Review for Exam3

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Chapter 8

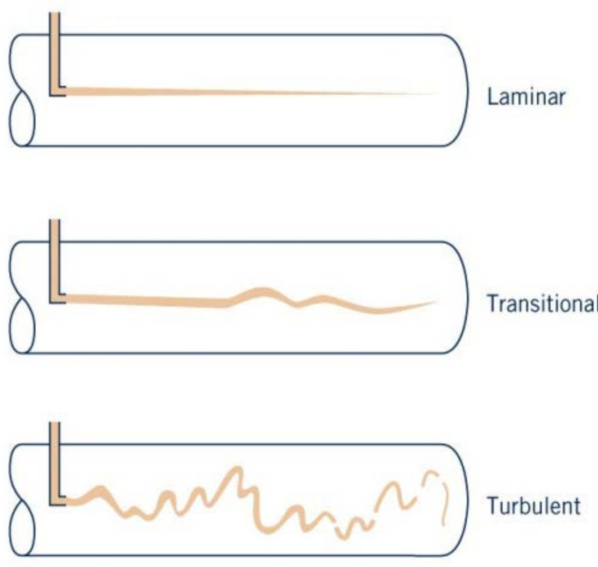
Flow in Conduits

- Internal flow: Confined by solid walls
- Basic piping problems:
  - Given the desired flow rate, what pressure drop (e.g., pump power) is needed to drive the flow?
  - Given the pressure drop (e.g., pump power) available, what flow rate will ensue?

Pipe Flow: Laminar vs. Turbulent

- Reynolds number regimes

$$\mathrm{Re} = \frac{\rho V D}{\mu}$$



$$\mathrm{Re} < \mathrm{Re}_{\mathrm{crit}} \sim 2{,}000$$

$$\mathrm{Re}_{\mathrm{crit}} < \mathrm{Re} < \mathrm{Re}_{\mathrm{trans}}$$

$$\mathrm{Re} > \mathrm{Re}_{\mathrm{trans}} \sim 4{,}000$$

Entrance Region and Fully Developed

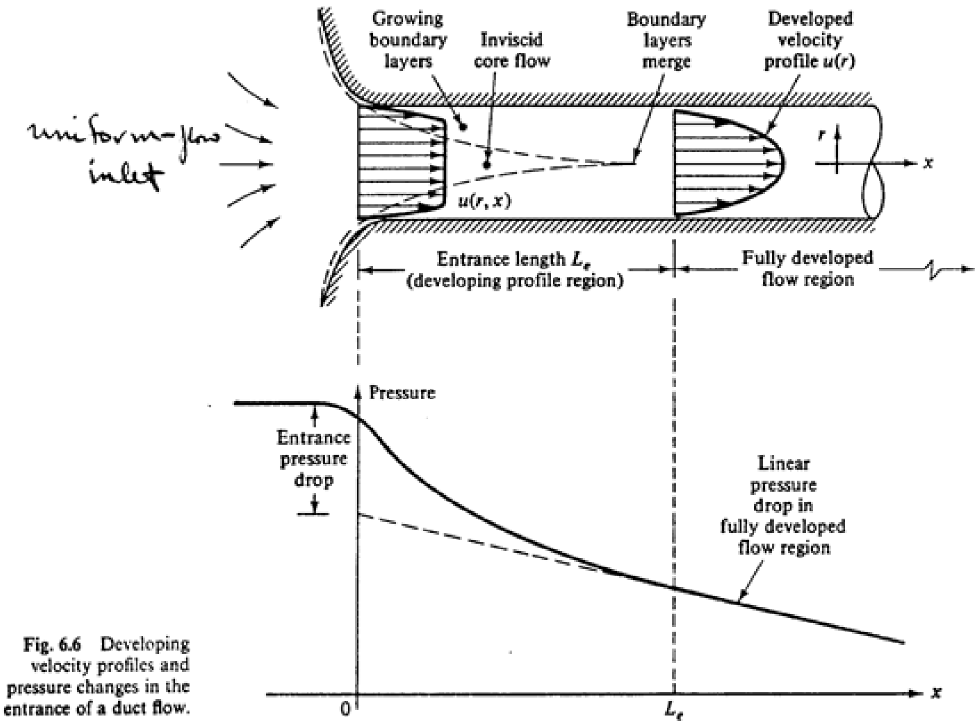


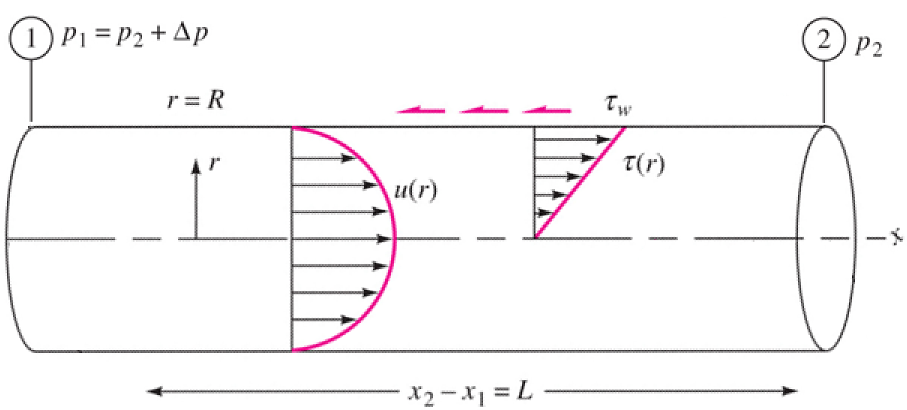
Fig. 6.6 Developing velocity profiles and pressure changes in the entrance of a duct flow.

