LAYOUT OF MACHINES AND FACILITIES

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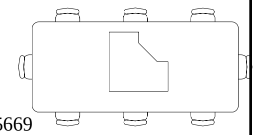


Outline

- LAYOUT EXAMPLES
- SINGLE-ROW MACHINE LAYOUT
- DOUBLE-ROW MACHINE LAYOUT
- MULTIROW FACILITY LAYOUT
- ALGORITHMS

Equipment Selection — Machine Cell Formation — Machine Layout — Cell Layout

Latest layout software *Proplanner*
http://www.proplanner.com/product/details/flowpath.aspx

Big Picture

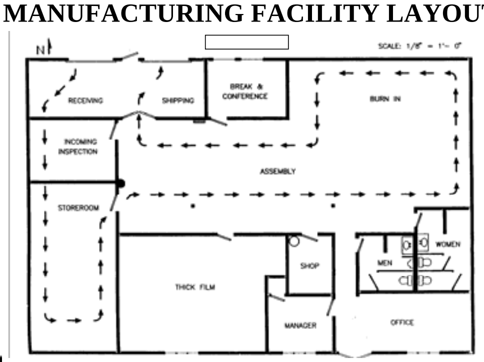
- Facility planning
  - Facility location
  - Facility design
    - Structural design
    - Machine layout design
    - Material handling system design
  - Facility utilization

Jobs: Facility Designer
Facility Planner

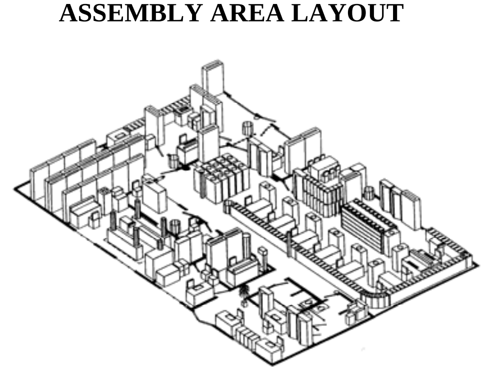
AutoCAD Software
http://www.autodesk.com/

LAYOUT EXAMPLES

MANUFACTURING FACILITY LAYOUT



ASSEMBLY AREA LAYOUT



Office Layout

LAYOUT OF MACHINES

Methods:

- **Visual, e.g., using templates**
- **Computer based (often an environment for evaluating various layout alternatives)**
- **Model based**

LAYOUT OF MACHINES

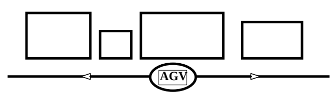
Visualization types:

- **2D**
- **3D**
- **Virtual reality**

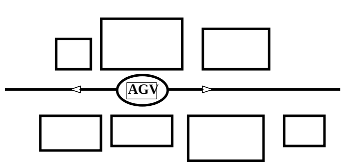
LAYOUT OF MACHINES
Model Based Approach

Basic Layout Patterns

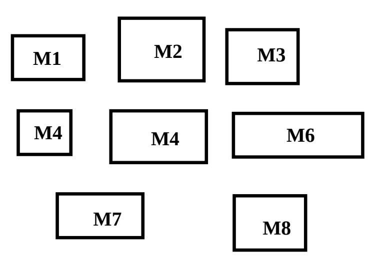
Single-Row Machine Layout



Double-Row Machine Layout



Multi-row Layout



BASIC DATA

- UNIT DISTANCE TRAVEL COST (c)
  - OR FREQUENCY OF TRIPS (f)
- DISTANCE (d)
  - OR TRAVEL TIME (t)

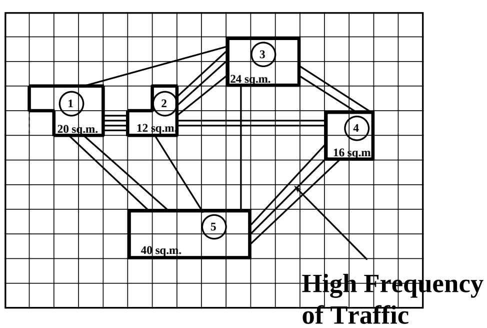
OBJECTIVE

Arrange machines on the shop floor in such a way that the total product of the travel cost per unit distance traveled [$/m] and the distance traveled [m] is minimum.

Travel cost is often replaced with the number (frequency - f) of trips per time unit [1/sec] between any two machines.
Distance is replaced with travel time - t [sec].

$$\text{Min} \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} f_{ik}\, t_{jl}\, x_{ij}\, x_{kl}$$

Example: Intuitive Machine Layout



SINGLE-ROW MACHINE LAYOUT

<u>Frequency of trips</u> (traffic flow)

$$f_{ij} = \sum_{k=1}^{n_{ij}} \left\lceil \frac{v_{ij}^{k}}{u_k} \right\rceil$$

where: v_{ij}^{k} = volume of part type k to be moved from machine i to machine j in a given time horizon (for example, 1 year)

n_{ij} = number of different parts to be moved from machine i to machine j in a given time horizon

u_k = number of parts to be moved in a single trip of the material handling equipment

