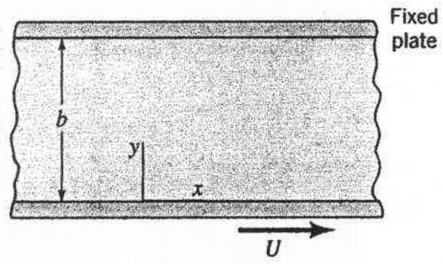
Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

6.96

6.96 The viscous, incompressible flow between the parallel plates shown in Fig. P6.96 is caused by both the motion of the bottom plate and a pressure gradient, $\partial p/\partial x$. As noted in Section 6.9.2, an important dimensionless parameter for this type of problem is $P = -(b^2/2\,\mu U)\,(\partial p/\partial x)$ where $\mu$ is the fluid viscosity. Make a plot of the dimensionless velocity distribution (similar to that shown in Fig. 6.32*b*) for $P = 3$. For this case where does the maximum velocity occur?




■ FIGURE P6.96

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)y^2 + C_1 y + C_2 \qquad \text{(Eq. 6.133)}$$

At $y = 0$, $u = U$ so that $C_2 = U$. At $y = b$, $u = 0$ and therefore

$$0 = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)b^2 + C_1 b + U$$

or

$$C_1 = -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)b - \frac{U}{b}$$

Thus,

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - by) + U\left(1 - \frac{y}{b}\right)$$

or in dimensionless form

$$\frac{u}{U} = \frac{b^2}{2\mu U}\left(\frac{\partial p}{\partial x}\right)\left(\frac{y}{b}\right)\left(\frac{y}{b} - 1\right) - \frac{y}{b} + 1 \qquad (1)$$

Since,

$$P = -\frac{b^2}{2\mu U}\left(\frac{\partial p}{\partial x}\right)$$

Eq. (1) can be written as

$$\frac{u}{U} = -P\left(\frac{y}{b}\right)\left(\frac{y}{b} - 1\right) - \frac{y}{b} + 1 \qquad (2)$$

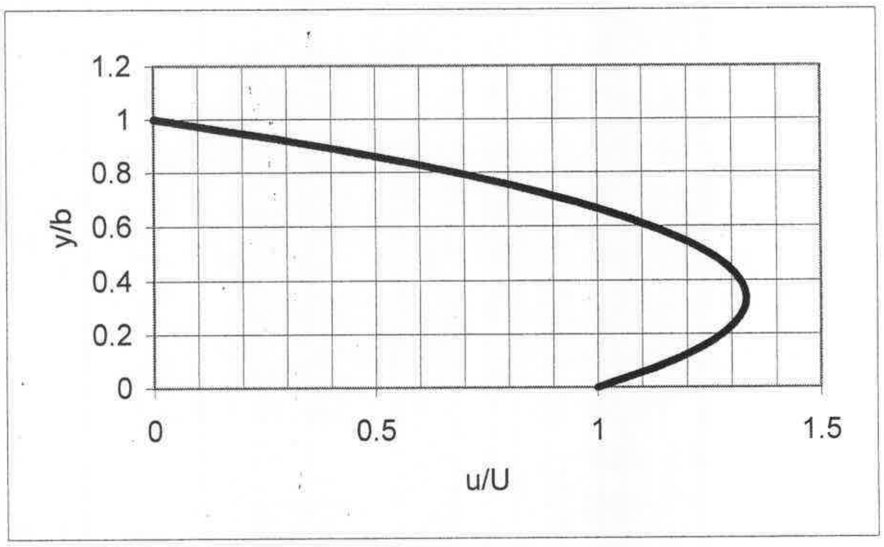
A plot of this velocity distribution for $P = 3$ is shown on the following page.

(con't)

6.96 (con't)

| u/U | y/b |
|---|---|
| 1 | 0 |
| 1.17 | 0.1 |
| 1.28 | 0.2 |
| 1.33 | 0.3 |
| 1.32 | 0.4 |
| 1.25 | 0.5 |
| 1.12 | 0.6 |
| 0.93 | 0.7 |
| 0.68 | 0.8 |
| 0.37 | 0.9 |
| 0 | 1 |

Calculated from Eq. (2) with P =3.




To determine where the maximum velocity occurs differentiate Eq. (2) and set equal to zero. Thus,

$$\frac{d(u/U)}{dy} = -P\left[2\left(\frac{y}{b^2}\right) - \frac{1}{b}\right] - \frac{1}{b} = 0$$

and with $P = 3$

$$\frac{d(u/U)}{dy} = -3\left[\frac{1}{b}\left(2\frac{y}{b} - 1\right)\right] - \frac{1}{b} = 0$$

so that

$$\underline{\underline{\frac{y}{b} = \frac{1}{3}}}$$