- Entrance Length, L_e:
  - o Laminar flow: $L_\mathrm{e}/D = 0.06Re$ $(L_{\mathrm{e,max}} = 0.06\mathrm{Re}_{\mathrm{crit}} \sim 138D)$
  - o Turbulent flow: $L_\mathrm{e}/D = 4.4\mathrm{Re}^{\frac{1}{6}}$ $(20D < L_\mathrm{e} < 30D$ for $10^4 < \mathrm{Re} < 10^5)$

Pressure Drop and Shear Stress

- Pressure drop, $\Delta p = p_1 - p_2$, is needed to overcome viscous shear stress.
- Considering force balance,

$$p_1\left(\frac{\pi D^2}{4}\right) - p_2\left(\frac{\pi D^2}{4}\right) = \tau_w(\pi D L) \;\Rightarrow\; \Delta p = 4\tau_w \frac{L}{D}$$



Head Loss and Friction Factor

- Energy equation

$$h_L = \frac{p_1 - p_2}{\gamma} + \frac{\alpha_1 V_1^2 - \alpha_2 V_2^2}{2g} + (z_1 - z_2) = \frac{\Delta p}{\gamma}$$

$$\Delta p = 4\tau_w \frac{L}{D} \quad \text{from force balance and } \gamma = \rho g$$

$$\therefore h_L = 4\tau_w \frac{L}{D} / \rho g = \left(\frac{8\tau_w}{\rho V^2}\right) \cdot \frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{V^2}{2g}$$

$\Rightarrow$Darcy – Weisbach equation

- Friction factor

$$f \equiv \frac{8\tau_w}{\rho V^2}$$

Fully-developed Laminar Flow

- Exact solution, $u(r) = V_c\left[1 - \left(\frac{r}{R}\right)^2\right]$

- Wall sear stress

$$\tau_w = -\mu \frac{du}{dr}\bigg)_{r=R} = \frac{8\mu V}{D}$$

where, $V = Q/A$

- Friction factor,

$$f = \frac{8\tau_w}{\rho V^2} = \frac{8}{\rho V^2} \cdot \frac{8\mu V}{D} = \frac{64}{\rho DV/\mu} = \frac{64}{\text{Re}}$$

Fully-developed Turbulent Flow

- Dimensional analysis

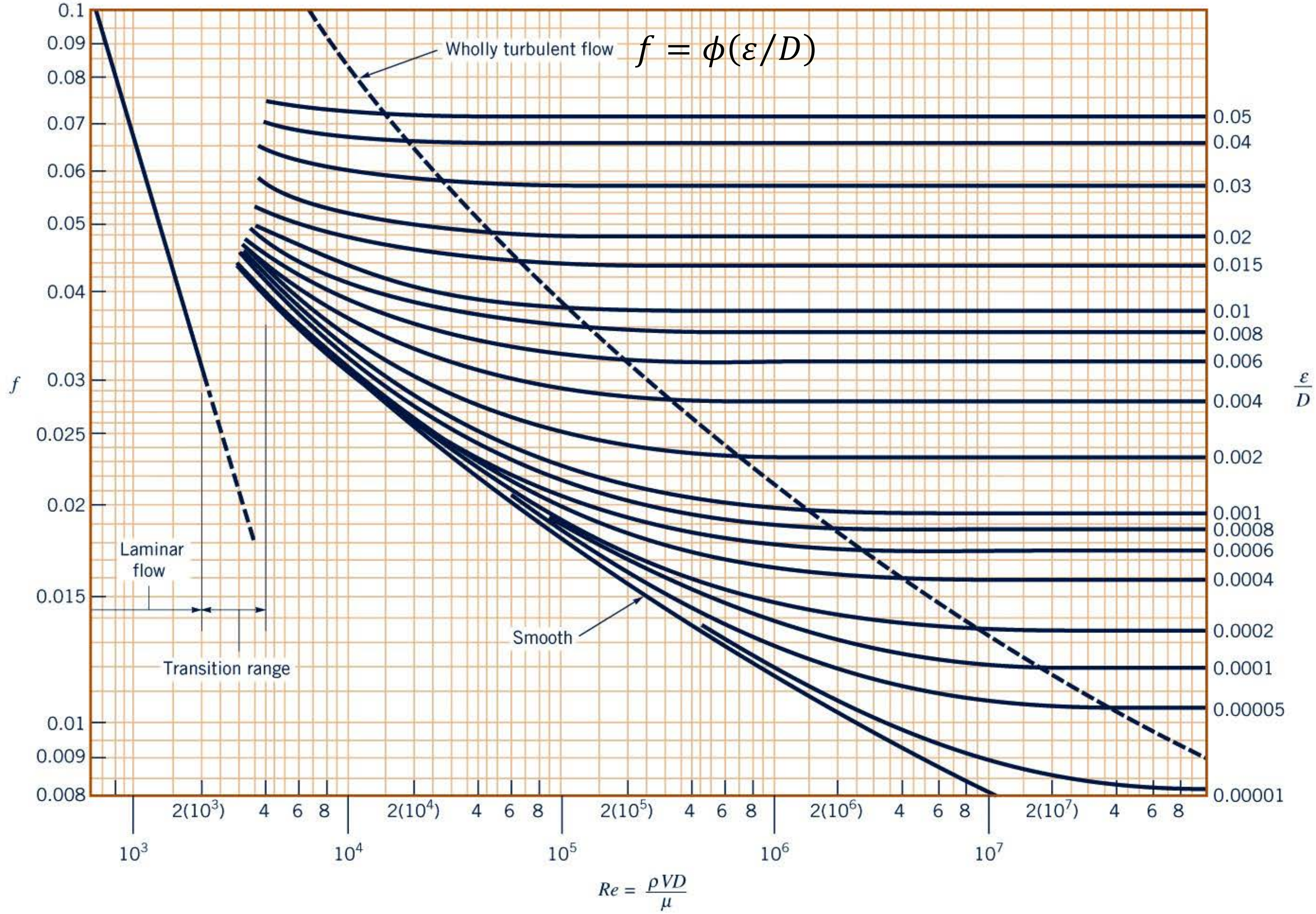
$$\tau_w = f(D, V, \mu, \rho, \varepsilon)$$