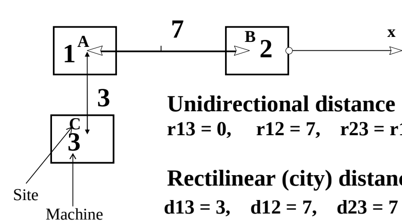
$\lceil \bullet \rceil$ = smallest integer greater than or equal to •

The AGV travel time involves:

- loading time
- acceleration time
- travel time
- deceleration time
- unloading time

DISTANCE CALCULATION

Consider 3 machines

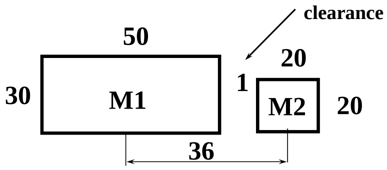


Unidirectional distance (along x axis)
r13 = 0, r12 = 7, r23 = r12 = 7

Rectilinear (city) distance
d13 = 3, d12 = 7, d23 = 7 + 3 = 10

Euclidean distance
e12 = 7, e13 = 7.48

Adjacent unidirectional distance r_{12}



$r_{12} = d_{12} = 25 + 10 + 1 = 36$

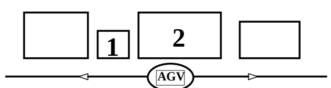
Time is the best measure to be used in the optimization of machine layout, however, if the travel speed is constant, an appropriate distance measure can be considered.

SINGLE ROW MACHINE LAYOUT

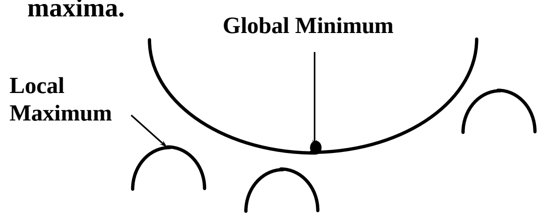
Algorithm 1 Overview

Two machines with the maximum flow between them (e.g., 1 and 2) are placed in the adjacent locations.

Other machines are placed to the left and right of the two machines based on the flow.



The minimization of the total product of flow x travel time is accomplished by local maxima.



$$\text{Min} \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} f_{ik}\, t_{jl}\, x_{ij}\, x_{kl}$$

Algorithm 1

Step 0. Set iteration number k = 1.

From the flow matrix $[f_{ij}]$ compute
$f_{i^*j^*} = \max\{f_{ij}: i, j = 1, 2, ..., m\}$.
If there is a tie, $f_{i^*j^*} = \max\{f_{ij} \bullet t_{ij}: i, j = 1, 2, ..., m\}$.
Connect i^*, j^* and include them in the solution set.
Set the solution set, $U = \{i^*, j^*\}$.

Step 1. Compute

$f_{p^*q^*} = \max\{ f_{i^*k}, f_{j^*l}: k,l \in \{\{1, 2, \ldots, m\} - U\}$.

Set $s^* = q^*$.

Consider two alternatives:

(a) Place machine s^* left of machine i^*;

(b) Place machine s^* right of machine j^*.

Compute:

$q_{s^*i^*} = \Sigma\ \{ f_{s^*r} \bullet t_{s^*r}: r \in U$ and machine s^* is placed left of machine $i^*\}$;

$q_{j^*s^*} = \Sigma\ \{ f_{s^*r} \bullet t_{s^*r}: r \in U$ and machine s^* is placed right of machine $j^*\}$

If $q_{s^*i^*} \le q_{j^*s^*}$,

(1) Select the alternative (a) above;

(2) Set $i^* = s^*$.

If $q_{s^*i^*} > q_{j^*s^*}$,

(1) Select the alternative (b) above;

(2) Set $j^* = s^*$.

$U = U + s^*$.

Step 2. Set iteration number k = k + 1.
Repeat Step 1 until the final solution is obtained (i.e.., until all the machines are included in the solution set U).

Example

Single-row machine layout

Given:

$$[f_{ij}] = \text{Machine}\ \begin{array}{c|cccc} & \multicolumn{4}{c}{\text{Machine}} \\ & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 10 & 15 & 15 \\ 2 & 10 & 0 & 0 & 5 \\ 3 & 15 & 0 & 0 & 40 \\ 4 & 15 & 5 & 40 & 0 \end{array}$$

fij = frequency of trips
tij = travel time
rij = unidirectional distance
dij = adjacent distance

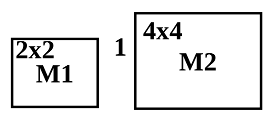
Machine number	Machine dimensions
1	2 x 2
2	4 x 4
3	6 x 6
4	2 x 2

Clearance = 1

Calculate adjacent distances

$$[d_{ij}] = \text{Machine}\ \begin{array}{c|cccc} & \multicolumn{4}{c}{\text{Machine}} \\ & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 4 & 5 & 3 \\ 2 & 4 & 0 & 6 & 4 \\ 3 & 5 & 6 & 0 & 5 \\ 4 & 3 & 4 & 5 & 0 \end{array}$$

