Ch9. Flow over Immersed Bodies (1)

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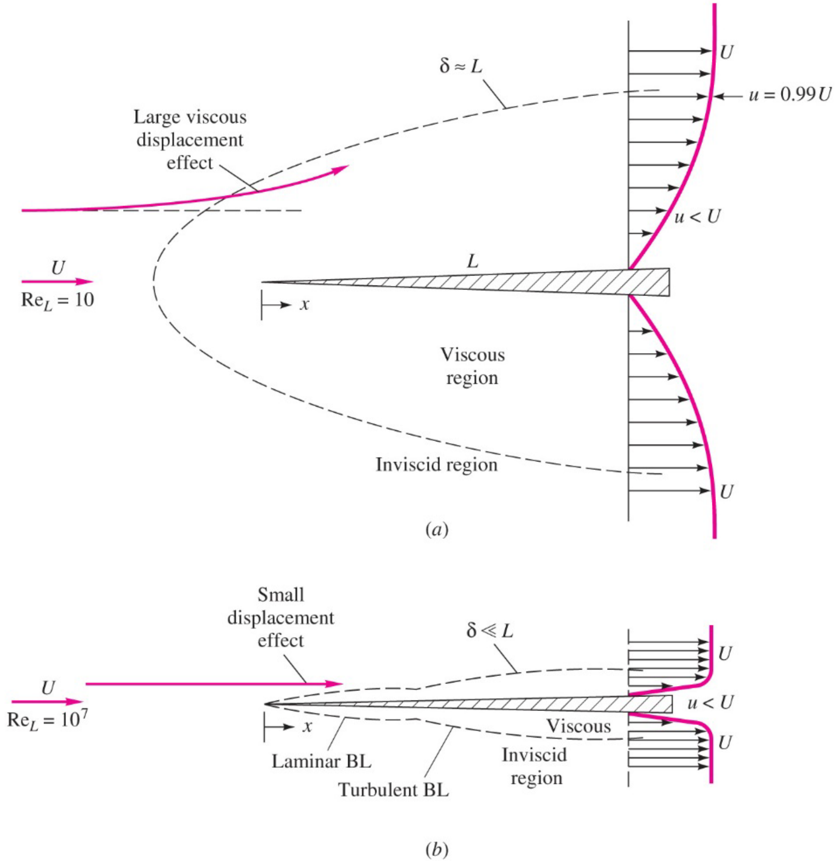
Internal vs. External Flow

Fluid flows are broadly categorized:

- **Internal flows** (Chapter 8)
  - <u>Bounded</u> by walls or fluid interfaces, such as ducts/pipes, turbomachinery, and Open channel/river flows.
  - Viscous boundary layers grow from the sidewalls, meet downstream, and fill the entire duct.
- **External flows** (Chapter 9)
  - <u>Unbounded or partially bounded</u>, such as boundary layer flows, bluff body flows, and free shear flows.
  - Flow fields can be decomposed into <u>viscous and inviscid regions.</u>
  - Free to expand no matter how thick the viscous layers grow.

External Flows

Boundary layer (BL) flow: High Reynolds number flow around streamlined bodies without flow separation.