$$\rightarrow \quad k - r = 6 - 3 = 3\ \Pi's$$

$$\frac{\tau_w}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right)$$

$$\therefore f = \phi(\mathrm{Re}, \varepsilon/D)$$

Moody Chart



Wholly turbulent flow
$f = \phi(\varepsilon/D)$
0.1
0.09
0.08
0.07
0.06
0.05
0.04
f
0.03
0.025
0.02
0.015
0.01
0.009
0.008
Laminar flow
Transition range
Smooth
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
$\frac{\varepsilon}{D}$
$2(10^3)$ 4 6 8
$2(10^4)$ 4 6 8
$2(10^5)$ 4 6 8
$2(10^6)$ 4 6 8
$2(10^7)$ 4 6 8
10^3
10^4
10^5
10^6
10^7
$Re = \frac{\rho VD}{\mu}$

Moody Chart – Contd.

- Colebrook equation

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

- Haaland equation

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.1} + \frac{6.9}{\mathrm{Re}}\right]$$

Minor Loss

- Loss of energy due to pipe system components (valves, bends, tees, and the like).
- Theoretical prediction is, as yet, almost impossible.
- Usually based on experimental data.

K_L: Loss coefficient

$$h_m = \sum K_L \frac{V^2}{2g}$$

E.g.)
Pipe entrance (sharp-edged), K_L=0.8
(well-rounded), K_L=0.04
Regular 90° elbows (flanged), K_L=0.3
Pipe exit, K_L=1.0

Pipe Flow Problems

- Energy equation for pipe flow:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$h_L = h_f + h_m = \left(f\frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$

  - Type I: Determine head loss h_L(or Δp)
  - Type II: Determine flow rate Q (or V)
  - Type III: Determine pipe diameter D

Iteration is needed for types II and III

Type I Problem

- Typically, V (or Q) and D are given → Find the pump power $\dot{W}_p$ required.
- For example, if $p_1 = p_2 = 0$ and $V_1 = V_2 = 0$, and $\Delta z = z_2 - z_1$,

$$0 + 0 + z_1 + h_p = 0 + 0 + z_2 + \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2\mathrm{g}}$$

Solve the energy equation for h_p,

$$h_p = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2\mathrm{g}} - \Delta z$$

From Moody Chart,

$$f = \phi\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right)$$

Then,

$$\dot{W}_p = h_p \cdot \gamma Q$$

Type II Problem

- Q (thus V) is unknown $\rightarrow$ Re is unknown
- Solve energy equation for V as a function of f. For example, if $p_1 = p_2, V_1 = V_2$ and $\Delta z = z_2 - z_1$,

$$0 + 0 + z_1 + h_p = 0 + 0 + z_2 + \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2\mathrm{g}}$$

$$\therefore V = \sqrt{\frac{2\mathrm{g}\left(h_p - \Delta z\right)}{f\frac{L}{D} + \sum K_L}}$$

Guess $f \rightarrow V \rightarrow \mathrm{Re} \rightarrow f_{\mathrm{new}}$; Repeat this until f converges $\Rightarrow V$

Type III Problem

- D is unknown $\rightarrow$ Re and ε/D are unknown
- Solve energy equation for D as a function of f. For example, if $p_1 = p_2, V_1 = V_2, \Delta z = z_1 - z_2$, and $\sum K_L$ = 0 and using $V = Q/(\pi D^2/4)$,

$$0 + 0 + z_1 + h_p = 0 + 0 + z_2 + \left(f\frac{L}{D} + 0\right)\frac{1}{2\mathrm{g}}\left(\frac{Q}{\pi D^2/4}\right)^2$$

$$\therefore D = \left[\frac{8LQ^2 \cdot f}{\pi^2 \mathrm{g}\left(h_p - \Delta z\right)}\right]^{\frac{1}{5}}$$

Guess $f \rightarrow D \rightarrow Re$ and $\varepsilon/D \rightarrow f_{\text{new}}$; Repeat this until f converges $\Rightarrow D$

Chapter 9

Flow over Immersed Bodies

- External flow: Unconfined, free to expand
- Complex body geometries require experimental data (dimensional analysis)