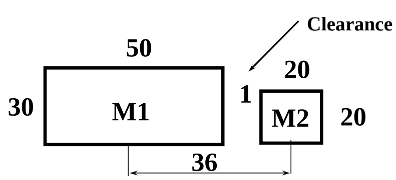
Machine number	Machine dimensions
1	2 x 2
2	4 x 4
3	6 x 6
4	2 x 2



d_{12} =
1 + 1 + 2 = 4

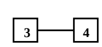
For simplicity assume time tij = distance dij

Distances



Adjacent distance d_{12} = 25 + 10 + 1 = 36
Unidirectional distance r_{12} = d_{12} = 36

Iteration 1
Step 0.
From the flow matrix [fij],
max {fij: i = 1, 2, 3, 4, j = 1, 2, 3, 4} = f34 = 40 is obtained. Thus i* = 3, j* = 4 is determined.
Machines 3 and 4 are connected and included in the solution, symbolically denoted as



The solution set is updated U = {3, 4} and the columns 3 and 4 of the matrix are marked with asterisks.

$$[f_{ij}] = \text{Machine}\ \begin{array}{c|cccc} & & & * & * \\ & \multicolumn{4}{c}{\text{Machine}} \\ & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 10 & 15 & 15 \\ 2 & 10 & 0 & 0 & 5 \\ 3 & 15 & 0 & 0 & 40 \\ 4 & 15 & 5 & 40 & 0 \end{array}$$

$$[f_{ij}] = \text{Machine}\ \begin{array}{c} \\ 1\\2\\3\\4 \end{array} \begin{array}{c} \text{Machine} \\ \begin{array}{cccc} 1 & 2 & 3^* & 4^* \end{array} \\ \begin{bmatrix} 0 & 10 & 15 & 15 \\ 10 & 0 & 0 & 5 \\ 15 & 0 & 0 & 40 \\ 15 & 5 & 40 & 0 \end{bmatrix} \end{array}$$

3 — 4

Step 1. Compute
Max { f3k, f4l: k = 1, 2; l = 1, 2} = f31 = f41 = 15.
Set s* = 1.
Consider two alternatives:
(a) Place machine 1 on the left hand side of machine 3

1 — 3 — 4

(b) Place machine 1 on the right hand side of machine 4

3 — 4 — 1

Min $\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}$ fik tjl xij xkl

1 — 3 — 4

qs*i* = q13 = f13 • t13 + f14 • t14 = f13 • r13 + f14 • r14
= f13 • d13 + f14 • (d13 + d34) = 15 • 4 + 15 • (4 + 5) = 195.

3 — 4 — 1

qj*s* = q41 = f14 • t14 + f13 • t13 = f14 • r14 + f13 • r13
= f14 • d14 +f13 • (d14 + d43) = 15 • 3 + 15 • (3 + 5) = 165.

Compute (repeated from the previous slide):
1 — 3 — 4 qs*i* = q13 = f13 • t13 + f14 • t14 = f13 • r13 + f14 • r14
= f13 • d13 + f14 • (d13 + d34) = 15 • 4 + 15 • (4 + 5) = 195.
3 — 4 — 1 qj*s* = q41 = f14 • t14 + f13 • t13 = f14 • r14 + f13 • r13
= f14 • d14 +f13 • (d14 + d43) = 15 • 3 + 15 • (3 + 5) = 165.

As q13 > q41,
(1) Alternative (b) (min) is selected where machine 1 is placed to the right of machine 4

3 — 4 — 1

(2) Set j* = 1;
(3) The set U is updated to U + s* = {1, 3, 4} and column 1 is marked with an aster.

Step 2. Since machine 2 is not included in the solution set U, go to Step 1.

3 — 4 — 1

$$[f_{ij}] = \text{Machine}\ \begin{array}{c} \\ 1\\2\\3\\4 \end{array} \begin{array}{c} \text{Machine} \\ \begin{array}{cccc} 1^* & 2 & 3^* & 4^* \end{array} \\ \begin{bmatrix} 0 & 10 & 15 & 15 \\ 10 & 0 & 0 & 5 \\ 15 & 0 & 0 & 40 \\ 15 & 5 & 40 & 0 \end{bmatrix} \end{array}$$

Iteration 2
Step 1. Compute Max { f3k, f1l : k = 2, l = 2} =
= f12 = 10.
Set s* = 2.
Consider two alternatives:
(a) Place machine 2 left of machine 3

2 — 3 — 4 — 1

(b) Place machine 2 right of machine 1

3 — 4 — 1 — 2

Compute:

2 — 3 — 4 — 1

qs*i* = q23 = f23 • t23 + f24 • t24 + f21 • t21 = f23 • r23 + f24 • r24 + f21 • r21
= f23 • d23 + f24 • (d23 + d34) + f21 • (d23 + d34 + d41)
= 0 • 6 + 5 • (6 + 5) + 10 • (6 + 5 + 3) = 195.

3 — 4 — 1 — 2

qj*s* = q12 = f21 • t21 + f24 • t24 + f23 • t23 = f21 • r21 + f24 • r24 +f23 • r23
= f21 • d21 + f24 • (d21 + d14) + f23 • (d21 + d14 + d43)
= 10 • 4 + 5• (4 + 3) + 0 • (4 + 3 + 5) = 75.

