/s
- This approach would have very different results depending on the site's wind variability around 6.3 m/s

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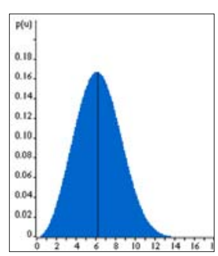
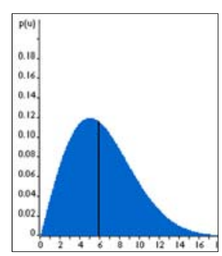
Characterizing Wind Variability





Characterizing Wind Variability

- Wind turbine designers use wind variability to optimize turbine designs to minimize energy generation costs and handle mechanical stress
- Wind farm designers use wind variability to locate wind farms and select wind turbines
- Investors use wind variability to estimate revenue from electricity generation

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Characterizing Wind Variability

Spatial Variability
- latitude
- topography
- surface roughness

Temporal Variability
- inter-annual
- annual (seasonal)
- diurnal
- short-term

Let's look at the long term temporal variability at one location

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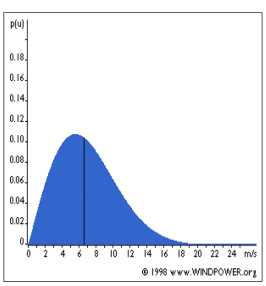
Weibull Distribution



- Wind variability is typically described using a Weibull distribution
- Weibull distributions can be asymmetrical
- Low wind speeds are more common than high wind speeds in many locations

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Weibull Distribution

Weibull distributions can be described by two parameters:
- $\lambda > 0$ is the scale parameter
- $k > 0$ is the shape parameter

$$f(x;k,\lambda) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}$$

For k=3.4, the Weibull distribution is similar to the normal distribution

For k=1, the Weibull distribution becomes the exponential distribution

http://en.wikipedia.org/wiki/Weibull_distribution

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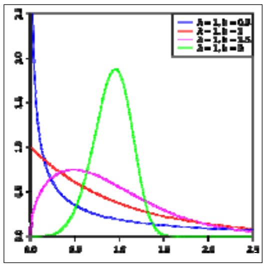
Weibull Distribution PDF



$$f(x;k,\lambda) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}$$

Median:

$$\lambda(\ln(2))^{1/k}$$

Mode:

$$\lambda\left(\frac{k-1}{k}\right)^{1/k}$$

Mean:

$$\lambda\Gamma\left(1+\frac{1}{k}\right)$$

where the gamma function is:

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\,dt$$

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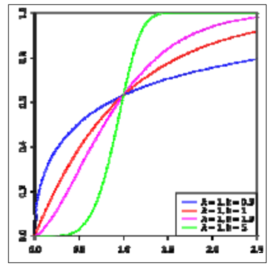
http://en.wikipedia.org/wiki/Weibull_distribution

Weibull Distribution CDF



$$f(x;k,\lambda) = 1 - e^{-(x/\lambda)^k}$$

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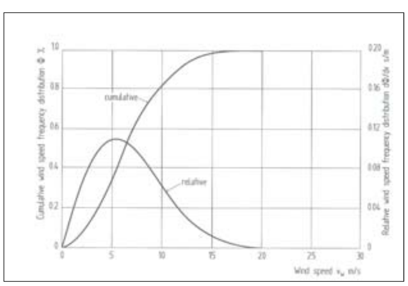
http://en.wikipedia.org/wiki/Weibull_distribution

PDF and CDF



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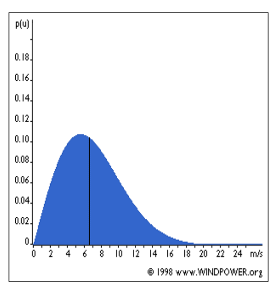
E. Hau (2006)

Weibull Distribution



- Mean = 7 m/s
- Median = 6.6 m/s
- 50% of the time the wind speed is less than 6.6 m/s, 50% of the time the wind speed is greater than 6.6 m/s
- The median does not equal the mean due to the asymmetry of the pdf

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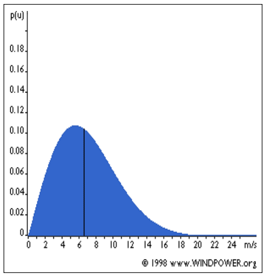
Weibull Distribution



- Mode = 5.5 m/s
- The mode is the most common wind speed which is located at the peak of the pdf curve
- The statistical distribution of wind speed varies by location and time scale
- The Weibull distribution is fit to a specific location using the shape and scale parameters