Drag

- Resultant force in the direction of the upstream velocity

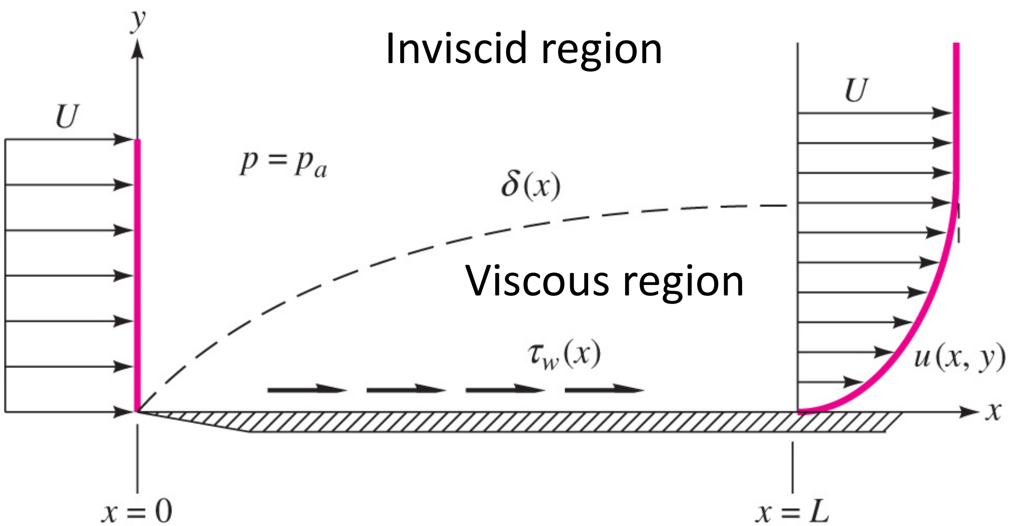
$$C_D = \frac{D}{\frac{1}{2}\rho V^2 A} = \frac{1}{\frac{1}{2}\rho V^2 A}\left\{\underbrace{\int_S (p - p_\infty)\underline{n} \cdot \hat{\boldsymbol{\imath}} dA}_{\substack{C_{Dp} = \text{ Pressure drag} \\ \text{(or Form drag)}}} + \underbrace{\int_S \tau_w \underline{t} \cdot \hat{\boldsymbol{\imath}} dA}_{C_f = \text{Friction drag}}\right\}$$

  - Streamlined body (t/ℓ << 1): C_f >> C_{Dp}
  - Bluff body (t/ℓ ~1): C_{Dp} >> C_f

where, t is the thickness and ℓ the length of the body

Boundary Layer

- High Reynolds number flow, $\mathrm{Re}_x = \frac{U_\infty x}{\nu} >> 1{,}000$
- Viscous effects are confined to a thin layer, δ
- $\frac{u}{U_\infty} = 0.99$ at $y = \delta$



Friction Coefficient

- Local friction coefficient

$$c_f(x) = \frac{2\tau_w(x)}{\rho U^2}$$

- Friction Drag

$$D_f = \int_A \tau_w(x) dA = \int_0^L \tau_w(x)(b \cdot dx)$$

- Friction drag coefficient

$$C_f = \frac{D_f}{\frac{1}{2}\rho U^2 A}$$

$$\therefore D_f = C_f \cdot \frac{1}{2}\rho U^2 A$$

Note: Darcy friction factor for pipe flow

$$f = \frac{8\tau_w}{\rho V^2}$$

Note:

$$C_f = \frac{1}{\frac{1}{2}\rho U^2 (bL)} \int_0^L \tau_w(x) b dx$$

$$= \frac{1}{L}\int_0^L \frac{2\tau_w(x)}{\rho U^2} dx$$

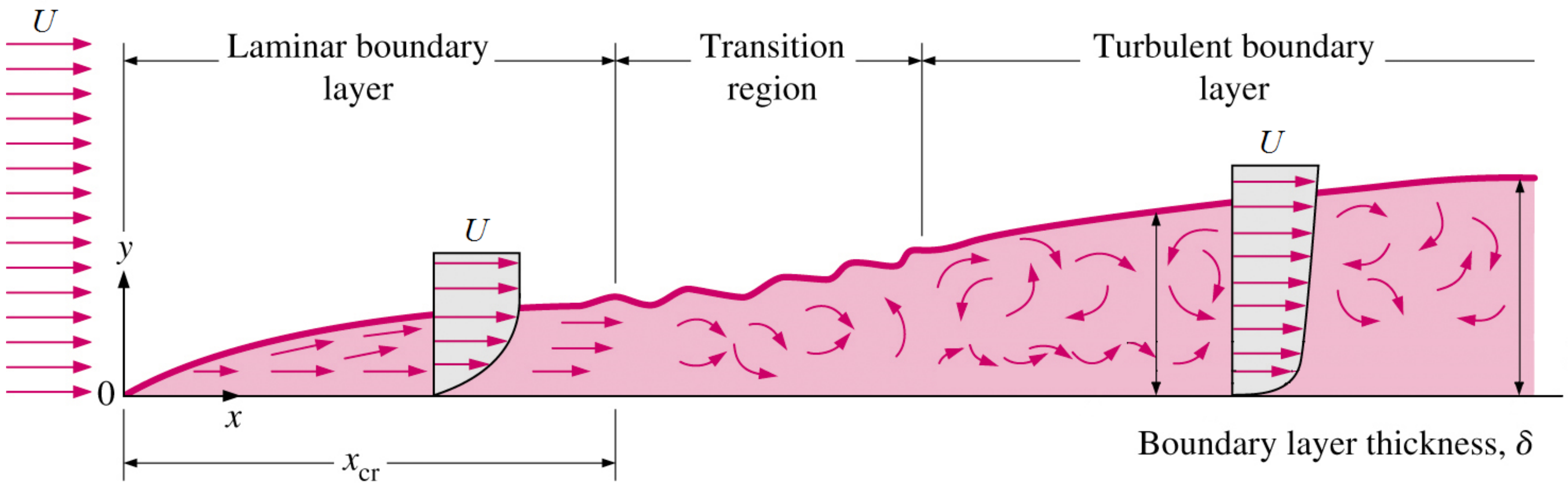
$$= \frac{1}{L}\int_0^L c_f(x) dx$$

$$\therefore C_f = \overline{c_f(x)}$$

Reynolds Number Regime

- Transition Reynolds number

$$\mathrm{Re}_{x,tr} = 5 \times 10^5$$



Boundary layer equation

- Assumptions
  - Dominant flow direction (x):
    $$u \sim U \quad \text{and} \quad v \ll u$$
  - Gradients across δ are very large in order to satisfy the no slip condition:
    $$\frac{\partial}{\partial y} \gg \frac{\partial}{\partial x}$$
- Simplified NS equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial y} = 0$$