Since q23 > q12,

(1) Alternative (b) [min]is selected, i.e.,

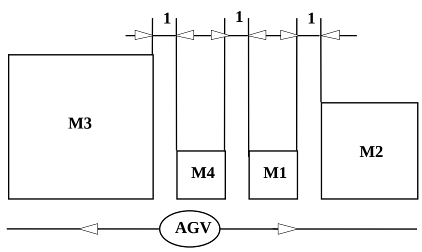
Best solution

3 — 4 — 1 — 2

(2) Row 1 and column 1 are deleted from [fij];
(3) Set j* = 2;
(4) The solution set U is updated to U + s* = {1, 2, 3, 4}.

Step 2. Since all machines have been included in the solution set U, stop.

Solution



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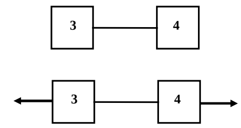
Algorithm 1 Summary

Pair (3, 4) selected

Look around



Machine 1 selected

Machine 1 placed

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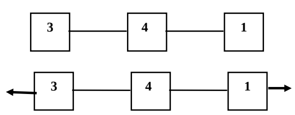
Algorithm 1 Summary

Alternative selected

Look around



Machine 2 selected

Machine 2 placed

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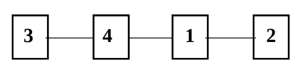
Algorithm 1 Summary

$$\text{Min} \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} f_{ik}\, t_{jl}\, x_{ij}\, x_{kl}$$

Lowest cost alternative selected



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Objective Function Revisited

$$\text{Min} \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} f_{ik}\, t_{jl}\, x_{ij}\, x_{kl}$$

f ⟶ No trips/sec

t ⟶ sec

$f_x t$ = No of trips

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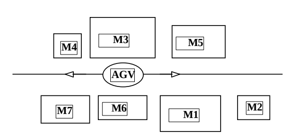
Double-Row Machine Layout

Example Machine Layout



Arrangement number	Machine pair
1	(M3, M6)
2	(M5, M1)
3	(M4, M7)
4	(M2)

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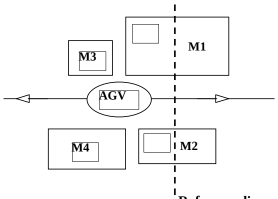
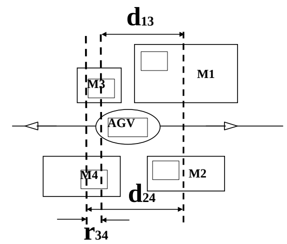
Computation of Unidirectional Distances Between Pairs of Machines



- - - - - : Reference line

Unidirectional distances

$r_{12} = 0$ $\quad r_{14} = d_{24}$
$r_{13} = d_{13}$ $\quad r_{23} = d_{13}$
$r_{24} = d_{24}$ $\quad r_{34} = | r_{13} - r_{24} | = | d_{13} - d_{24} |$



$r_{34} = | r_{24} - r_{13} | = | d_{13} - d_{24} |$

Algorithm 2: Double-row machine layout

- A pair of machines with the largest value of the flow is selected
- The machines selected are placed on the opposite sites of the isle (or an AGV track)
- Other pairs of machines are placed to the left and right of the pairs selected.

Algorithm 2

Step 0. Set iteration number k = 1.
From the flow matrix [fij], compute
fi*j* = Max {fij: i, j = 1, 2, ..., m }.
If there is a tie, fi*j* = Max {fij • tij: i, j = 1, 2, ..., m}.
Place i*, j* on the opposite sites of the AGV path and include them in the solution.
Set the solution set, U = {i*, j*}.
Remove columns i* and j* of matrix [fij] from further consideration.

Step 1. Compute
fp*q* = Max {fi*k, fj*l: k, l ∈ {1, 2, ..., m} - U}.
Set s* = q* and remove q* from further consideration.
Compute
fx*y* = Max { fi*k, fj*l, fs*v: k, l, v ∈ {{1, 2, ..., m} - U - q* }.
Set t* = y* and remove y* from further consideration.
Consider two alternatives:
(a) Place s* right of i* and t* right of j*.
(b) Place t* right of i* and s* right of j*.

Compute:

qi*s*j*t* =∑{ fs*r • ts*r + ft*r • tt*r: r U } + fs*t* • ts*t*,
where s* is placed right of i* and t* is placed right of j*;
qi*t*j*s* = { fs*r • ts*r + ft*r • tt*r: r U } + fs*t* • ts*t*,
where t* is placed right of i* and s* is placed right of j*.

If qi*s*j*t* < qi*t*j*s* , select the alternative (a);
Otherwise, select the alternative (b).
Set U = U + {s*, t*}.

Step 2. If only one machine has not been assigned, go to Step 4;
Otherwise go to Step 3.

Step 3. Compute
fp*q* = Max { fi*k, fj*l, fs*v, ft*w: k, l, v, w ∈ {1, 2, ..., m} - U}.
Set c* = q* and remove q* from further consideration.

Compute
fx*y* = max { fi*k, fj*l, fs*v, ft*w, fc*z: k , l, v, w, z ∈ {{1, 2, ..., m} - U - q*}.

Set d* = y* and remove y* from further consideration.
Consider four alternatives:

(a) Place c* left of i* and d* left of j*;
(b) Place d* left of i* and c* left of j*;
(c) Place c* right of s* and d* right of t*;
(d) Place d* right of s* and c* right of t*.