(a) Low Re flow

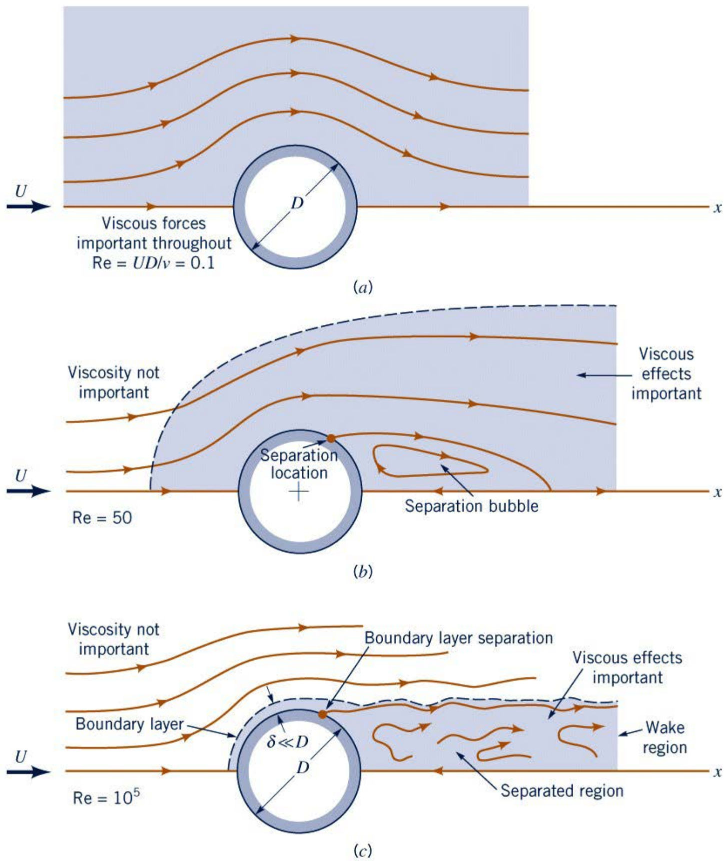
- The viscous effects are important.
- Viscous region is broad and extends far ahead and to the sides of the plate.
- The streamlines are displaced from their original uniform upstream conditions.

(b) High Re flow

- The flow is dominated by inertial effects and the viscous effects are negligible everywhere except in a thin region very close to the plate.
- The displacement effect on the outer inviscid layer is negligible.

External Flows – Contd.

Bluff body flow: Flow around bluff bodies with flow separation



(a) Creeping flow (Re <1): Viscous effects are important throughout a relatively large portion of the flow field (several diameters in any direction from the cylinder).

(b) Low Re flows: Viscous effects are convected downstream. Also, the flow cannot follow the curved path around to the rear of the body and separates from the body at some location on the body and forms a separation bubble.

(c) High Re flows: The area under viscous effects involves a thin boundary layer on the front portion and an irregular, unsteady wake region that extends far downstream of the cylinder

External Flows – Contd.

Free shear flows: Shear flows such as jets, wakes, and mixing layers, which are also characterized by absence of walls and development and spreading in an unbounded or partially bounded ambient domain. Advanced topic and also uses boundary layer theory.

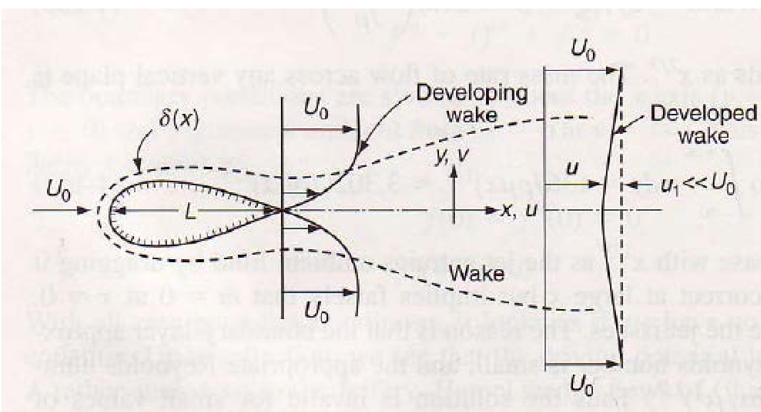


FIGURE 4-19
Flow in the wake of a body immersed in a stream.