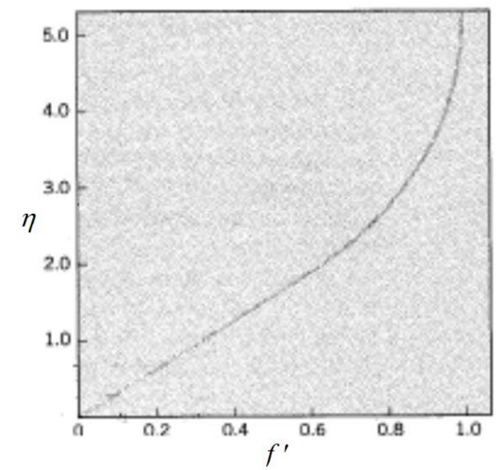
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Laminar boundary layer

- Blasius introduced coordinate transformations

$$\eta \equiv y\sqrt{\frac{U_\infty}{\nu x}}$$

$$\Psi \equiv \sqrt{\nu x U_\infty} f(\eta)$$



Then, rewrote the BL equations as a simple ODE,

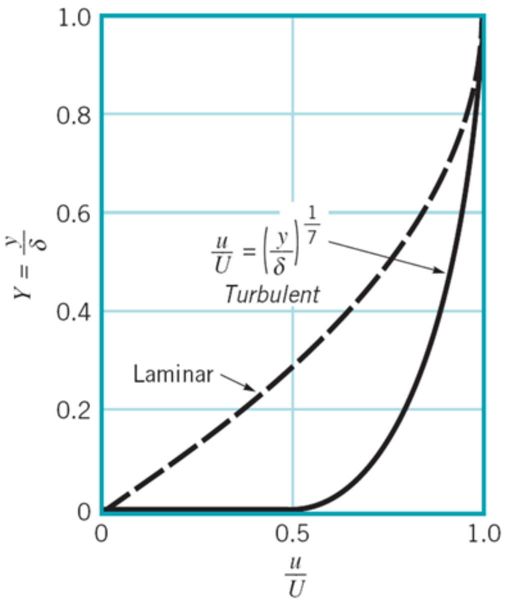
$$f f'' + 2f''' = 0$$

From the solutions,

$$\frac{\delta(x)}{x} = \frac{5}{\sqrt{Re_x}}\ ; \quad c_f(x) = \frac{0.664}{\sqrt{Re_x}}; \quad C_f = \frac{1.328}{\sqrt{Re_L}}$$

Turbulent boundary layer

- $\frac{u}{U} \approx \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$ one − seventh − power law
- $c_f \approx 0.02 Re_\delta^{-\frac{1}{6}}$ power − law fit
- $\frac{\delta(x)}{x} = \frac{0.16}{Re_x^{\frac{1}{7}}}$; $c_f(x) = \frac{0.027}{Re_x^{\frac{1}{7}}}$; $C_f = \frac{0.031}{Re_L^{\frac{1}{7}}}$



- Valid for a fully turbulent flow over a smooth flat plate from the leading edge.
- Better results for sufficiently large $Re_L > 10^7$

Turbulent boundary layer – Contd.

- Alternate forms by using an experimentally determined shear stress formula:
- $\tau_w = 0.0225\rho U^2 \left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}}$
- $\frac{\delta(x)}{x} = 0.37\mathrm{Re}_x^{-\frac{1}{5}};\quad c_f(x) = \frac{0.058}{\mathrm{Re}_x^{\frac{1}{5}}};\quad C_f = \frac{0.074}{\mathrm{Re}_L^{\frac{1}{5}}}$
- Valid only in the range of the experimental data; $\mathrm{Re}_L = 5 \times 10^5 \sim 10^7$ for smooth flat plate

Turbulent boundary layer – Contd.

- Other formulas for smooth flat plates are by using the logarithmic velocity-profile instead of the 1/7-power law:

$$\frac{\delta}{L} = c_f(0.98 \log \mathrm{Re}_L - 0.732)$$

$$c_f = (2 \log \mathrm{Re}_x - 0.65)^{-2.3}$$

$$C_f = \frac{0.455}{(\log_{10} \mathrm{Re}_L)^{2.58}}$$

These formulas are valid in the whole range of $\mathrm{Re}_L \leq 10^9$

Turbulent boundary layer – Contd.