Compute:

qc*i*d*j* = Σ { fc*r • tc*r + fd*r • td*r: r ∈ U } + fc*d* • tc*d*,
where c* is placed left of i* and d* is placed
left of j*;

qd*i*c*j* = Σ { fc*r • tc*r + fd*r • td*r: r ∈ U } + fc*d* • tc*d*,
where d* is placed left of i* and c* is placed left of j*;
qs*c*t*d* = Σ { fc*r • tc*r + fd*r • td*r: r ∈ U } + fc*d* • tc*d*,
where c* is placed right of s* and d* is placed
right of t*;
qs*d*t*c* = Σ { fc*r • tc*r + fd*r • td*r: r ∈ U } + fc*d* • tc*d*,
where d* is placed right of s* and c* is placed
right of t*.

If qc*i*d*j* = min { qc*i*d*j*, qd*i*c*j*, qs*c*t*d*, qs*d*t*c* },
(1) Select alternative (a);
(2) Set i* = c* and j* = d*.
If qd*i*c*j* = min { qc*i*d*j*, qd*i*c*j*, qs*c*t*d*, qs*d*t*c* },
(1) Select alternative (b);
(2) Set i* = d* and j* = c*.
If qs*c*t*d* = min { qc*i*d*j*, qd*i*c*j*, qs*c*t*d*, qs*d*t*c* },
(1) Select alternative (c);
(2) Set s* = c* and t* = d*.
If qs*d*t*c* = min { qc*i*d*j*, qd*i*c*j*, qs*c*t*d*, qs*d*t*c* },
(1) Select alternative (d);
(2) Set s* = d* and t* = c*.
Set U = U + {c*, d*}.
Go to Step 5.

Step 4. Set c* to the last machine which has not been
assigned.
Consider four alternatives:
(a) Place c* left of i*;
(b) Place c* left of j*;
(c) Place c* right of s*;
(d) Place c* right of t*.
Compute:
qc*i*= Σ { fc*r • tc*r: r ∈ U },
where c* is placed left of i*;
qc*j* = Σ { fc*r • tc*r: r ∈ U },
where c* is placed left of j*;
qs*c* = Σ { fc*r • tc*r: r ∈ U },
where c* is placed right ofs*;
qt*c* = Σ { fc*r • tc*r: r ∈ U },
where c* is placed right of t*.

If qc*i* = min { qc*i*, qc*j*, qs*c*, qt*c* },
select alternative (a);
If qc*j* = min { qc*i*, qc*j*, qs*c*, qt*c* },
select alternative (b);
If qs*c* = min { qc*i*, qc*j*, qs*c*, qt*c* },
select alternative (c);
If qt*c* = min { qc*i*, qc*j*, qs*c*, qt*c* },
select alternative (d).
Set U = U + { c* }.
Go to Step 5.

Step 5. Set iteration number k = k + 1.
Repeat Steps 2 through 4 until the final solution
is obtained
(i.e., all the machines are included in the solution set U).

Example

Frequency of trips

$[f_{ij}]$ = Machine

Machine	1	2	3	4	5
1	0	5	1	4	1
2	5	0	3	0	2
3	1	3	0	0	0
4	4	0	0	0	5
5	1	2	0	5	0

Clearances

$[c_{ij}]$ = Machine

Machine	1	2	3	4	5
1	0	1	1	1	2
2	1	0	1	1	1
3	1	1	0	1	1
4	1	1	1	0	3
5	2	1	1	3	0

Machine Dimensions

Machine Number	Machine Dimension
1	50x30
2	20x20
3	25x20
4	60x35
5	30x15

Distances

Distance d_{ij} is calculated from machine dimensions and clearance

$[d_{ij}]$ = Machine

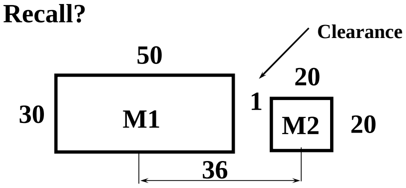
Machine	1	2	3	4	5
1	0	36	38.5	56	42
2	36	0	23.5	41	26
3	38.5	23.5	0	43.5	28.5
4	56	41	43.5	0	48
5	42	1	28.5	48	0

Machine Dimensions

Machine Number	Machine Dimension
1	50x30
2	20x20
3	25x20
4	60x35
5	30x15

Based on d_{ij}, r_{ij} is computed in the algorithm
Assume $t_{ij} = r_{ij}$

Recall?



Adjacent distance d_{12} = 25 + 10 + 1 = 36
Unidirectional distance $r_{12} = d_{12}$ = 36

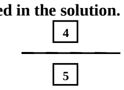
Example

$[f_{ij}]$ = Machine

Machine	1	2	3	4	5
1	0	5	1	4	1
2	5	0	3	0	2
3	1	3	0	0	0
4	4	0	0	0	5
5	1	2	0	5	0

Iteration 1

Step 0. For the flow matrix above, compute
max {fij: i, j = 1, 2, ..., 5} = f12 = f45 = 5 is determined.
The flow value f45 is selected because f45 • t45 =
f45 • d45 = 5 • 48 = 240 > f12 • t12 = f12 * d12 = 5 • 36 = 180.
Thus i* = 4 and j* = 5.
Machines 4 and 5 are assigned to the opposite sites of the AGV path and are included in the solution.