Wake flow

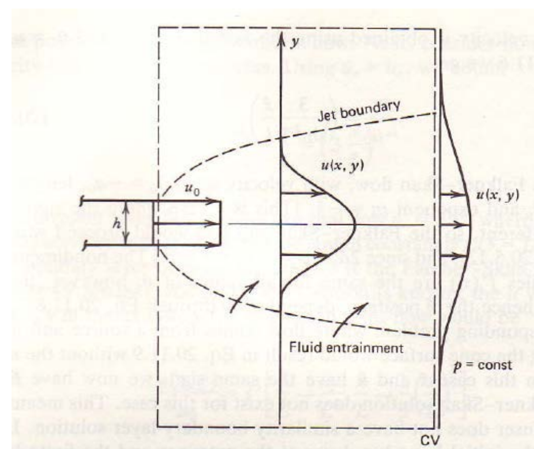
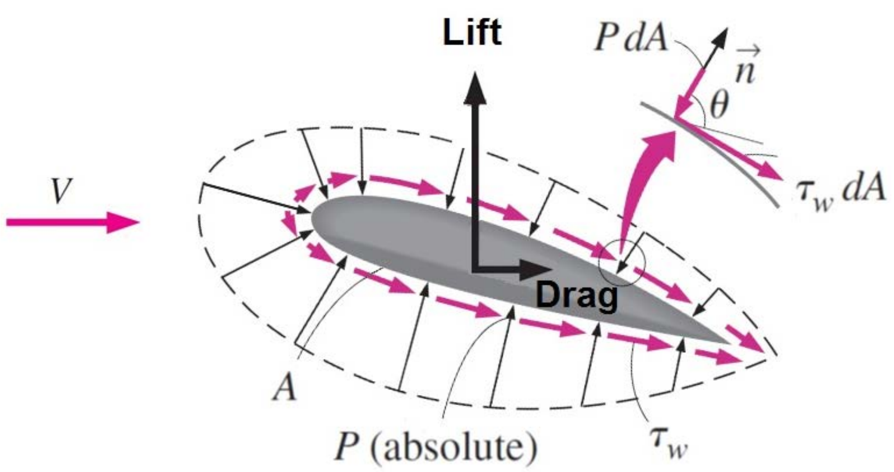


Figure 20.16 Plane laminar jet into an infinite medium.

Jet flow

Basic Considerations

- The effect of the interaction between the body and the fluid can be given in terms of the wall shear stresses due to the viscous effects and normal stresses due to the pressure.
- The resultant force in the flow direction is called **DRAG ($\boldsymbol{D}$)** and the force in the direction normal to the flow is called **LIFT ($\boldsymbol{L}$)**.



Note: t = thickness and c = chord of the wing

Drag force

$$D = \int_A \Bigg[\underbrace{(p - p_\infty)\underline{n}\cdot\hat{\boldsymbol{\imath}}}_{\substack{\text{pressure}\\ \text{(or form)drag,}\\ D_p}} + \underbrace{\tau_w \underline{t}\cdot\hat{\boldsymbol{\imath}}}_{\substack{\text{friction}\\ \text{drag,}\\ D_f}}\Bigg] dA$$

$\frac{t}{c} \ll 1 \qquad D_f \gg D_p$ streamlined body

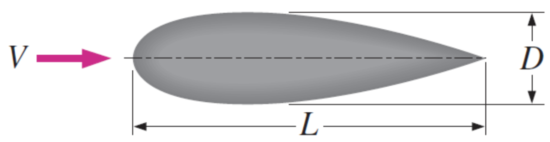
$\frac{t}{c} \sim 1 \qquad D_p \gg D_f$ bluff body