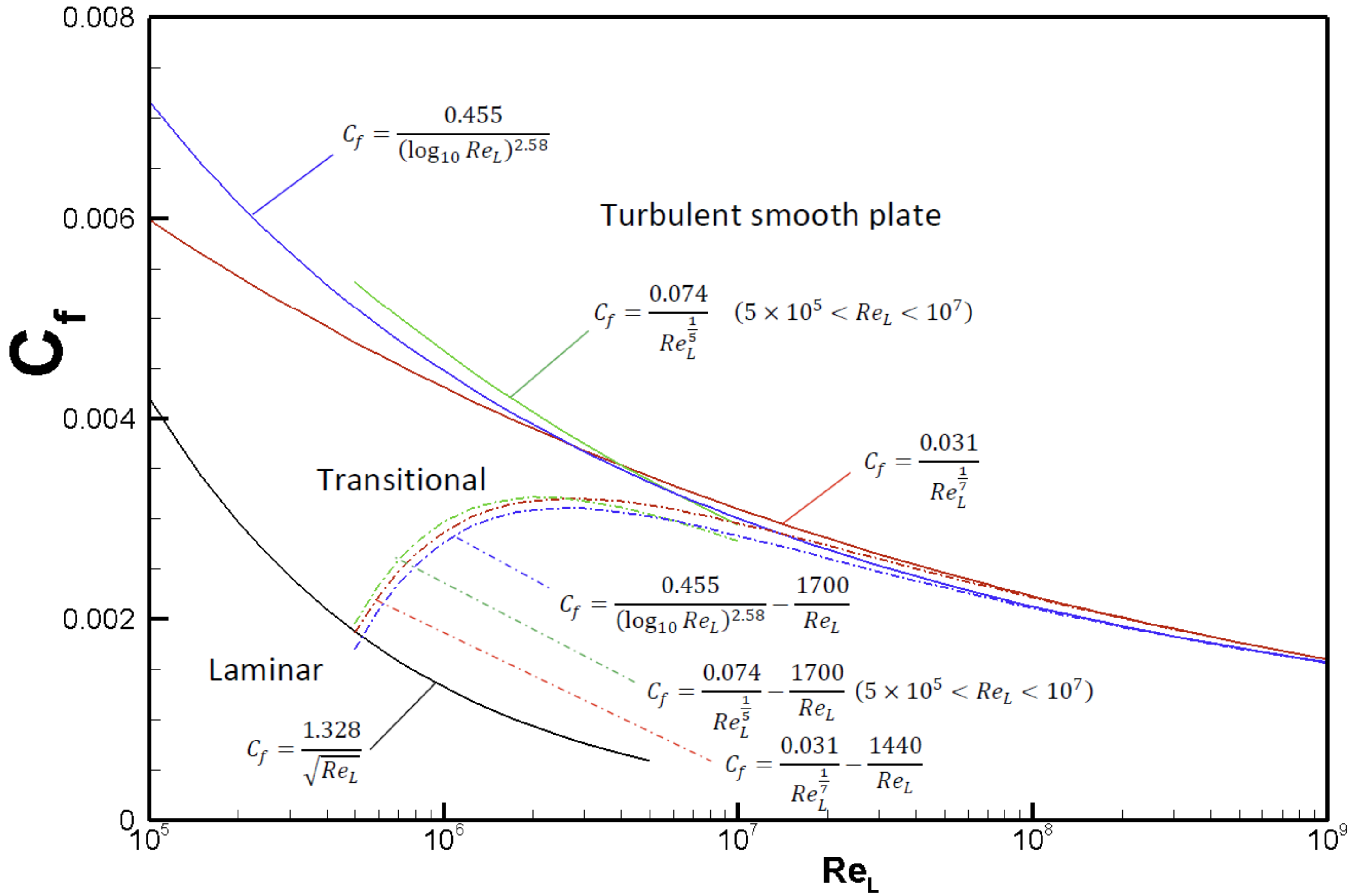
- Composite formulas (for flows initially laminar and subsequently turbulent with $\mathrm{Re}_t = 5 \times 10^5$):

$$C_f = \frac{0.031}{\mathrm{Re}_L^{\frac{1}{7}}} - \frac{1440}{\mathrm{Re}_L}$$

$$C_f = \frac{0.074}{\mathrm{Re}_L^{\frac{1}{5}}} - \frac{1700}{\mathrm{Re}_L}$$

$$C_f = \frac{0.455}{(\log_{10} \mathrm{Re}_L)^{2.58}} - \frac{1700}{\mathrm{Re}_L}$$

Turbulent boundary layer – Contd.



Bluff Body Drag

- In general,

$$D = f(V, L, \rho, \mu, c, t, \varepsilon, \dots)$$

- Drag coefficient:

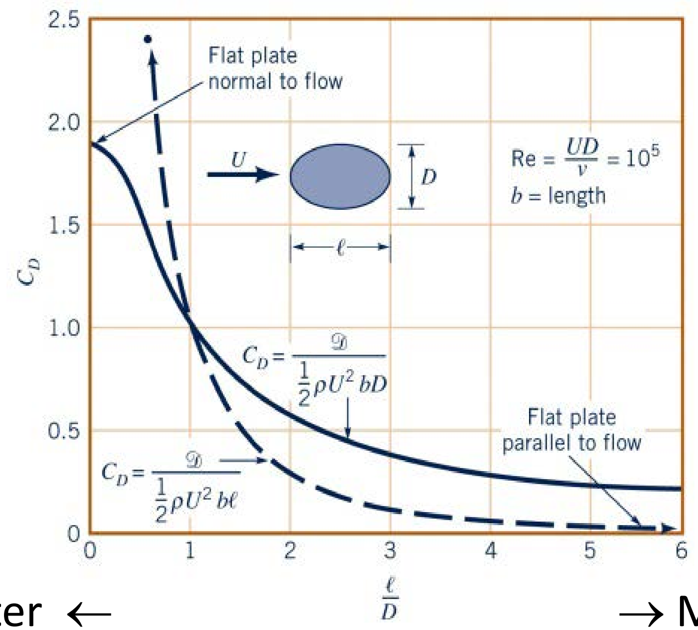
$$C_D = \frac{D}{\frac{1}{2}\rho V^2 A} = \phi\left(AR, \frac{t}{L}, \mathrm{Re}, \frac{c}{V}, \frac{\varepsilon}{L}, \dots\right)$$