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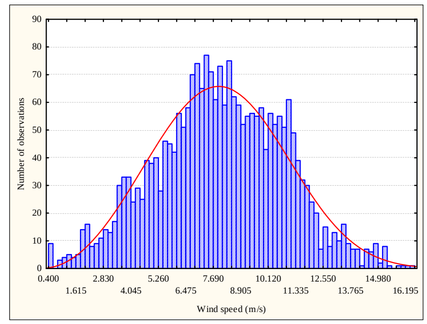
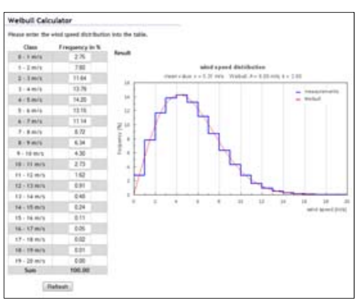
Wind Speed Histogram

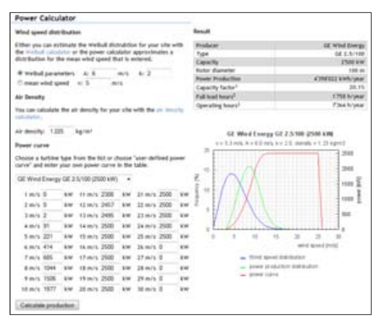


Weibull Calculator



http://www.wind-data.ch/tools/weibull.php?lng=en

Power Calculator



http://www.wind-data.ch/tools/powercalc.php?lng=en

Betz's Law

$$\rho A_1 v_1 = \rho A_2 v_2$$

$$P = \frac{1}{2}\dot{m}(v_1^2 - v_2^2)$$

- As kinetic energy is extracted from the wind, the wind slows down
- If all kinetic energy were extracted from the wind, the wind speed would reduce to zero and could not leave the turbine
- If the wind speed at the turbine was zero, new air could not enter the rotor and no energy could be extracted

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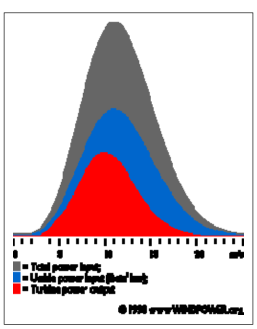
Betz's Law

$$\rho A_1 v_1 = \rho A_2 v_2$$

$$P = \frac{1}{2}\dot{m}(v_1^2 - v_2^2)$$

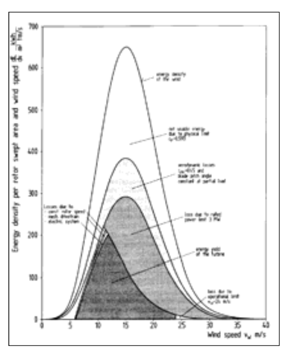
- An ideal wind turbine slows down the wind by $\frac{2}{3}$ of its original speed $\left(v_2 = \frac{1}{3}v_1\right)$
- Betz's Law (1919) says that the theoretical maximum kinetic energy that can be converted into mechanical energy using an ideal wind turbine is $\frac{16}{27}$ or approximately 59.3%
- Commercial wind turbines can only achieve 70-80% of the Betz limit due to losses

Power Density Function



- The y-axis represents power density at each wind speed for a particular site
- The gray area is the total power available: $P = 0.5 \times \rho \times A \times v^3$
- The blue area is the theoretical power that can be extracted by Betz's Law
- The red area is the actual power a particular turbine can produce, determined by the power curve

Power Density Function



E. Hau (2006)

- This more detailed power density function illustrates:

$$E = E_{rot} \times E_{gear} \times E_{gen} \times E_{pconv}$$

where
E=wind turbine system efficiency
E_{rot}=rotor efficiency
E_{gear}=gearbox efficiency
E_{gen}=generator efficiency
E_{pconv}=power converter efficiency

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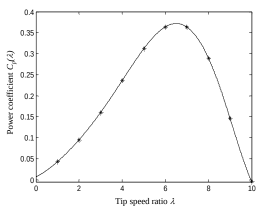
Mechanical Power Summary

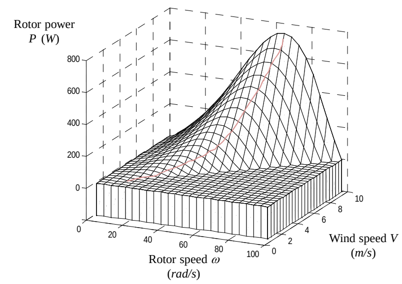


$$P = \frac{1}{2}\rho A C_p(\lambda) v^3$$

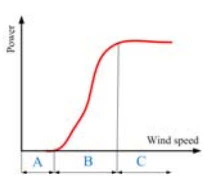
$$\lambda = \frac{r\omega}{v}$$

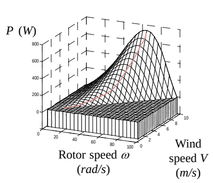
where
- P =power produced by rotor
- ρ=air density
- A=rotor swept area
- v=wind speed
- λ=tip speed ratio
- C_p=power coefficient
- r=rotor radius
- ω=rotor speed