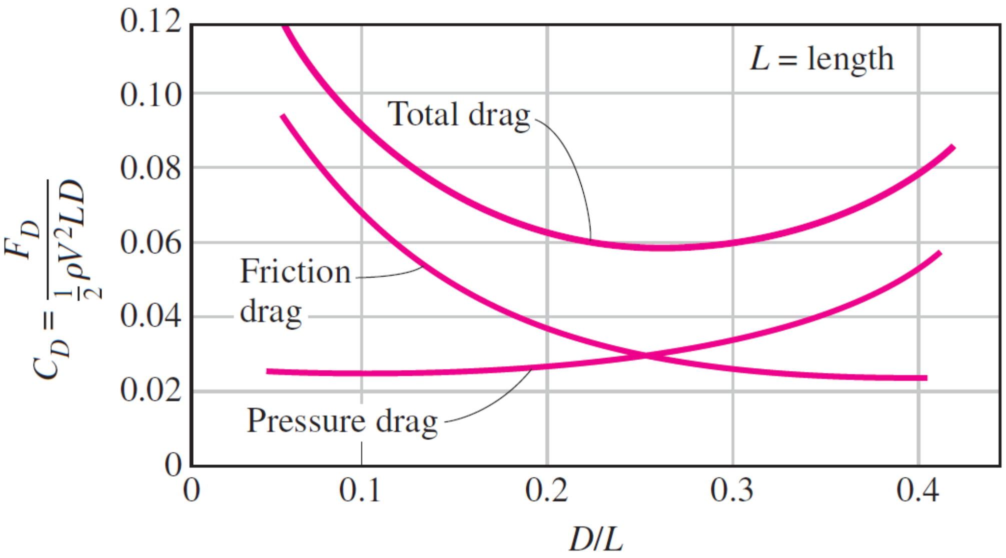
Lift force

$$L = \int_A \Bigg[(p - p_\infty)\underline{n}\cdot\hat{\boldsymbol{\jmath}} + \underbrace{\tau_w \underline{t}\cdot\hat{\boldsymbol{\jmath}}}_{\substack{\text{usually small}\\ \text{contribution}}}\Bigg] dA$$

Basic Considerations – Contd.

- <u>Streamlining</u>: One way to reduce drag
  - reduces flow separation $\rightarrow$ reduces the pressure drag
  - increases surface area $\rightarrow$ increases the friction drag





- o Trade-off relationship between pressure drag and friction drag
- o Benefit of streamlining: reduces vibration and noise

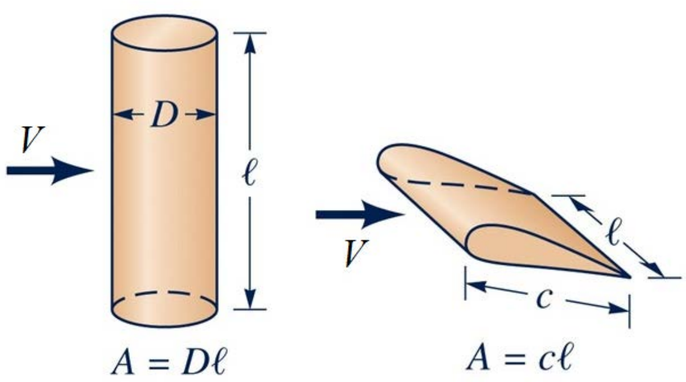
Basic Considerations – Cont.

Without detailed information concerning the shear stress and pressure distribution on a body, the widely used alternative is to define dimensionless lift and drag coefficients:

$$C_D = \frac{D}{\frac{1}{2}\rho V^2 A}$$

and

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 A}$$



Frontal area

Planform area

Note: The characteristic area A can be either the <u>frontal area</u> or the <u>planform area</u>, which must be clearly stated.

Boundary Layer Theory

The theory assumes that viscous effects are confined to a thin layer close to the surface within which:

- There is a dominant flow direction (x) such that $u \sim U$ and $v \ll u$
- However, the gradients across δ are very large, thus $\frac{\partial}{\partial y} \gg \frac{\partial}{\partial x}$