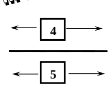


The solution set is updated, U = {4, 5}. Exclude columns 4 and 5 from further consideration.

Max Flow

$[f_{ij}]$ = Machine

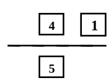
Machine	1	2	3	4 *	5 *
1	0	5	1	4	1
2	5	0	3	0	2
3	1	3	0	0	0
4	4	0	0	0	5
5	1	2	0	5	0



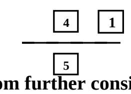
Step 1. Compute
max { f4k, f5l: k, l = 1, 2, 3} = f41.
Set s* = 1 and exclude column 1 from further consideration.

$[f_{ij}]$ = Machine

Machine	1 *	2	3	4 *	5 *
1	0	5	1	4	1
2	5	0	3	0	2
3	1	3	0	0	0
4	4	0	0	0	5
5	1	2	0	5	0



Compute:



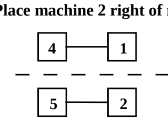
max {f5*l*, f1v: *l*, v = 2, 3} = f12.
Set t* = 2 and exclude column 2 from further consideration.

$[f_{ij}]$ = Machine

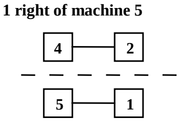
Machine	1 *	2 *	3	4 *	5 *
1	0	5	1	4	1
2	5	0	3	0	2
3	1	3	0	0	0
4	4	0	0	0	5
5	1	2	0	5	0

Consider two alternatives:
(a) Place machine 1 right of machine 4 and machine 2 right of machine 5
(b) Place machine 2 right of machine 4 and machine 1 right of machine 5

(a)

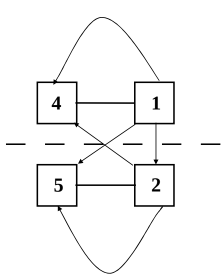


(b)



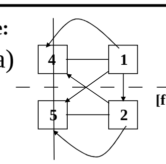
Distance Pattern

(a)



Compute:

(a)



$[f_{ij}]$ = Machine

Machine	1	2	3	4	5
1	0	5	1	4	1
2	5	0	3	0	2
3	1	3	0	0	0
4	4	0	0	0	5
5	1	2	0	5	0

$[d_{ij}]$ = Machine

Machine	1	2	3	4	5
1	0	36	38.5	56	42
2	36	0	23.5	41	26
3	38.5	23.5	0	43.5	28.5
4	56	41	43.5	0	48
5	42	1	28.5	48	0

tij = rij

qs*i*t*j* = q4152 = f14 • t14 + f15 • t15 + f24 • t24 + f25 • t25 + f12 • t12 = f14 • r14 + f15 • r15 + f24 • r25 + f25 • r25 + f12 • r12 = f14 • d14 + f15 • d14 + f24 • d25 + f25 • d25 + f12 • | d14 - d25 | = 4 • 56 + 1 • 56 + 0 • 26 + 2 • 26 + 5 • 30 = 482.

(b)

4 — 2

— — — — — —

5 — 1

qs*j*t*i* = q4251 = f24 • t24 + f25 • t25 + f14 • t14 + f15 • t15 + f12 • t12
= f24 • r24 + f25 • r25 + f14 • r14 + f15 • r15 + f12 • r12 = f24 • d24 + f25 • d24
+ f14 • d15 + f15 • d15 + f12 • | d24 - d15 | = 0 • 41 + 2 • 41 + 4 • 42 + 1 • 42
+ 5 • |41 - 42 | = 297.

Since q4251 < q4152 , alternative (b) is selected.
Set U = U + {1, 2}.

Step 2. Since only machine 3 has not been assigned, go to Step 4.

Step 4. Set c* = 3.

$[f_{ij}]$ = Machine

Machine	1*	2*	3	4*	5*
1	0	5	1	4	1
2	5	0	3	0	2
3	1	3	0	0	0
4	4	0	0	0	5
5	1	2	0	5	0

Consider four alternatives:

(a) Place machine 3 left of machine 4

3 — 4 — 2

5 — 1

(b) Place machine 3 left of machine 5

4 — 2

3 — 5 — 1

(c) Place machine 3 right of machine 2

4 — 2 — 3

5 — 1

(d) Place machine 3 right of machine 1

4 — 2

5 — 1 — 3

Compute

(a)

3 — 4 — 2

5 — 1

qc*i* = q34 = f34 • t34 + f35 • t35 + f32 • t32 + f31 • t31 = f34 • r34 + f35 • r35
+ f32 • r32 + f31 • r31 = f34 • d34 + f35 • d34 + f32 • (d34 + d42) + f31•
(d34 + d51)
= 0 • 43.5 + 0 • 43.5 + 3 • (43.5 + 41) + 1 • (43.5 + 42) = 109.5 • 1 + 67 • 3
= 339.