- For bluff bodies experimental data are used to determine C_D

$$D = \frac{1}{2}\rho V^2 A \cdot C_D$$

Shape dependence

- The blunter the body, the larger the drag coefficient
- The amount of streamlining can have a considerable effect

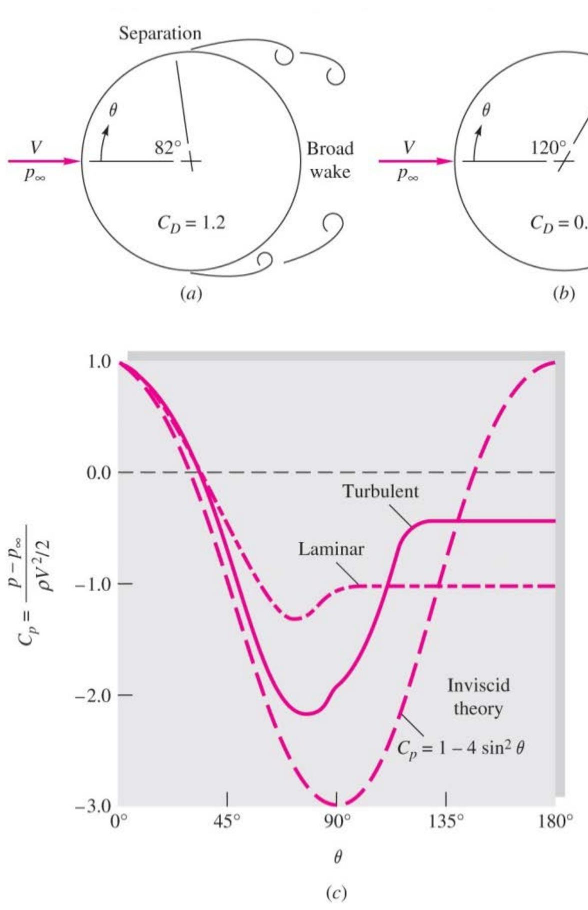


Blunter ← → More streamlined

Separation

- Fluid stream detaches from a surface of a body at sufficiently high velocities.
- Only appears in viscous flows.
- Inside a separation region: low-pressure, existence of recirculating /backflows; viscous and rotational effects are the most significant

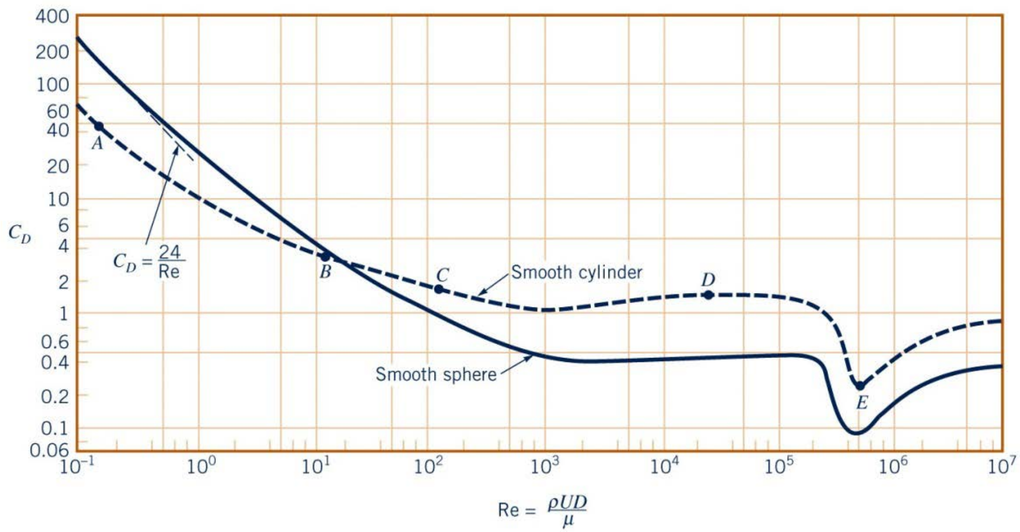
Laminar Turbulent

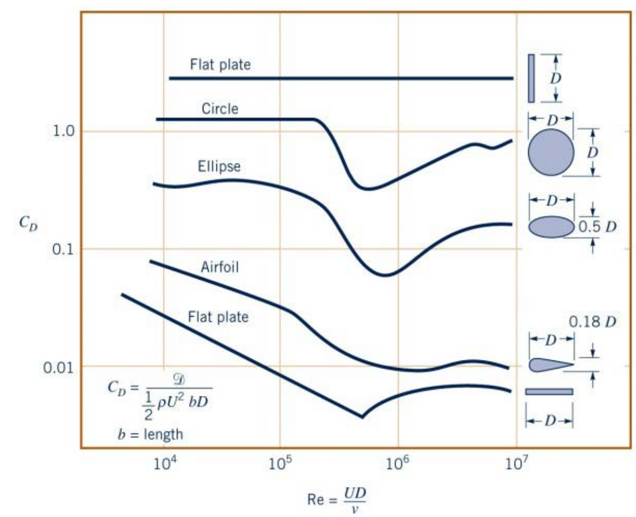


Reynolds number dependence

- Very low Re flow (Re < 1)
  - Inertia effects are negligible (creeping flow)
  - $C_D \sim \mathrm{Re}^{-1}$
  - Streamlining can actually increase the drag (an increase in the area and shear force)
- Moderate Re flow ($10^3 < \mathrm{Re} < 10^5$)
  - For streamlined bodies, $C_D \sim \mathrm{Re}^{-\frac{1}{2}}$
  - For blunt bodies, $C_D \sim \text{constant}$
- Very large Re flow (turbulent boundary layer)
  - For streamlined bodies, C_D increases
  - For relatively blunt bodies, C_D decreases when the flow becomes turbulent ($10^5 < \mathrm{Re} < 10^6$)