Mechanical Power Summary



Mechanical Power Summary





A:

Insufficient power to overcome friction and initial torque

B:

Turbine operates to maximize efficiency

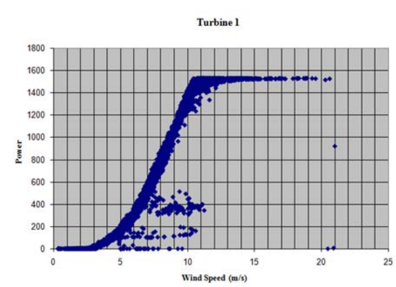
C:

Fixed (rated) power operation

$$P = \frac{1}{2}\rho A C_p(\lambda) v^3$$

$$\lambda = \frac{r\omega}{v}$$

Actual Power Curve



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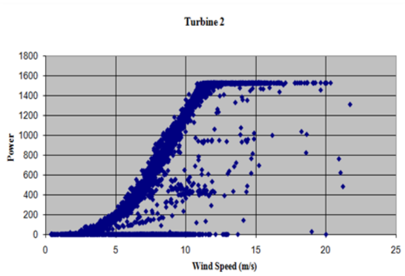
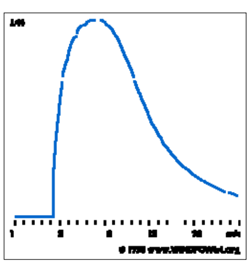
Actual Power Curve



Validating Power Curves

- Power curves are constructed using measurements in areas with low turbulence intensity, and with the wind coming directly towards the front of the turbine
- Local turbulence and complex terrain (e.g., turbines placed on a rugged slope) may mean that wind gusts hit the rotor from varying directions
- It may therefore be difficult to reproduce the power curve exactly at any given location

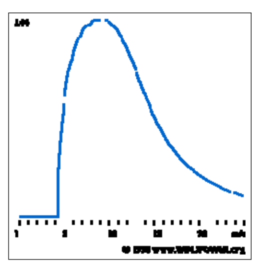
The Power Coefficient



- The power coefficient indicates how efficiently a turbine converts wind energy into electricity (turbine efficiency)
- Efficiency varies with wind speed

$$Power\ Coefficient = \frac{Electrical\ Power\ Output}{Wind\ Energy\ Input}$$

The Power Coefficient



- This turbine was designed for the maximum efficiency of 44% at a wind speed of approximately 9 m/s
- The efficiency at low speeds is not as important because there is not much energy to harvest
- At wind speeds above the rated speed the turbine wastes excess energy
- Efficiency matters most in regions of wind speed where the most energy is found

The Capacity Factor

$$Turbine\ Capacity\ Factor = \frac{Actual\ Annual\ Energy\ Output}{Theoretical\ Max\ Energy\ Output}$$

- Here the theoretical max energy output is defined as a turbine running at its maximum rated power for 8766 hours of the year ($24 \times 365.25 = 8766$)
- The capacity factor could vary from 0 to 100% but in practice is usually between 20% to 70%
- Example: suppose a 600 kW turbine produced 1.5 million kWh in a year. The capacity factor is:

$$\frac{1500000}{(24 \times 365.25 \times 600)} = 0.285\ or\ 28.5\%$$

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The Capacity Factor Paradox

- Although a large capacity factor is preferred, it may not always be the most profitable
- For example, in a very windy location, it may be an advantage to use a large generator for a given rotor diameter
- This would tend to lower the capacity factor (using less of the capacity of a relatively larger generator), but it may imply a substantially larger annual production

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Comparison

$$Power\ Coefficient = \frac{Electrical\ Power\ Output}{Wind\ Energy\ Input}$$

$$Availability\ Factor = \frac{Time\ Turbine\ is\ Available\ in\ a\ Year}{Total\ Time\ in\ a\ Year}$$

$$Turbine\ Capacity\ Factor = \frac{Actual\ Annual\ Energy\ Output}{Theoretical\ Max\ Energy\ Output}$$

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Acknowledgement

The material included in the presentation comes largely from the Danish Wind Industry Association

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Weibull and Power Calculators

http://www.wind-data.ch/tools/weibull.php?lng=en

http://www.wind-data.ch/tools/powercalc.php?lng=en

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