(b)

4 — 2

3 — 5 — 1

qc*j* = q35 = f35 • t35 + f34 • t34 + f32 • t32 + f31 • t31 = f35 • r35 + f34 • r34
+ f32 • r32 + f31 • r31 = f35 • d35 + f34 • d35 + f32 • (d35 + d42) + f31•
(d35 + d51) = 0 • 28.5 + 0 • 28.5 + 3 • (28.5 + 41) + 1 • (28.5 + 42) = 279.

Compute (continued)

(c)

4 — 2 — 3

5 — 1

qs*c* = q23 = f32 • t32 + f31 • t31 + f34 • t34 + f35 • t35 = f32 • r32 + f31 • r31
+ f34 • r34 + f35 • r35 = f32 • d32 + f31 • d32 + f34 • (d32 + d24) + f35•
(d32 + d15) = 3 • 23.5 + 1 • 23.5 + 0 • (23.5 + 41) + 0 • (23.5 +42) = 94.

(d)

4 — 2

5 — 1 — 3

qt*c* = q13 = f31 • t31 + f32 • t32 + f34 • t34 + f35 • t35 = f31 • r31 + f32 • r32
+ f34 • r34 + f35 • r35 = f31 • d31 + f32 • d31 + f34 • (d31 + d24) + f35•
(d31 + d15)
= 1 • 38.5 + 3 • 38.5 + 0 • (38.5 + 41) + 0 • (38.5 +42) = 154.

Since q23 = min {q34, q35, q23, q13 }, alternative (c) is selected

Best solution

4 — 2 — 3

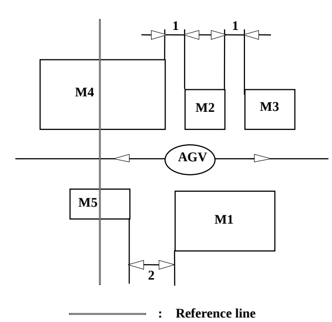
— — — — — — — —

5 — 1

Set U = U + {3}.

Step 5. **Since all machines have been included in the set U, stop.**

Solution

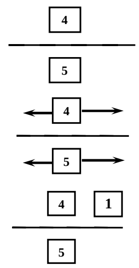


Algorithm 2 Summary

Pair (4, 5) selected

Looking around

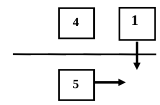
Machine 1 selected



Algorithm 2 Summary

Looking for

Machine 2 selected

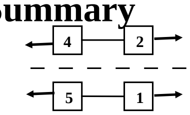


Evaluate cost

Alternative (b) selected

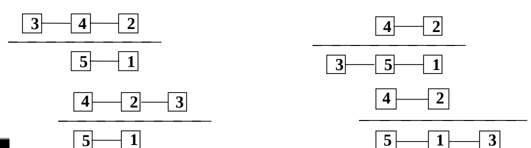
Algorithm 2 Summary

Looking around



Machine 3 selected

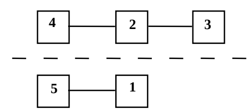
Four alternatives evaluated



Algorithm 2 Summary

Lowest cost alternative selected

Best solution



MULTIROW FACILITY AND MACHINE LAYOUT

Quadratic Assignment Model

n = number of facilities and their locations
aij = cost of locating and operating facility i at location j
fik = flow of material between facility i and facility k
cjl = cost per unit flow of material between location j and location l.

$$x_{ij} = \begin{cases} 1 & \text{if facility } i \text{ is at location } j \\ 0 & \text{otherwise} \end{cases}$$

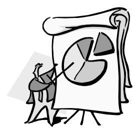
$$\text{Min} \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} f_{ik}\, c_{jl}\, x_{ij}\, x_{kl} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\, x_{ij}$$

subject to:

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1, \ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1, \ldots, n$$

$$x_{ij} = 0 \text{ or } 1 \qquad i = 1, \ldots, n,\; j = 1, \ldots, n$$



MULTIROW FACILITY(MACHINE) LAYOUT

CRAFT Algorithm

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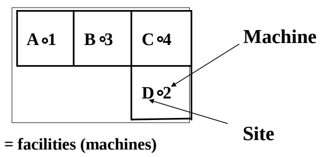
MULTIROW FACILITY(MACHINE) LAYOUT

Example

Determine the layout of four facilities (machines) 1, 2, 3, and 4 at sites A, B, C, and D

Location of Four Facilities (Machines) at Four Sites



1, 2, 3, 4 = facilities (machines)
A, B, C, D = sites (beacons)
• = site center

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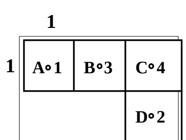
Flow Matrix
(between facilities (machines))

Assumptions:

- **Symmetric flow matrix**
- **Square facilities**

$$[f_{ij}] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} - & 50 & 20 & 100 \\ 50 & - & 30 & 10 \\ 20 & 30 & - & 70 \\ 100 & 10 & 70 & - \end{array}\right] \end{array}$$

Potential problem?