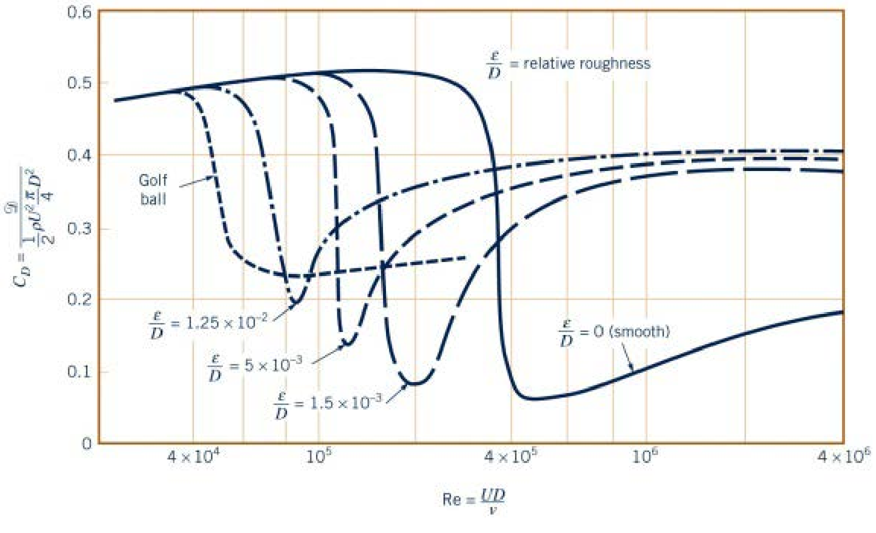
- For extremely blunt bodies, $C_D \sim \text{constant}$





Surface roughness

- For streamlined bodies, the drag increases with increasing surface roughness
- For blunt bodies, an increase in surface roughness can actually cause a decrease in the drag.
- For extremely blunt bodies, the drag is independent of the surface roughness

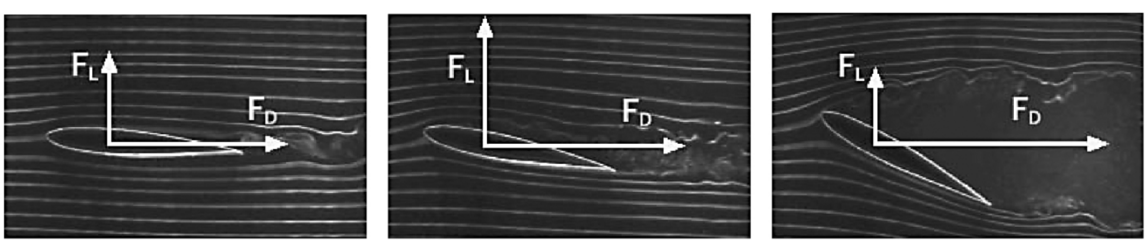


Lift

- Lift, L: Resultant force normal to the upstream velocity

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$$

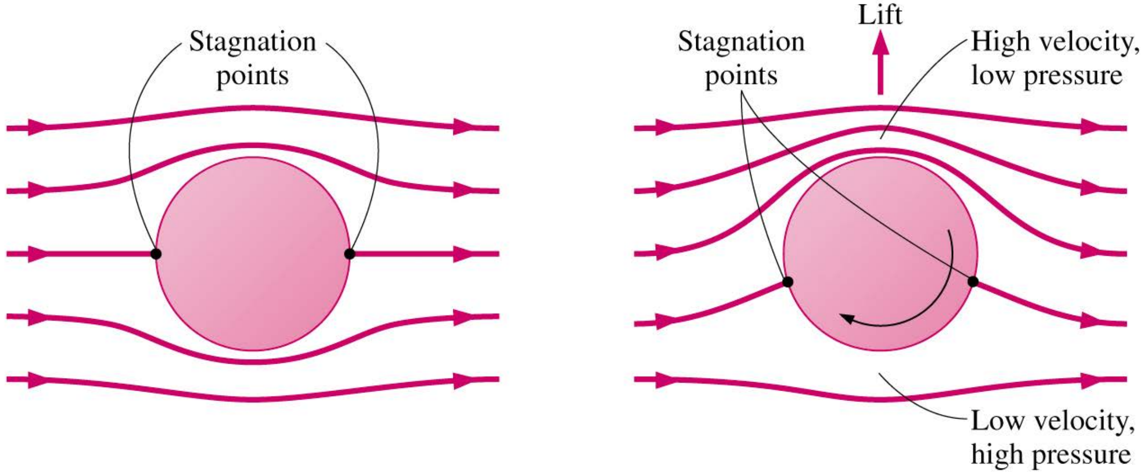
$$L = C_L \cdot \frac{1}{2}\rho U^2 A$$



(a) 5° (b) 15° (c) 30°

Magnus Effect

- Lift generation by spinning
- Breaking the symmetry causes a lift



(*a*) Potential flow over a stationary cylinder (*b*) Potential flow over a rotating cylinder

Minimum Flight Velocity

- Total weight of an aircraft should be equal to the lift

$$W = F_L = \frac{1}{2} C_{L,max} \rho V_{min}^2 A$$

Thus,

$$V_{min} = \sqrt{\frac{2W}{\rho C_{L,max} A}}$$