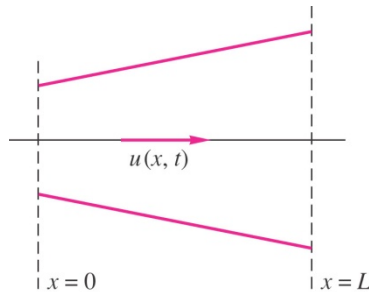NAME

Quiz 5. When a valve is opened, fluid flows in the expansion duct shown below according to the approximation

$$\underline{V} = u\hat{\imath} = U\left(1 - \frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\hat{\imath}$$

for t $\ll$ L/U. If L = 1 m and U = 1 m/s, then at $(x, t) = (L, L/2U)$,

      1) Find the unsteady (local) acceleration of a_x

      2) Find the convective acceleration of a_x

      3) Find the total acceleration a_x

Acceleration:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$u(x, t)$

$x = 0$ $x = L$

Note: Attendance (+2 points), format (+1 point)

Solution:

1) local acceleration

$$(a_x)_{local} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left[U\left(1 - \frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\right] = \frac{U^2}{L}\left(1 - \frac{x}{2L}\right)$$

(+2 points)

at (x, t) = $(L, L/2U)$,

$$(a_x)_{local} = \frac{U^2}{L}\left(1 - \frac{L}{2L}\right) = \frac{U^2}{2L} = \frac{(1 \text{ m/s})^2}{2 \times 1 \text{ m}} = 0.5\frac{\text{m}}{\text{s}^2}$$

(+1 points)

2) convective acceleration

$$(a_x)_{conv} = u\frac{\partial u}{\partial x} = U\left(1 - \frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\frac{\partial}{\partial x}\left[U\left(1 - \frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\right] = -\frac{U^2}{2L}\left(1 - \frac{x}{2L}\right)\left(\frac{Ut}{L}\right)^2$$

(+2 points)

at (x, t) = $(L, L/2U)$,

$$(a_x)_{conv} = -\frac{U^2}{2L}\left(1 - \frac{L}{2L}\right)\left(\frac{U}{L} \cdot \frac{L}{2U}\right)^2 = -\frac{U^2}{16L} = -\frac{(1 \text{ m/s})^2}{16 \times 1 \text{ m}} = -0.0625\frac{\text{m}}{\text{s}^2}$$

(+1 points)

3) total acceleration

$$a_x = (a_x)_{local} + (a_x)_{conv} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{U^2}{2L}\left(1 - \frac{x}{2L}\right)\left(1 - \frac{1}{2}\left(\frac{Ut}{L}\right)^2\right)$$ (+0.5 points)

at $(x, t) = (L, L/2U)$,

$$a_x = \frac{U^2}{L}\left(1 - \frac{L}{2L}\right)\left(1 - \frac{1}{2}\left(\frac{U}{L}\cdot\frac{L}{2U}\right)^2\right) = \frac{7U^2}{16L} = \frac{7 \times (1\ \text{m/s})^2}{16 \times 1\ \text{m}} = 0.4375\frac{\text{m}}{\text{s}^2}$$ (+0.5 points)