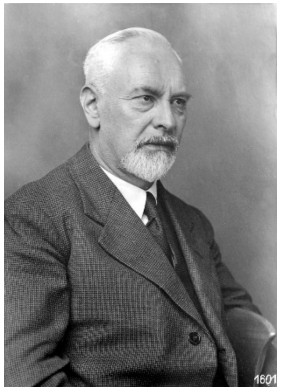
For 2D flows, the NS equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

are simplified to the following boundary layer equations,

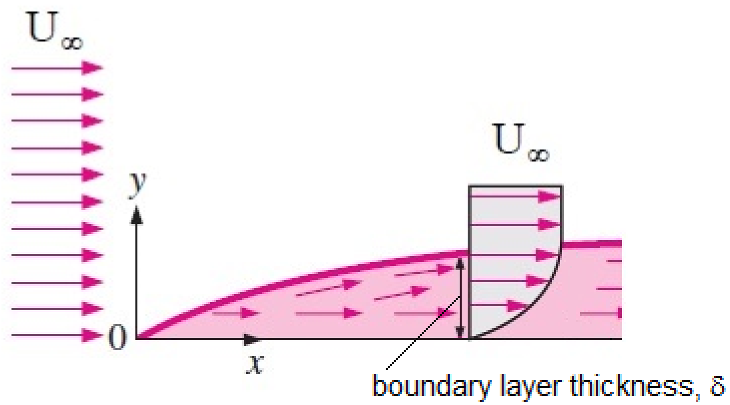
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$
$$\frac{\partial p}{\partial y} = 0$$

It is noted that the continuity equation is unaffected.



Ludwig Prandtl (1875-1953), a German engineer who first defined the boundary layer for aerodynamics (Wikipedia).

Laminar BL over a Flat Plate



Continuity eq.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum eq. (BL equation)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Note: $\frac{\partial p}{\partial x} = 0$ for a flat plate

B.C.

$$u = v = 0 \quad @\, y = 0$$
$$u = U_\infty \quad @\, y \geq \delta$$

Note: The symbol U_∞ is used interchangeably with either U_0 or U to represent the oncoming stream velocity in the remaining slides.

Blasius Solution for Laminar BL

Introduce a dimensionless transverse coordinate and a stream function,

$$\eta \equiv y\sqrt{\frac{U_\infty}{\nu x}}$$

$$\Psi \equiv \sqrt{\nu x U_\infty} f(\eta)$$

Thus,

$$u = \frac{\partial \Psi}{\partial y} = \frac{\partial \Psi}{\partial \eta}\frac{\partial \eta}{\partial y} = U_\infty f'(\eta)$$

$$v = -\frac{\partial \Psi}{\partial x} = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}(\eta f' - f)$$

Note: Boundary conditions

At y = 0 (or η = 0), u = v = 0, thus,

$$U_\infty f'(0) = 0$$

$$\therefore f'(0) = 0$$

and

$$\frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\big(\eta f'(0) - f(0)\big) = 0$$

$$\therefore f(0) = 0$$

Also, as $y \rightarrow \infty$ (or, $\eta \rightarrow \infty$), u = U_∞, thus,

$$U_\infty f'(\infty) = U_\infty$$

$$\therefore f'(\infty) = 1$$

Blasius Solution for Laminar BL – Contd.

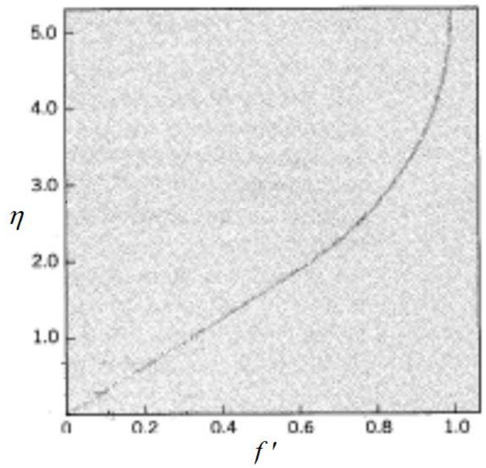
Substitution into the BL equations yields

$$f f'' + 2f''' = 0$$

with B.C.'s

$$f = f' = 0 \quad @\ \eta = 0$$
$$f' = 1 \qquad @\ \eta \to \infty$$

Numerical solution of the Blasius equation:



Note:

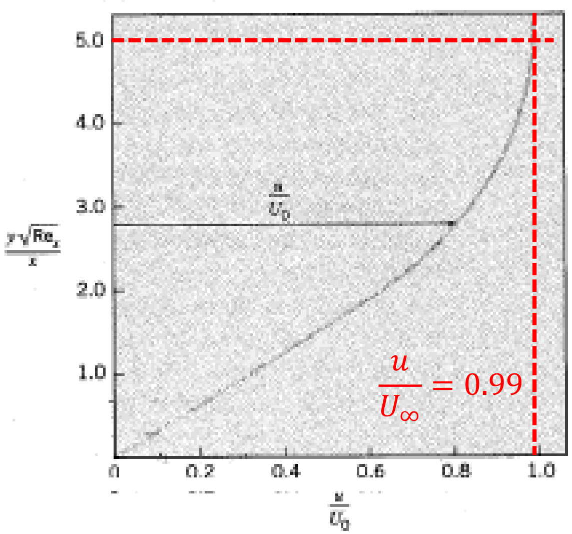
$$\mathrm{Re}_x \equiv \frac{U_\infty x}{\nu}$$

$$\therefore \eta = y\sqrt{\frac{U_\infty}{\nu x}} = y\sqrt{\frac{U_\infty x}{\nu}\cdot\frac{1}{x^2}} = \frac{y\sqrt{\mathrm{Re}_x}}{x}$$

$$u = U_\infty f'(\eta)$$
$$\therefore f'(\eta) = \frac{u}{U_\infty}$$

Blasius Solution for Laminar BL – Contd.

- Boundary layer thickness, δ:



By definition, $y = \delta$ is where $\frac{u}{U_\infty} = 0.99$,

$$\frac{\delta\sqrt{\text{Re}_x}}{x} = 5$$

$$\therefore\ \delta = \frac{5x}{\sqrt{\text{Re}_x}}$$

Thus, δ is proportional to $\sqrt{x}$,

$$\delta(x) \sim \sqrt{x}$$

and the proportionality is

$$C = 5\sqrt{\frac{\nu}{U_\infty}}$$

Blasius Solution for Laminar BL – Contd.

- Wall shear stress

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg)_{y=0} = \mu \cdot U_\infty f''(0) \cdot \frac{\sqrt{\mathrm{Re}_x}}{x}$$

$$\therefore \tau_w = 0.332\mu \frac{U_\infty \sqrt{\mathrm{Re}_x}}{x}$$

Note:

$$\frac{\partial u}{\partial y}\bigg)_{y=0} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y}\bigg)_{\eta=0}$$

- o $\frac{\partial u}{\partial \eta} = \frac{\partial}{\partial \eta}(U_\infty f') = U_\infty \eta''$
- o $\frac{\partial \eta}{\partial y} = \frac{\partial}{\partial y}\left(\frac{y\sqrt{\mathrm{Re}_x}}{x}\right) = \frac{\sqrt{\mathrm{Re}_x}}{x}$

$f''(0) = 0.332$ (Blasius solution)

- Local friction coefficient

$$c_f(x) = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = \frac{2}{\rho U_\infty^2} \cdot 0.332\mu \frac{U_\infty \sqrt{\mathrm{Re}_x}}{x}$$

$$\therefore c_f(x) = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

Blasius Solution for Laminar BL – Contd.

- Friction drag coefficient

$$C_f = \frac{D_f}{\frac{1}{2}\rho U_\infty^2 A}$$

Where,

$$D_f = \int_A \tau_w dA$$

Thus,

$$\frac{D_f}{\frac{1}{2}\rho U_\infty^2 bL} = \frac{1}{bL}\int_0^L \frac{2\tau_w}{\rho U_\infty^2} b dx = \frac{1}{L}\int_0^L c_f(x) dx = 2c_f(L)$$

$$\therefore C_f = \frac{1.328}{\sqrt{\mathrm{Re}_L}}$$

Note:
$A = bL$
$dA = bdx$
b = plate width
L = plate length
$\mathrm{Re}_L = \frac{U_\infty L}{\nu}$