Matrix of Rectilinear Distances
(between sites)

$$[d_{ij}] = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \left[\begin{array}{cccc} - & 1 & 2 & 3 \\ 1 & - & 1 & 2 \\ 2 & 1 & - & 1 \\ 3 & 2 & 1 & - \end{array}\right] \end{array}$$

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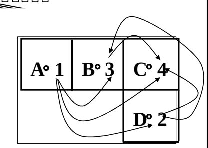
CRAFT Algorithm

Begin with an initial assignment

Site Machine

Cost 510 A •1 B • 3 C • 4
D • 2

=



$$[f_{ij}] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} - & 50 & 20 & 100 \\ 50 & - & 30 & 10 \\ 20 & 30 & - & 70 \\ 100 & 10 & 70 & - \end{array}\right] \end{array} \qquad [d_{ij}] = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \left[\begin{array}{cccc} - & 1 & 2 & 3 \\ 1 & - & 1 & 2 \\ 2 & 1 & - & 1 \\ 3 & 2 & 1 & - \end{array}\right] \end{array}$$

COST =

= f12dAD + f13dAB + f14dAC + f23dDB + f24dDC + f34dBC
= 50•3 + 20•1 + 100•2 + 30•2 + 10•1 + 70•1 = 510.

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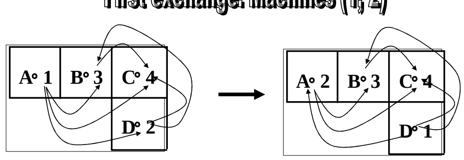
Pairwise Exchanges

First exchange: Machines (1, 2)



Cost = f12dAD + f13dBD + f14dCD + f23dAB + f24dAC+ f34dBC
=50•3+20•2+100•1+30•1+10•2+70•1= 410.

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Cost computation

Cost 510 A •1 B • 3 C • 4
D • 2

Exchange facilities (machines) 1 and 2

Cost = 410 A • 2 B • 3 C • 4
D • 1

The cost change is then 410 - 510 = -100.

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The Result

All pairs exchanged

Exchanged Pair	(1, 2)	(1, 3)	(1, 4)	(2, 3)	(2, 4)	(3, 4)
Cost Change	-100	-50	-80	-60	90	-100

A° 2	B° 3	C° 4
		D° 1

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Cost 410

A •2 B •3 C •4
D •1

Exchange Pair	(1,2)	(1,3)	(1,4)	(2,3)	(2,4)	(3,4)
Cost Change	100	-40	10	30	100	100

Solution with facilities (machines) 1 and 3 exchanged is selected

Cost 370

A •2 B •1 C • 4
D •3

Exchange Pair	(1,2)	(1,3)	(1,4)	(2,3)	(2,4)	(3,4)
Cost Change	80	40	90	90	40	60

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Final Solution

Best solution

Cost 370

A •2 B • 1C • 4
D • 3

A° 2	B° 1	C° 4
		D° 3

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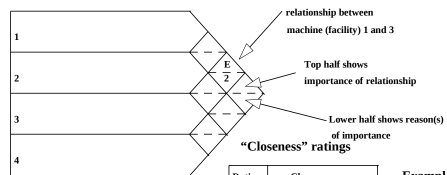
Machine Relationship Constraints



"Closeness" ratings

Rating	Closeness
A	Absolutely Necessary
E	Especially Important
I	Important
O	Ordinary Closeness
U	Unimportant
X	Not desirable

Example of reasons behind the "closeness" rating

Code	Reason
1	Personal contact
2	Convenience
3	Noise, Disturbance
4	Light

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EXAMPLE: Flow data and relationship constraints

From \ To	1	2	3	4	5
1		1500 E	100 O	_	800 I
2	_		1200 E	200 O	_
3	_	_		100 O	_
4	_	_	50 O		2500 A
5	_	_	_	_	

"Closeness" rating

Rating	Closeness
A	Absolutely Necessary
E	Especially Important
I	Important
O	Ordinary Closeness
U	Unimportant
X	Not desirable

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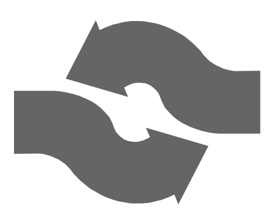
Process Layout: Medical Application



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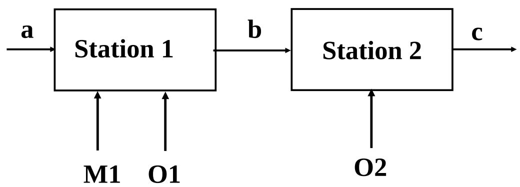
Data Collection

Process 1

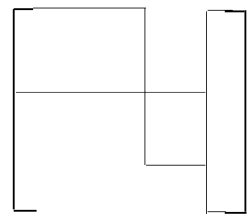


Resources

Possible Groupings

Resource

Station

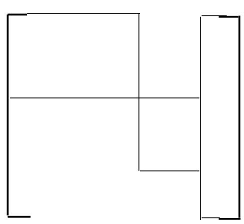


Note: Precedences are ignored in this matrix

Possible Groupings

Station

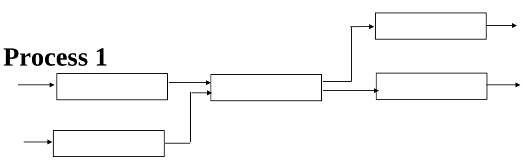
Process



System Layout

Based on the process-station grouping

Process 1



Process 2